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THE PARTICLE-WAKE INTERACTIONS IN A HEXAGONAL TWO-DIMENSIONAL DUST LATTICE

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Dust lattice (DL) modes are studied in a hexagonal two-dimensional dust lattice in plasma crystal, associated with the particle-wake interaction. The longitudinal and horizontal transverse modes are coupled with the vertical transverse mode for dust grains, due to the particle-wake interaction.

Introduction

In fundamental physics the most surprising discovery was the observation of crystal-like structures. Such plasma crystal structures can support longitudinal and transverse vibrational modes, which have been extensively studied in the last years [1-20]. Dust lattice waves (DLW's) are produced by the oscillations of regularly spaced charged micrometer sized particles suspended in a plasma (plasma crystals), and there have been many publications on experimental and theoretical investigations of the DL modes in one- and two-dimensional (1D and 2D) crystals[3-13]. Unlike the weakly coupled plasmas, both the longitudinal wave and transverse wave can exist in plasma crystals.

Although plasma crystals structures include 1D chains, 2D hexagonal monolayers and 3D cubic lattices has investigated extensively. Crystalline complex plasma structures has been observed in recent rf discharge experiments [21], in which the plasma sheath was embedded in an external magnetic field.

The coupling between the transverse and longitudinal dust lattice modes due to the particle-wake interaction in a one dimensional string, studied in Ref.22. The influence of magnetic field direction, ion focusing effect and equilibrium charge gradient on the propagation of DL modes in a 1D string is considered, in Ref. 20, and they founded the modification of DL waves and a new coupling between modes.

In this paper the influence of the ion flow in the sheath, in particular ‘ion wakes’ on dust lattice waves in a hexagonal two-dimensional dusty plasma crystal, is studied. It is shown that the longitudinal and horizontal transverse modes are coupled with the vertical transverse mode for dust grains, due to the particle-wake interaction, as three modes may be coupled simultaneously. A novel type of oscillatory mode is found.

Model Description

In this paper, we consider a 2D monolayer of micro particles forming the hexagonal-type 2D crystal, and investigate the propagation of the dust lattice waves in this system theoretically, including effects relevant for the sheath region, namely, anisotropy of interactions caused by ion focusing (ion wake). In this model, we have used the excess positive charge of the wake (along the vertical axis) as a point-like

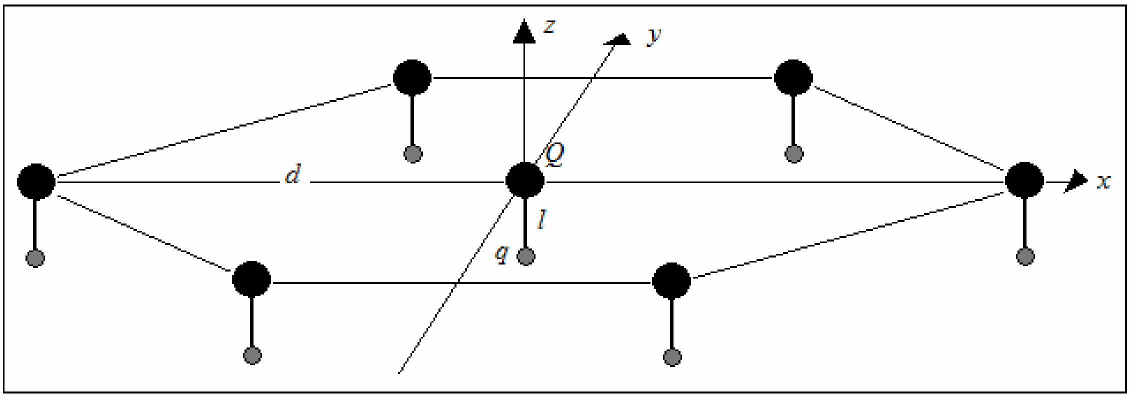

Figure 1. Hexagonal crystal structure.

effective charge q, located at some distance l beneath the particle [16].

For simplicity we assume that dust grians has equal mass, with their centers evently separated by the distance a in the equilibrium. For the geometrical modeling of this crystalline structure, we use the so-called particle-string model, allowing for the z -direction as vertical vibrations.

The schematics of the model is shown in Fig. 1. This crystal structure is formed by particles of the same mass M, separated by the average distance a and carrying negative equilibrium charge. Letting the particle at the site (n,m) be the origin of the plane, then the position of dust particles at equilibrium are $(a,0,0)$, $(-a,0,0)$, $(a/2,\sqrt{3}a/2,0)$, $(-a/2,\sqrt{3}a/2,0)$, $(a/2,-\sqrt{3}a/2,0)$, $(-a/2,-\sqrt{3}a/2,0)$; and the position of the wakes at equilibrium are $(a,0,-l)$, $(-a,0,-l)$, $(a/2,\sqrt{3}a/2,-l)$, $(-a/2,\sqrt{3}a/2,-l)$, $(a/2,-\sqrt{3}a/2,-l)$ and $(-a/2,-\sqrt{3}a/2,-l)$. If the particles exit from equilibrium positions, we may define the six length variables $d_{Q1}, d_{Q2}, d_{Q3}, d_{Q4}, d_{Q5}$, and d_{Q6}, which represent the distances from the particle (n,m) to the nearest particles, and the six length variables for distance between original particle and wake-charge as $d_{q1}, d_{q2}, d_{q3}, d_{q4}, d_{q5}$, d_{q6}; which these length variables can define as follow:

$$d_{Q1} = \sqrt{(a+\Delta u_1)^2 + (\Delta \upsilon_1)^2 + (\Delta w_1)^2}\ , \tag{1}$$

$$d_{Q2} = \sqrt{(a+\Delta u_2)^2 + (\Delta \upsilon_2)^2 + (\Delta w_2)^2}\ , \tag{2}$$

$$d_{Q3} = \sqrt{(a/2+\Delta u_3)^2 + (\sqrt{3}a/2+\Delta \upsilon_3)^2 + (\Delta w_3)^2}\ , \tag{3}$$

$$d_{Q4} = \sqrt{(a/2+\Delta u_4)^2 + (\sqrt{3}a/2+\Delta \upsilon_4)^2 + (\Delta w_4)^2}\ , \tag{4}$$

$$d_{Q5} = \sqrt{(a/2+\Delta u_5)^2 + (\sqrt{3}a/2+\Delta \upsilon_5)^2 + (\Delta w_5)^2}\ , \tag{5}$$

$$d_{Q6} = \sqrt{(a/2+\Delta u_6)^2 + (\sqrt{3}a/2+\Delta \upsilon_6)^2 + (\Delta w_6)^2}\ , \tag{6}$$

$$d_{q1} = \sqrt{(a+\Delta u_1)^2 + (\Delta \upsilon_1)^2 + (-l+\Delta w_1)^2}\ , \tag{7}$$

$$d_{q2} = \sqrt{(a+\Delta u_2)^2 + (\Delta \upsilon_2)^2 + (l+\Delta w_2)^2}\ , \tag{8}$$

$$d_{q3} = \sqrt{(a/2+\Delta u_3)^2 + (\sqrt{3}a/2+\Delta \upsilon_3)^2 + (l+\Delta w_3)^2}\ , \tag{9}$$

$$d_{q4} = \sqrt{(a/2 + \Delta u_4)^2 + (\sqrt{3}a/2 + \Delta \upsilon_4)^2 + (l + \Delta w_4)^2}\ , \qquad (10)$$

$$d_{q5} = \sqrt{(a/2 + \Delta u_5)^2 + (\sqrt{3}a/2 + \Delta \upsilon_5)^2 + (-l + \Delta w_5)^2}\ , \qquad (11)$$

$$d_{q6} = \sqrt{(a/2 + \Delta u_6)^2 + (\sqrt{3}a/2 + \Delta \upsilon_6)^2 + (l + \Delta w_6)^2}\ , \qquad (12)$$

where u_i, υ_i and w_i (for i = 1,2,…,6) denote the displacements of the respective particles from their equilibrium positions in both x , y and z directions, and

$$\Delta u_1 = u_{n+1,m} - u_{n,m}, \Delta u_2 = u_{n,m} - u_{n-1,m}, \Delta u_3 = u_{n+1/2,m+\sqrt{3}/2} - u_{n,m},$$

$$\Delta u_4 = u_{n,m} - u_{n-1/2,m+\sqrt{3}/2}, \Delta u_5 = u_{n,m} - u_{n-1/2,m-\sqrt{3}/2}, \Delta u_6 = u_{n+1/2,m-\sqrt{3}/2} - u_{n,m},$$

$$\Delta v_1 = v_{n+1,m} - v_{n,m}, \Delta v_2 = u_{n,m} - v_{n-1,m}, \Delta v_3 = v_{n+1/2,m+\sqrt{3}/2} - v_{n,m},$$

$$\Delta v_4 = v_{n-1/2,m+\sqrt{3}/2} - v_{n,m}, \Delta v_5 = v_{n,m} - v_{n-1/2,m-\sqrt{3}/2}, \Delta v_6 = v_{n,m} - v_{n+1/2,m-\sqrt{3}/2},$$

$$\Delta w_1 = w_{n+1,m} - w_{n,m}, \Delta w_2 = w_{n-1,m} - w_{n,m}, \Delta w_3 = w_{n+1/2,m+\sqrt{3}/2} - w_{n,m},$$

$$\Delta w_4 = w_{n-1/2,m+\sqrt{3}/2} - w_{n,m}, \Delta w_5 = w_{n-1/2,m-\sqrt{3}/2} - w_{n,m}, \Delta w_6 = w_{n-1/2,m+\sqrt{3}/2} - w_{n,m}.$$

The components of the force exerted upon the particle at (n,m) by the nearest particles are

$$\ddot{u}_n + 2\gamma\dot{u}_n = \Omega_{11}^2(\Delta u_1 - \Delta u_2) + \Omega_{12}^2(\Delta u_3 - \Delta u_4 - \Delta u_5 + \Delta u_6) + \Omega_{13}^2(\Delta v_3 - \Delta v_4 - \Delta v_5 + \Delta v_6) + \Omega_{14}^2(\Delta w_1 - \Delta w_2) + \Omega_{15}^2(\Delta w_3 - \Delta w_4 - \Delta w_5 + \Delta w_6) \qquad (13)$$

$$\ddot{v}_n + 2\gamma\dot{v}_n = \Omega_{21}^2(\Delta u_3 + \Delta u_4 - \Delta u_5 - \Delta u_6) + \Omega_{22}^2(\Delta v_1 - \Delta v_2) + \Omega_{23}^2(\Delta v_3 + \Delta v_4 - \Delta v_5 - \Delta v_6) + \Omega_{24}^2(\Delta w_3 + \Delta w_4 - \Delta w_5 - \Delta w_6) \qquad (14)$$

$$\ddot{w}_n + 2\gamma\dot{w}_n = -\Omega_v^2 w_n + \Omega_{31}^2(\Delta u_1 + \Delta u_2) + \Omega_{32}^2(\Delta u_3 + \Delta u_4 + \Delta u_5 + \Delta u_6) + \Omega_{33}^2(\Delta v_3 + \Delta v_4 + \Delta v_5 + \Delta v_6) - \Omega_{34}^2(\Delta w_1 + \Delta w_2) - \Omega_{35}^2(\Delta w_3 + \Delta w_4 + \Delta w_5 + \Delta w_6) \qquad (15)$$

where

$$\Omega_o^2 = Q^2 e^{-\kappa}/Ma^3,\ \Omega_v^2 = -(QE - Mg)/M,$$

$$\Omega_{11}^2 = \Omega_o^2[(2 + 2\kappa + \kappa^2) - \frac{\bar{q}e^{-(K-1)\kappa}}{K^5}(K\kappa + (3 - K^2) + K^2\kappa^2 + (2 - K^2)K\kappa)],$$

$$\Omega_{12}^2 = \frac{1}{4}\Omega_o^2[(-1 - \kappa + \kappa^2) - \frac{\bar{q}e^{-(K-1)\kappa}}{K^5}(K\kappa - (4K^2 - 3) + K^2\kappa^2 - (4K^2 - 2)K\kappa)],$$

$$\Omega_{13}^2 = \frac{\sqrt{3}}{4}\Omega_o^2[(3 + 3\kappa + \kappa^2) - \frac{\bar{q}e^{-(K-1)\kappa}}{K^5}(3 + 3K\kappa + K^2\kappa^2)],$$

$$\Omega_{14}^2 = \Omega_o^2\frac{\bar{q}\bar{l}e^{-(K-1)\kappa}}{K^2}(3 + 3K\kappa + K^2\kappa^2), \Omega_{15}^2 = \frac{1}{2}\Omega_o^2\frac{\bar{q}\bar{l}e^{-(K-1)\kappa}}{K^5}(3 + 3K\kappa + K^2\kappa^2),$$

$$\Omega_{21}^2 = \frac{\sqrt{3}}{4}\Omega_o^2[(3 + 3\kappa + \kappa^2) - \frac{\bar{q}e^{-(K-1)\kappa}}{K^5}(3 + 3K\kappa + K^2\kappa^2)],$$

$$\Omega_{22}^2 = \Omega_o^2[(-1 - \kappa) + \frac{\bar{q}e^{-(K-1)\kappa}}{K^3}(1 + K\kappa)],$$

$$\Omega_{23}^2 = \frac{1}{4}\Omega_o^2[(5 + 5\kappa + 3\kappa^2) - \frac{\bar{q}e^{-(K-1)\kappa}}{K^5}(3 - 4K^2 + 3K\kappa + (6 - 4K^2)K\kappa + 3K^2\kappa^2)],$$

$$\Omega_{24}^2 = \Omega_o^2\frac{\sqrt{3}}{2}\frac{\bar{q}\bar{l}e^{-(K-1)\kappa}}{K^5}(3 + 3K\kappa + K^2\kappa^2), \Omega_{31}^2 = \Omega_o^2\frac{\bar{q}\bar{l}e^{-(K-1)\kappa}}{K^5}(3 + 3K\kappa + K^2\kappa^2),$$

$$\Omega_{32}^2 = \frac{1}{2}\Omega_o^2 \frac{\bar{q}\bar{l}e^{-(K-1)\kappa}}{K^5}(3+3K\kappa+K^2\kappa^2), \Omega_{33}^2 = \frac{\sqrt{3}}{2}\Omega_o^2 \frac{\bar{q}\bar{l}e^{-(K-1)\kappa}}{K^5}(3+3K\kappa+K^2\kappa^2),$$

$$\Omega_{34}^2 = \Omega_o^2[(1+\kappa) - \frac{\bar{q}e^{-(K-1)\kappa}}{K^5}((1+K\kappa)K^2 - (1-K^2)(3+3K\kappa+K^2\kappa^2)],$$

$$\Omega_{35}^2 = \Omega_o^2[(1+\kappa) - \frac{\bar{q}e^{-(K-1)\kappa}}{K^5}((1+K\kappa)K^2 - (1-K^2)(3+3K\kappa+K^2\kappa^2)].$$

The gravity force compensated with electric force in *w=0* position

$$QE - Mg = -M\omega_g^2 w. \tag{16}$$

Waves can propagate along an arbitrary direction, denoted by an angle α, which represents the angle between the wave vector k and a primitive translation vector(along the x axis), i.e.,

$$u_{n,m}, v_{n,m}, w_{n,m} \propto u_0, v_0, w_0 e^{-i\omega t} e^{i(k_x x + k_y y)}, \tag{17}$$

where $k_x = k\cos\alpha$, $k_y = k\sin\alpha$. By letting Eq.(17) into Eqs.(13-15) and neglecting nonlinear terms, we thus obtain the system of equations

$$a_{11}u_{n,m} + a_{12}v_{n,m} + a_{13}w_{n,m} = 0, \tag{18}$$

$$a_{21}u_{n,m} + a_{22}v_{n,m} + a_{23}w_{n,m} = 0, \tag{19}$$

$$a_{31}u_{n,m} + a_{32}v_{n,m} + a_{33}w_{n,m} = 0, \tag{20}$$

where

$$a_{11} = \omega^2 - 4\Omega_{11}^2 \sin^2(\frac{k_x a}{2}) - 4\Omega_{12}^2(\sin^2(\frac{k_x a + \sqrt{3}k_y a}{4}) + \sin^2(\frac{k_x a - \sqrt{3}k_y a}{4})),$$

$$a_{12} = -4\Omega_{13}^2(-\sin^2(\frac{k_x a + \sqrt{3}k_y a}{4}) + \sin^2(\frac{k_x a - \sqrt{3}k_y a}{4})),$$

$$a_{13} = 2i[\Omega_{14}^2 \sin(k_x a) + \Omega_{15}^2(\sin(\frac{k_x a + \sqrt{3}k_y a}{2}) + \sin(\frac{k_x a - \sqrt{3}k_y a}{2}))],$$

$$a_{21} = -4\Omega_{21}^2(\sin^2(\frac{k_x a + \sqrt{3}k_y a}{4}) + \sin^2(\frac{k_x a - \sqrt{3}k_y a}{4})),$$

$$a_{23} = 2i\Omega_{24}^2(\sin(\frac{k_x a + \sqrt{3}k_y a}{2}) + \sin(\frac{k_x a - \sqrt{3}k_y a}{2})),$$

$$a_{32} = 2i[\Omega_{33}^2(\sin(\frac{k_x a + \sqrt{3}k_y a}{2}) - \sin(\frac{k_x a - \sqrt{3}k_y a}{2}))],$$

$$a_{33} = \omega^2 - \Omega_v^2 + 4\Omega_{34}^2 \sin^2(\frac{k_x a}{2}) + 4\Omega_{35}^2(\sin^2(\frac{k_x a + \sqrt{3}k_y a}{4}) + \sin^2(\frac{k_x a - \sqrt{3}k_y a}{4})).$$

Then we set the determinant of coefficients of Eqs.(17-19), it gives the dispersion relation as a third rank polynomial versus ω^2. In limitation conditions when eliminating the wake ion effect, results of this paper converges to the results of, B. Farokhi *et al* [17] and S. V. Vladimirov *et al* [18].

Propagation of wave in x-direction

We consider the propagation of wave in x-direction, as a special case. In this case the horizontal case is independent of two other modes, and vertical mode is coupled with longitudinal mode. By using of Eq. 17, the components of equation of motion leads to

$$-(\omega^2+2i\gamma\omega)w=[2i\Omega_{31}^2\sin(ka)+4i\Omega_{32}^2\sin^2(\frac{ka}{4})]u$$
$$+[-\Omega_v^2+4\Omega_{34}^2\sin^2(\frac{ka}{2})+8\Omega_{35}^2\sin^2(\frac{ka}{4})]w$$
$$-(\omega^2+2i\gamma\omega)v=[-4\Omega_{22}^2\sin^2(\frac{ka}{2})-8\Omega_{23}^2\sin^2(\frac{ka}{4})]v \qquad (21)$$
$$-(\omega^2+2i\gamma\omega)w=[2i\Omega_{31}^2\sin(ka)+4i\Omega_{32}^2\sin^2(\frac{ka}{4})]u$$
$$+[-\Omega_v^2+4\Omega_{34}^2\sin^2(\frac{ka}{2})+8\Omega_{35}^2\sin^2(\frac{ka}{4})]w$$

Dispersion relation in absence of damping term is

$$\omega^2=\frac{1}{2}\Omega_v^2+2(\Omega_{11}^2-\Omega_{34}^2)\sin^2(\frac{ka}{2})+4(\Omega_{12}^2-\Omega_{35}^2)\sin^2(\frac{ka}{4})$$
$$\pm\{[\frac{1}{2}\Omega_v^2+2(\Omega_{11}^2-\Omega_{34}^2)\sin^2(\frac{ka}{2})+4(\Omega_{12}^2-\Omega_{35}^2)\sin^2(\frac{ka}{4})]^2$$
$$+4[\Omega_{11}^2\sin^2(\frac{ka}{2})+2\Omega_{12}^2\sin^2(\frac{ka}{4})][\Omega_v^2-4\Omega_{34}^2\sin^2(\frac{ka}{2}) \qquad (22)$$
$$-8\Omega_{35}^2\sin^2(\frac{ka}{4})]-4[\Omega_{14}^2\sin^2(ka)+2\Omega_{15}^2\sin^2(\frac{ka}{2})]^2\}^{1/2}$$
$$\omega^2=4\Omega_{22}^2\sin^2(\frac{ka}{2})+8\Omega_{23}^2\sin^2(\frac{ka}{4}).$$

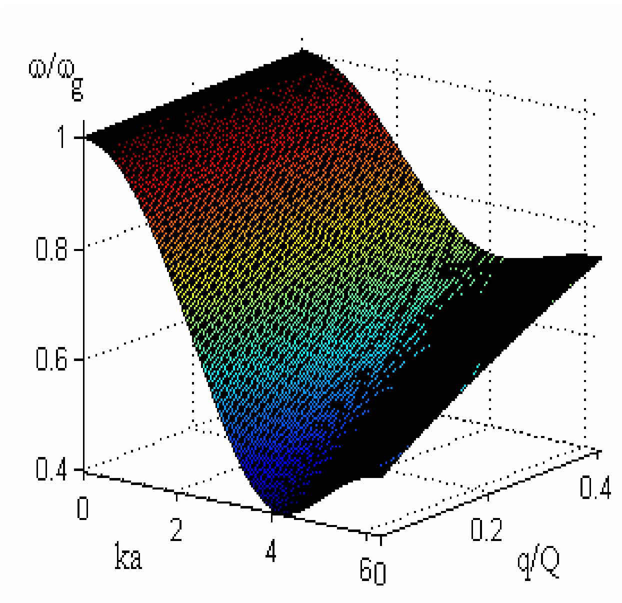

Figure 2. The normalized vertical transverse frequency, for $4\Omega_o^2(1+\kappa)=0.3721\omega_g^2$, $K=1$ and for propagation along *x*-axis($\theta=0$).

The coupling between two modes leads to an instability, which to get the growth rate of this instability we can expand determinant of Eqs. (21-a) and (21-b), around intersection point of these two modes (ω_o, k_o), and obtain $\mathrm{Re}\,\omega\cong\omega_o$,

$$\mathrm{Im}\omega\cong-\gamma+\frac{\Omega_{14}\Omega_{31}}{\omega_o}\sin(k_o a)+\frac{2\Omega_{15}\Omega_{32}}{\omega_o}\sin(\frac{k_o a}{2}) \qquad (23)$$

for the lowest order in $|\omega-\omega_o|/\omega_o \ll 1$. Eq. 23, indicating that this hybrid mode is unstable, when the neutral gas friction is sufficiently weak.

Conclusion

The wake-particle interactions in hexagonal crystal, leads to coupling of three modes in a general case. Also the frequency of vertical transverse, horizontal transverse and longitudinal modes is dependent on wake-particle interaction. Then a dispersion relation can obtain as a polynomial of ω^2, which is include of three modes. When wave propagate along x-direction, coupling between longitudinal and horizontal transverse modes will be vanish, but longitudinal and vertical transverse modes remain coupled. Fig.2 shows increasing of frequency with wake charge. The coupling between two modes leads to an instability, when the neutral gas friction is sufficiently weak.

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«PARTICLE-WAKE» ВЗАЄМОДІЇ В ГЕКСАГОНАЛЬНІЙ ДВОМІРНІЙ ПИЛОВІЙ ҐРАТЦІ

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Вивчаються моди пилової гратки у гексагональній двовимірній пиловій гратці у кристалі плазми, пов'язані з взаємодією типу "пробудження частинок". Відбувається взаємодія поздовжніх і горизонтальних поперечних мод з вертикальною поперечною модою для пилових зерен внаслідок взаємодії типу "пробудження частинок".