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EXCITED NUCLEON AS A VAN DER WAALS SYSTEM OF PARTONS

Saturation in deep inelastic scattering (DIS) and deeply virtual Compton scattering (DVCS) is associated with a phase transition between the partonic gas, typical of moderate x and Q^2, and partonic fluid appearing at increasing Q^2 and decreasing Bjorken x. In this article we review recent developments in the studies of the collective properties of the nucleon excited in DIS.

Key words: nucleon, quarks, gluons, van der Waals system, Compton scattering.

Introduction

Modern high-energy, relativistic-invariant physics is essentially non-Euclidean. The theoretical basis is local quantum field theory, in particular quantum chromo-dynamics (QCD), assuming the existence of point-like particles – quarks and gluons, comprised in a finite volume of hadrons. The internal structure of the hadrons can be scrutinized in deep-inelastic scattering (DIS) of leptons on hadrons.

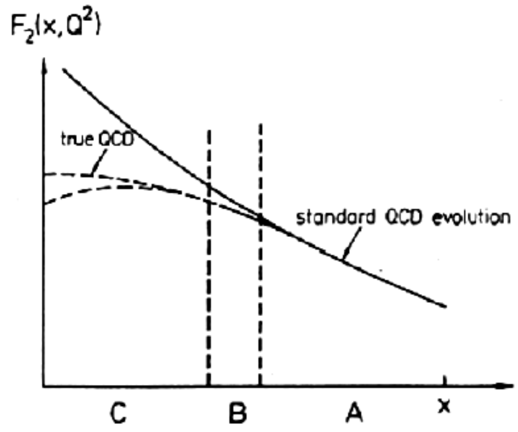

Fig. 1. Change of regime in the behavior of the SF is visible around $x \sim 10^{-2}$.

The idea [1], based on the observed behavior of the DIS structure functions, is that the partonic matter in a nucleon (or nucleus) as seen in DIS, undergoes a change of phase from a nearly perfect gas, typical of the Bjorken scaling region, to a liquid, where the logarithmic scaling violation is replaced by a power (see Fig. 1). The presence of two regions, namely that of Bjorken scaling and beyond it (call them for the moment “dilute” and “dense”), via an intermediate mixed phase, are visible in this figure. They can be quantified in various ways that will be discussed in the next section. The relevant variable here are the fraction of the nucleon momentum, or the Bjorken variable x and the incident photon’s virtuality Q^2.

The coexistence of two phases, gaseous and fluid, can be described e.g. by the van der Waals equation, valid in a tremendous range of its variables and applicable to any system (e.g. molecular, atomic or nuclear) with short range repulsion and long range attraction between the constituents, see e.g. Fig. 2.

Dis structure functions, geometrical scaling and saturation

Most generally, DIS structure functions (SF) are sums of a singlet (S) and non-singlet (NS) terms, $F_2(x,Q^2) = F_2^S(x,Q^2) + F_2^{NS}(x,Q^2)$, each a product of a low-x, $\sim x^{\alpha}$ and high-x, $(1-x)^n$ factor, more specifically:

$$F_2^S(x,Q^2)=A_0\left(\frac{Q^2}{Q^2+\alpha}\right)^{1+\Delta(Q^2)} x^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}, \quad (1)$$

where $\Delta_0 = 0.1$, $b \approx 0.4$, etc.

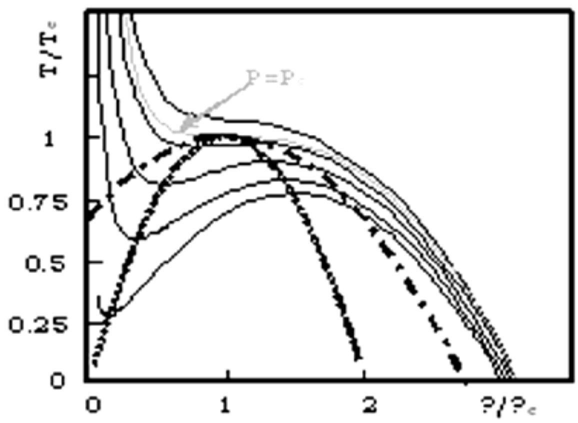

Fig. 2. Universal van der Waals curves.

A related phenomenon in DIS is the so-called geometrical scaling (GS) (not to be confused with GS in hadronic processes!), implying the existence of a "saturation radius" $R_0(x)$, given by

$$R_0^2(x)=\left(\frac{x}{x_0}\right)^{\lambda}/Q_0^2, \qquad (2)$$

where $Q_0^2 = 1$ GeV^2, $\lambda = 0.29$ and $x_0 = 3\times10^{-4}$.

Substitution of (2) into the effective intercept of the low-x factor in Eq. (1) (the Pomeron contribution),

$$F_2(x,Q^2) \sim x^{\Delta\left[Q_S^2(x)\sim x^{-0.3}\right]}. \qquad (3)$$

Formally, saturation can be treated in the context of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) eqiation and its modifi-cations. The production of partons in a nucleon by splitting of partons, resulting in the increase of their number N, is described by the asymptotic BFKL evolution equation

$$\frac{\partial N(x,k_T^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N\left(x,K_T^2\right), \quad (4)$$

where K_{BFKL} is the BFKL integral kernel (splitting function). It results in a power-like increase of the SF toward smaller x, $F_2(x) \sim x^{-\alpha p+1}$, α_p obeying the BFKL pomeron intercept, $\alpha_p - 1 = (4\alpha_s N_c \ln 2)/\pi > 0$, α_s is the ("running") QCD coupling and N_c is the number of colors. Since the number of partons increases with energy, at certain "saturation" values of x and Q^2, an inverse process, namely the recombination of pairs of partons comes into play, and the BFKL equation is replaced by the following (approximate) one

$$\frac{\partial N(x,k_T^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x,K_T^2) - \alpha_s\left[K_{BFKL} \otimes N(x,K_T^2)\right]^2, \quad (5)$$

based on the simple idea that the number of recombinations is roughly proportional to the the number of parton pairs, N^2.

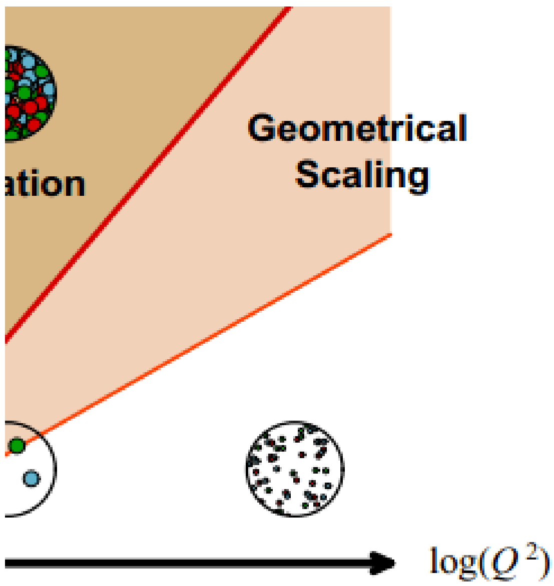

Fig. 3. Phase diagram of DIS.

Statistical models of SF

The idea that DIS structure functions (SF) can be treated thermodynamically by means of statistical mechanics is not new, moreover it continues attract attention, although several subtle points remain to be clarified.

For simplicity, we focus our attention on the small-x singlet (gluon) component of the SF, the extension to low-x and/or the non-singlet (valence quark) contributions being straight-forward

$$xG(x,Q^2) \sim \frac{X_0 x^b}{\exp[(x-X_0)/\bar{x}]+1}, \qquad (6)$$

where x is the Bjorken (light-cone) variable, X_0 is the chemical potential, that for the gluon component can be set zero, and $\bar{x}$ is interpreted as the temperature inside the proton.

The low-*x* factor in Eq. (6), will affect the ideal Stefan-Boltzmann equation of state (ES) only when it will be written in Eq. (6). As a consequence, the ideal Stefan-Boltzmann ES will be modified as

$$P(T) \sim T^{4+b}. \qquad (7)$$

The relative contribution of this correction is negligible at small x, but it increases with Q^2 and decreasing x, resulting in a gas-liquid phase transition.

Gas-fluid phase transition in the van der Waals equation of state

Having defined the statistical properties of the SF, we now proceed to ES describing the transition between a parton gas, via a mixed foggy phase, to the partonic liquid. To this end, we use the van der Waals equation

$$\left(P+\frac{N^2\alpha}{V^2}\right)(V-Nb)=NT, \qquad (8)$$

where *a* and *b* are parameters depending on the properties of the system, *N* is the number of particles and *V* is the volume of the "container".

Alternatively, Eq. (10) can be written as

$$\left(P+\frac{\alpha}{V^2}\right)(V-b)=RT,$$

or, equivalently,

$$P=\frac{RT}{V-b}-\frac{\alpha}{V^2}.$$

Following Ref. [1], we present two example of ES, one based on the Skyrme effective interaction and finite-temperature Hartree-Fock theory, and the other one is the van der Waals ES. Jaqaman et al. start with the ES

$$P=\rho kT-\alpha_0\rho^2+\alpha_3(1+\sigma)\rho^{(2+\sigma)}, \qquad (9)$$

where $\rho = N/V$ is the density and α_0, α_3 and σ are parameters, $\sigma = 1$ corresponding to the usual Skyrme interaction.

Let us now write the van der Waals (vdW) ES in the form

$$P(T,N,V)=-\left(\frac{\partial F}{\partial V}\right)_{TN}=\frac{NT}{V-bN}-\alpha\left(\frac{N}{V}\right)^2=\frac{nT}{1-bn}-an^{\ 2}, \qquad (10)$$

where $n = N/V$ is the particle number density, α is the strength of the mean-field attraction, and b governs the short-range repulsion. We identify the particle number density with the SF $F_2(x,Q^2)$ of Sec. II. Fig. 4 shows the pressure-density dependence calculated from Eq. (9) with a $\alpha = 2$ GeV^{-2} and $b = 0.2$ GeV^{-3}. Representative isotherms are shown in this figure: the dark blue line (second from the top) is the critical one, $T_c = 8\alpha/(27b)$. Above this temperature (top line, in pale blue), the pressure rises uniformly with density, corresponding to a single thermo-dynamical state for each P and T. By contrast, for subcritical temperatures $(0<T<T_c)$ (red lines) the function $P(n)$ has a maximum followed by a minimum, see Fig. 4. Below the critical value P_c, three density regimes exists. The smallest density region lies in the the gaseous phase below the spinodal region, while the highest densities lie in the liquid phase, above the spinodal region. The coexistence phase can be determined by a Maxwell construction.

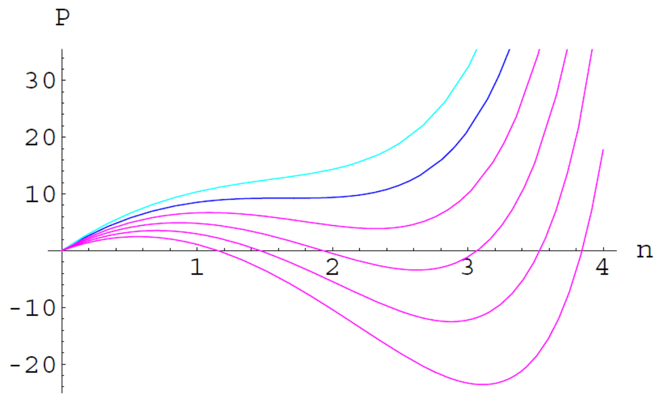

Fig. 4. The pressure-to-density dependence calculated, in arbitrary units, from Eq. (9).

Mapping the saturation region in DIS onto the spinodal region in the vdW ES; metastability, overheating and supercooling

The correspondence between the ES with its variables P, T, μ and the observables, depending on the reaction kinematics, is the most delicate and complicated point in the thermo-dynamical description of any high-energy collisions, especially of DIS. It needs caution, further studies and numerical tests. Attempts to link two different approaches to hadron dynamics, one based on the S matrix (scattering amplitude, cross sections) and the other one on their collective properties (statistical mechanics, thermodynamics, equation of state) are known from the literature [2, 3].

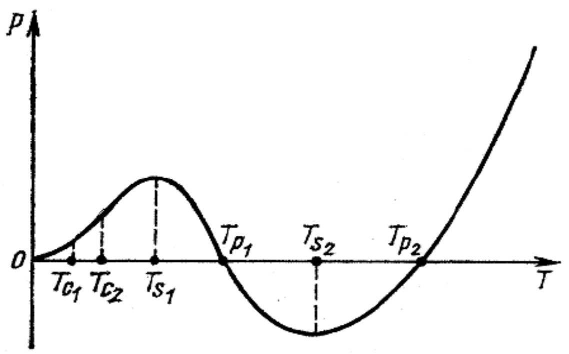

Fig. 5: $P(T)$ and $P(c)$, where c is the sound velocity, dependence calculated from Eq.(11).

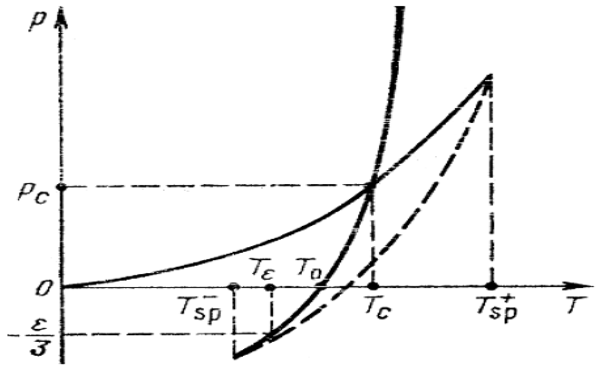

Fig. 6. Meta-stable super-cooled and overheated states calculated from a modification of the bag ES, see [2].

An original ES of the ultra-relativistic nuclear matter based on the S matrix formulation of statistical physics was derived in Ref. [2, 3]. For vanishing chemical potential, $\mu = 0$, it reads

$$P(t) = AT^4 - BT^5 + CT^6, \qquad (11)$$

where the values of the coefficients A, B and C (all positive!) were determined by the hadron scattering data. Similarities between the vdW ES and Eq. (11) were quantified in Refs. [2,3], where the parameters a and b appearing in the van der Waals ES were assumed to be temperature-dependent, $a = a_0 / T^{\alpha}$, $b = T^{\beta}$. Note that the limiting particle density is $1/b$.

Conclusions

• The aim of our paper was not just another parameterization of DIS structure functions; instead we propose a new insight into the properties of the interior of the nucleon.

• For simplicity, we have concentrated on the singlet component of the SF (gluons and sea quarks), dominating the low-x region, where saturation takes place. The inclusion the nonsinglet and higher-x components (valence quarks), according to the prescriptions given in Sec. II, are straightforward.

• We omitted any discussion of the large-x $(1-x)^n$ factor in the SF. This is because the statistical approach is not valid in the $x \to 1$ limit, on the one hand, and for simplicity, on the other hand.

•Any phase transition may produce fluctuations in the observed spectra of produced particles. These fluctuations may originate either from the gas-fluid-gas phase transition under discussion, or from the (de)confinement transition, beyond the scope of the present paper.

•The gas-fluid transition does not necessarily follow the van der Waals ES. Possible alternatives are e.g. percolation or clustering of partons similar to the case of the molecule.

•Whatever the details of the transition, the important point to realize is the existence of a dense partonic substance, different from the perfect partonic gas, associated with Bjorken scaling in DIS or the so-called quark-gluon plasma, predicted by perturbative QCD and expected in high-energy hadronic and/or nuclear collisions.

•A general remark, concerning collective properties of the nuclear matter: matter is made of quarks, while gluons are binding forces between them. In that sense, strictly speaking, statistics should be applied to quarks rather than gluons.

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ЗБУДЖЕНИЙ НУКЛОН ЯК ВАН ДЕР ВААЛЬСОВСЬКА СИСТЕМА ПАРТОНІВ

Насичення в глибоко непружному розсіянні (ГНР) і глибоко віртуальному комптонівському розсіянні (ГВКР) пов'язане з фазовим переходом між партонним газом з типовими середніми x та Q^2 й партонною рідиною, що появляється при збільшенні Q^2 і зменшенні бйоркенівської *x*. У цій статті розглядаються останні досягнення в дослідженнях колективних властивостей нуклонів, насичених у ГНР.

Ключові слова: нуклон, кварки, глюон, системи Ван дер Ваальса, комптонівське розсіяння.

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ВОЗБУЖДЕННЫЙ НУКЛОН КАК ВАН ДЕР ВААЛЬСОВСКАЯ СИСТЕМА ПАРТОНОВ

Насыщение в глубоко неупругом рассеянии (ГНР) и глубоко виртуальном комптоновском рассеянии (ГВКР) связано с фазовым переходом между партонным газом с типичными средними x и Q^2 и партонной жидкостью, появляющимся при увеличении Q^2 и уменьшении бьеркеновской x. В этой статье рассмотрены последние достижения в исследованиях коллективных свойств нуклонов, насыщенных в ГНР.

Ключевые слова: нуклон, кварки, глюон, системы Ван дер Ваальса, комптоновское рассеяние.