

EXPERIMENTAL PHYSICS

INSTITUTE OF EXPERIMENTAL PHYSICS

SLOVAK ACADEMY OF SCIENCES



**THE 18th SMALL TRIANGLE MEETING
on theoretical physics**

October 16–19, 2016 | Ptíče

Y OF SCIENCES



The Physical Implementation of the Two-Qubit Logical Operator CNOT

V.Yu. Lazur, O.K. Reity, S.I. Myhalyna, O.F. Pavlyk

*Department of Theoretical Physics, Uzhhorod National University,
54 Voloshyna Street, Uzhhorod 88000, Ukraine*

Abstract

The optical properties of a system of two identical two-level atoms in collective (symmetric Ψ_s and antisymmetric Ψ_a) Bell states are studied for arbitrary interatomic distances. The closed-form analytical expressions for shifts and widths of considered states are obtained taking into account the retarded dipole-dipole interaction between atoms. It is shown that under certain conditions the retarded interaction of atoms of this quantum diatomic system can lead to both additional widening of energy levels of atoms compared to radiation width, and the opposite effect – narrowing of the natural width of atomic levels. We can identify three types of quantum information transfer processes associated with either a resonant (non-resonant) absorption of a photon by one of the atoms of the system or with variation of interatomic distance.

1 Introduction

The unit of quantum information (quantum bit or qubit) is a superposition of two base functions of a quantum system [1]. A method for writing quantum information on an individual double-level atom of a system taking a region whose linear dimensions are substantially smaller than the wavelength of photons has been proposed, where intense quasi-resonance laser radiation is used and the incidence angle of the external radiation on the system is varied [2]–[4]. Of great importance in this method of writing and reading information is resonance transfer of quantum information from one atom to another. On the other hand, transfer of quantum information for arbitrary distances plays an important role in creating quantum communication systems. In this paper, we propose a unified way of describing resonance transfer of quantum information for arbitrary distances, including the effect of quantum teleportation. Quantum teleportation was discovered experimentally [5] with the use of a three-photon scheme for transferring quantum information from one observer to another taking into account two types of polarization of photons. Other schemes of quantum teleportation have also been proposed, for instance, those using a system of three electrons [6] or three atomic beams of two-level atoms [7]. The schemes of quantum teleportation described in [5]–[7] are, in fact, based on the superposition principle for nonlocal quantum

systems. In this case, the effect of quantum teleportation does not come into conflict with the principle of relativity since not energy, but information, which acquires an additional sense in quantum processes as compared to the information in classical systems, is transferred instantaneously. In this paper, it is shown that the information about two possible states of atoms can be transferred from atom to atom within a short time interval. For this case, the value of this interval depending on the type of interatomic interaction is calculated. In a system of two identical interacting atoms, absorption (emission) of actual photons is nonlocal in character, namely, the event of disappearance (birth) of a photon may take place at the site of location of one atom, while the quantum transition will occur between the states of the other atom. The classical and quantum methods for information transfer with corresponding characteristic times are also considered.

2 Energy of the resonant interaction of atoms located at arbitrary distances

Let us write the operator of electric dipole-dipole interaction of two electrons located at an arbitrarily large distance from each other, near various nuclei [8, 9]:

$$\hat{V}_{dip}^{(\pm)} = \exp\left(\frac{i}{c}\omega_0 R\right) \left\{ \frac{\vec{d}_1 \vec{d}_2 - 3(\vec{n}_R \vec{d}_1)(\vec{n}_R \vec{d}_2)}{R^3} \pm \frac{e}{2mc} \left[\frac{\vec{d}_1 \hat{p}_2 - 3(\vec{n}_R \vec{d}_1)(\vec{n}_R \hat{p}_2)}{R^2} - \frac{\vec{d}_2 \hat{p}_1 - 3(\vec{n}_R \vec{d}_2)(\vec{n}_R \hat{p}_1)}{R^2} \right] - \frac{e^2}{m^2 c^2} \frac{\hat{p}_1 \hat{p}_2 - 3(\vec{n}_R \hat{p}_1)(\vec{n}_R \hat{p}_2)}{R} \right\}, \quad (1)$$

where $\vec{n}_R = \vec{R}/R$, $\omega_0 \equiv \omega_{n0} = (E_n - E_0)/\hbar$ is the resonance frequency in the spectrum of the atoms, $\vec{d}_1 = e\vec{r}_1$ and $\vec{d}_2 = e\vec{r}_2$ are the operators of the electric dipole moments of single-electron atoms, \vec{p}_1 and \vec{p}_2 are the momentum operators of the first and the second atomic electrons, respectively.

For the system of two interacting atoms having one electron, the Hamiltonian \hat{H} can be represented in the form of sum of Hamiltonians of isolated atoms, $\hat{H}_0 = \hat{H}_1(\vec{r}_1) + \hat{H}_2(\vec{r}_2)$, and operator of electric dipole-dipole interaction between them, $\hat{V}_{dip}^{(\pm)}$:

$$\hat{H} = \hat{H}_0 + \hat{V}_{dip}^{(\pm)} = \hat{H}_1(\vec{r}_1) + \hat{H}_2(\vec{r}_2) + \hat{V}_{dip}^{(\pm)}(\vec{r}_1, \vec{r}_2; R). \quad (2)$$

Let $E_{n_1 n_2} = E_{n_1} + E_{n_2}$ and $|n_1 n_2\rangle$ are the eigenvalue and eigenfunction of “unperturbed” operator of energy $\hat{H}_0 = \hat{H}_1 + \hat{H}_2$ without interatomic interaction, respectively. The unperturbed wave function of the initial state of two atoms is of the form

$$|00\rangle = \varphi_0(1)\varphi_0(2) \exp(-iE_0 t_1/\hbar) \exp(-iE_0 t_2/\hbar) \equiv \tilde{\varphi}_0(1)\tilde{\varphi}_0(2). \quad (3)$$

Here E_0 is the energy of the initial states of the first and second atoms, the indices 1 and 2 correspond to the coordinates and times for the first and the second atom, respectively. For atoms having no constant dipole moment in the first order of perturbation theory, the energy correction is equal to zero, i.e., $\langle 00 | \hat{V}_{dip}^{(\pm)} | 00 \rangle = 0$.

The interaction between the atoms distorts the wave functions of the atomic states, and the disturbed wave function for a system of two atoms in the ground state is of the form [10, 1]):

$$\Psi_0(1)\Psi_0(2) = \tilde{\varphi}_0(1)\tilde{\varphi}_0(2) + \sum_{n_1 n_2} \frac{\langle \tilde{\varphi}_{n_1}(1)\tilde{\varphi}_{n_2}(2) | \hat{V}_{dip}^{(\pm)} | \tilde{\varphi}_0(1)\tilde{\varphi}_0(2) \rangle}{2E_0 - E_{n_1} - E_{n_2}} \tilde{\varphi}_{n_1}(1)\tilde{\varphi}_{n_2}(2), \quad (4)$$

where the indices n_1 and n_2 correspond to intermediate states of the atoms. We consider the state $\Psi_0(1)\Psi_0(2)$ as an initial state of atoms interacting with field of real photons.

Let the final state $|n0\rangle$ of two resonant atoms corresponds to the excited state $|n\rangle$ of atom $A(1)$ with wave function $\tilde{\varphi}_n(1) = \varphi_n(1) \exp(-iE_n t_1/\hbar)$ and energy E_n as well as the ground state $|0\rangle$ of atom $A(2)$ with wave function $\tilde{\varphi}_0(2) = \varphi_0(2) \exp(-iE_0 t_2/\hbar)$ and energy E_0 .

In the zeroth approximation, the stationary state of the system corresponds to two wave functions:

$$\Phi_s(1, 2) = \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) + \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)], \quad (5)$$

$$\Phi_a(1, 2) = \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) - \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)]. \quad (6)$$

In order to obtain corrections to the energy of symmetrical (6) and antisymmetrical (7) states of the system of two resonant atoms in the first order of perturbation theory one has to calculate the mean value of the perturbation operator $\hat{V}_{dip}^{(\pm)}$ (2) in these states, i.e.,

$$\Delta E_s = \langle \Phi_s | \hat{V}_{dip}^{(\pm)} | \Phi_s \rangle, \quad \Delta E_a = \langle \Phi_a | \hat{V}_{dip}^{(\pm)} | \Phi_a \rangle. \quad (7)$$

Substituting the expressions (2), (6) and (7) into the matrix elements (8) we obtain that

$$\begin{aligned} \Delta E_s(R) = -\Delta E_a(R) &= \omega_0^3 e^{i\omega_0 R/c} |\langle n | \vec{d} | 0 \rangle|^2 \\ &\times \left[\left(\frac{1}{\omega_0^3 R^3} - \frac{i}{c\omega_0^2 R^2} \right) \Phi(1, 2) - \frac{\Phi'(1, 2)}{c^2 \omega_0 R} \right] \equiv \Delta E_{AA}(R). \end{aligned} \quad (8)$$

Here

$$\begin{aligned} \Phi(1, 2) &\equiv \cos \theta_1^x \cos \theta_2^x + \cos \theta_1^y \cos \theta_2^y - 2 \cos \theta_1^z \cos \theta_2^z, \\ \Phi'(1, 2) &\equiv \cos \theta_1^x \cos \theta_2^x + \cos \theta_1^y \cos \theta_2^y \end{aligned}$$

are the geometric factors, θ_i^x , θ_i^y , θ_i^z ($i = 1, 2$) are the angles between one of corresponding axes and the direction of dipole transition in i -th atom $A(i)$.

Note that the quantity ΔE_s (ΔE_a) is complex shift of energy E_s (E_a) of symmetrical (antisymmetrical) state Φ_s (Φ_a) of atoms:

$$\Delta E_s = \delta E_s - \frac{i}{2} \hbar \gamma_s, \quad \Delta E_a = \delta E_a - \frac{i}{2} \hbar \gamma_a. \quad (9)$$

The first-order corrections to the energy $E_n + E_0$ of states (6) and (7) give only real parts δE_s and δE_a of the complex shifts:

$$\delta E_{s,a}(R) = \pm \text{Re} \Delta E_{AA}(R) = \pm e^2 |\langle n | \vec{r} | 0 \rangle|^2 F(1, 2; R). \quad (10)$$

Here

$$F(1, 2; R) = \left[\frac{\Phi(1, 2)}{R^3} - \frac{\omega_0^2 \Phi'(1, 2)}{c^2 R} \right] \cos\left(\frac{\omega_0 R}{c}\right) + \frac{\omega_0 \Phi(1, 2)}{c R^2} \sin\left(\frac{\omega_0 R}{c}\right). \quad (11)$$

The expressions for δE_s and δE_a can be simplified at $\omega_0 R/c \rightarrow 0$, i.e., when size of two-atomic quantum system is much smaller than characteristic wave length $\lambda_0 = 2\pi c/\omega_0$ in spectrum of interacting atoms ($R \ll \lambda_0$). In this case one can neglect retardation of dipole-dipole interaction of atoms, that makes it possible to substitute $\cos(\omega_0 R/c) = 1$, $\sin(\omega_0 R/c) = \omega_0 R/c$ in the expression (12) for F and to omit terms containing velocity of light c . Then for δE_s and δE_a we obtain

$$\delta E'_{s,a} = \pm \frac{e^2}{R^3} |\langle n | \vec{r} | 0 \rangle|^2 \Phi(1, 2). \quad (12)$$

As it is expected, this expression coincides with the known formula [10] for the energy of resonant exchange of excitations between two neutral atoms located closely one from other.

Therefore, we see that at account of dipole-dipole interatomic interaction (2) the symmetrical and antisymmetrical states of atoms are characterized by the energies

$$E_s = E_n + E_0 + \delta E_s = E_n + E_0 + \frac{3e^2 \hbar f_{n0}}{2m\omega_0} F(1, 2; R), \quad (13)$$

$$E_a = E_n + E_0 + \delta E_a = E_n + E_0 - \frac{3e^2 \hbar f_{n0}}{2m\omega_0} F(1, 2; R). \quad (14)$$

and wave functions

$$\Psi_s = \Phi_s \exp(-i\delta E_s t_1/\hbar), \quad \Psi_a = \Phi_a \exp(-i\delta E_a t_1/\hbar), \quad (15)$$

where t_1 is the local time for the atomic pair associated with the position of the atom $A(1)$.

3 Width of the symmetric and antisymmetrical states of two identical atoms

It is apparent that states (15) are entangled states of a pair of interacting atoms because in these states separate atom-qubit has not certain energy. Implement a formal transition in the formulas (15) to the complex energy:

$$\delta E_s \rightarrow \delta E_s - \frac{i}{2} \tilde{\Gamma}_s = \delta E_s - \frac{i}{2} \hbar \Gamma_s = \delta E_s - \frac{i}{2} \hbar (\gamma_0 + \gamma_s), \quad (16)$$

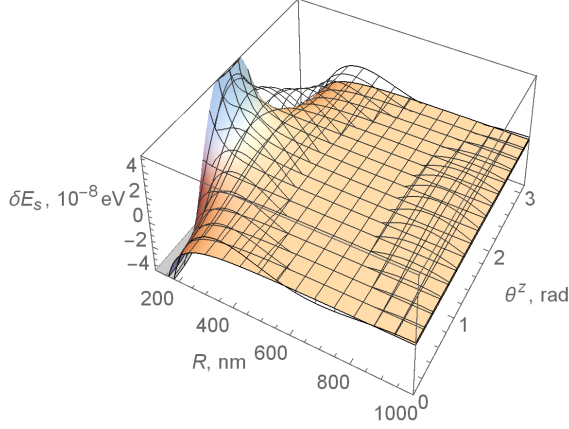


Figure 1: The dependencies of the energy shifts $\delta E_s(R, \theta^z)$ (11) and $\delta E'_s(R, \theta^z)$ (13) (transparent and opaque surfaces, respectively) on the interatomic distance R and angle $\theta^z = \theta_1^z = \theta_2^z$ of orientation of the dipole moments of the transition of the atoms that are parallel each to other ($\theta_1^x = \theta_2^x$, $\theta_1^y = \theta_2^y$)

$$\delta E_a \rightarrow \delta E_a - \frac{i}{2}\tilde{\Gamma}_a = \delta E_a - \frac{i}{2}\hbar\Gamma_a = \delta E_a - \frac{i}{2}\hbar(\gamma_0 + \gamma_a). \quad (17)$$

In the first order of perturbation theory, the contribution of dipole interaction in the total width Γ_s (Γ_a) is determined by imaginary part γ_s (γ_a) of doubled complex shift ΔE_s (ΔE_a):

$$\gamma_s(R) = -\gamma_a(R) = -\frac{2}{\hbar}\text{Im}\Delta E_{AA}(R) = -\frac{2\omega_0^3 e^2 |\langle n|\vec{r}|0\rangle|^2}{\hbar} \tilde{F}(1, 2; R), \quad (18)$$

where

$$\tilde{F}(1, 2; R) = \left[\frac{\Phi(1, 2)}{\omega_0^3 R^3} - \frac{\Phi'(1, 2)}{c^2 \omega_0 R} \right] \sin\left(\frac{\omega_0 R}{c}\right) - \frac{\Phi(1, 2)}{c\omega_0^2 R^2} \cos\left(\frac{\omega_0 R}{c}\right), \quad (19)$$

γ_0 is the radiation rate of an excited state E_n of the isolated atom.

The total widths of symmetrical and antisymmetrical states are of the form:

$$\Gamma_s = \gamma_0 + \gamma_s \approx 2\gamma_0, \quad \Gamma_a = \gamma_0 + \gamma_a \approx \text{O}\left(\frac{\omega_0^2 R^2}{c^2}\right). \quad (20)$$

We can see that under the condition $R \ll \lambda_0$ the retarded interaction of atoms in the symmetrical state Ψ_s (15) leads to doubling of natural width of atomic levels. For antisymmetrical state the widths γ_a and γ_0 compensate each other almost completely, and the total width Γ_a is small when comparing with γ_0 . It means that when $R \ll \lambda_0$ the retarded interaction of atoms completely suppresses radiation decay of excited states of atoms. This property of mutual influence of closely located atoms was interpreted in [12] as the near-field effect.

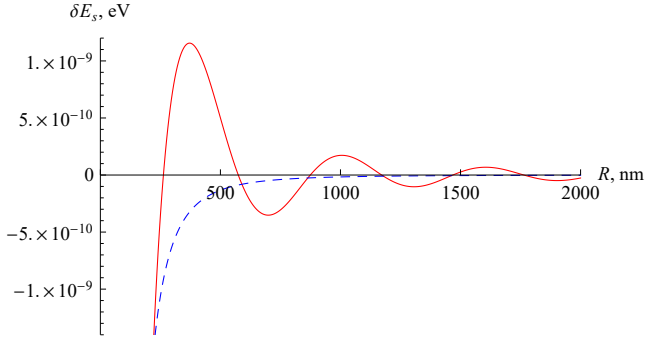


Figure 2: The dependencies of the energy shifts δE_s (11) and $\delta E'_s$ (13) (solid and dotted lines, respectively) on the interatomic distance R when the dipole moments of the transition of the atoms are oriented along the axis Oz : $\theta^z = \theta_1^z = \theta_2^z = 0$

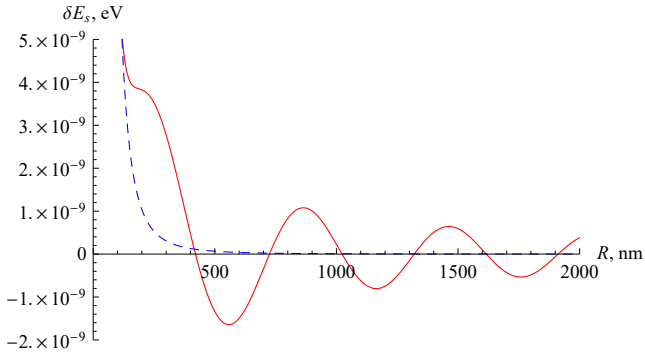


Figure 3: The dependencies of the energy shifts δE_s (11) and $\delta E'_s$ (13) (solid and dotted lines, respectively) on the interatomic distance R when the dipole moments of the transition of the atoms are perpendicular to axis Oz : $\theta^z = \theta_1^z = \theta_2^z = \pi/2$

In paper [13] the near-field effect was proposed as a basis for writing the quantum information on separate two-level atoms of two-qubit quantum computer by means of intensive quasiresonant radiation at a modification of an angle of incidence of external wave. Reading the quantum information (after calculations) can be performed by means of weakly intensive probe radiation using solutions of equations system for dipole oscillators obtained in [12].

In the case $R \gg \lambda_0$

$$\gamma_s(R) = -\gamma_a(R) = \frac{3\gamma_0 c}{2\omega_0 R} \Phi'(1, 2) \sin\left(\frac{\omega_0 R}{c}\right). \quad (21)$$

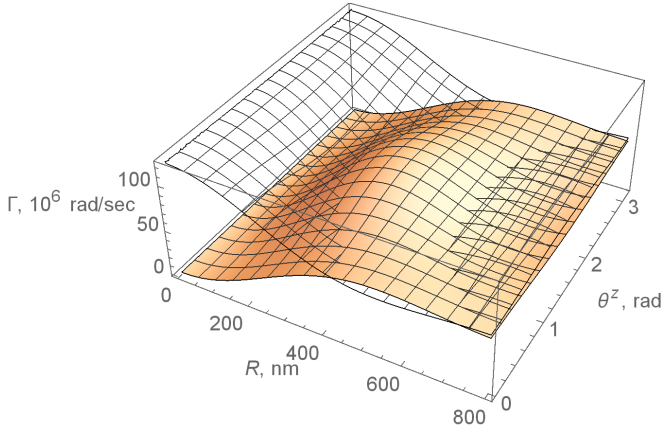


Figure 4: The dependencies of the widths $\Gamma_s(R, \theta^z)$ and $\Gamma_a(R, \theta^z)$ (transparent and opaque surface, respectively), calculated by means of (16), (17) and (18), on the interatomic distance R and angle $\theta^z = \theta_1^z = \theta_2^z$ of orientation of the dipole moments of the transition of the atoms that are parallel each to other ($\theta_1^x = \theta_2^x$, $\theta_1^y = \theta_2^y$)

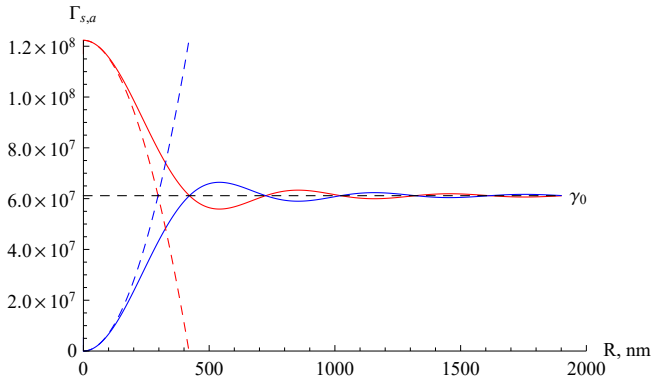


Figure 5: The dependencies of the widths Γ_s and Γ_a (solid lines), calculated by means of (16), (17) and (18), on the interatomic distance R when the dipole moments of the transition of the atoms are oriented along the axis Oz : $\theta^z = \theta_1^z = \theta_2^z = 0$. The dashed lines represent $\Gamma_s^{(||)}$ and $\Gamma_a^{(||)}$ calculated by means of the asymptotic formulas (22) and (24)

In the case when $R \ll \lambda_0$ ($\omega_0 R/c \ll 1$) and the dipole moments is parallel to the axis Oz , the expression (21) takes the form

$$\gamma_s^{(||)}(R) = -\gamma_a^{(||)}(R) = \gamma_0 \left(1 - \frac{\omega_0^2 R^2}{10c^2} \right), \quad (22)$$

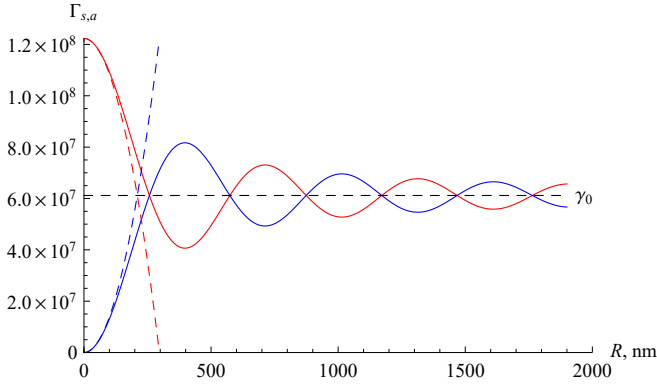


Figure 6: The dependencies of the widths Γ_s and Γ_a (solid lines), calculated by means of (16), (17) and (18), on the interatomic distance R when the dipole moments of the transition of the atoms are perpendicular to axis Oz : $\theta^z = \theta_1^z = \theta_2^z = \pi/2$. The dashed lines represent $\Gamma_s^{(\perp)}$ and $\Gamma_a^{(\perp)}$ calculated by means of the asymptotic formulas (23) and (29).

where $\gamma_s^{(\parallel)}(0) = -\gamma_a^{(\parallel)}(0) = \gamma_0$.

Similarly, in the case of the perpendicular direction of the dipole moments

$$\gamma_s^{(\perp)}(R) = -\gamma_a^{(\perp)}(R) = \gamma_0 \left(1 - \frac{\omega_0^2 R^2}{5c^2} \right). \quad (23)$$

Substituting the asymptotic expansions (22), (23) for $\gamma_s(R)$, $\gamma_a(R)$ into (16) and (17) we obtain for the total widths $\Gamma_{s(a)}^{(\parallel)}$ and $\Gamma_{s(a)}^{(\perp)}$ the following estimates

$$\Gamma_s^{(\parallel)} = \gamma_0 + \gamma_s^{(\parallel)} \approx 2\gamma_0, \quad \Gamma_a^{(\parallel)} = \gamma_0 + \gamma_a^{(\parallel)} \approx \frac{\gamma_0 \omega_0^2 R^2}{10 c^2}, \quad (24)$$

$$\Gamma_s^{(\perp)} = \gamma_0 + \gamma_s^{(\perp)} \approx 2\gamma_0, \quad \Gamma_a^{(\perp)} = \gamma_0 + \gamma_a^{(\perp)} \approx \frac{\gamma_0 \omega_0^2 R^2}{5 c^2}. \quad (25)$$

4 Quantum teleportation and resonant transmission of quantum information between two atom-qubits

Let us assume that at the initial point in time $t_1 = 0$ the system of two considered atoms is in the state $\Psi_m^{(0)} = \Psi_0(1)\Psi_0(2)$ (see (5)). Let us consider the quantum transition $\Psi_m^{(0)} \rightarrow \Psi_n^{(0)} = \Psi_{s(a)}$, where the wave functions $\Psi_{s(a)}$ are given by formulas (15). We define the transition energy for this case as

$$E_m^{(0)} - E_n^{(0)} = E_0 - E_n \mp \delta E_s = -\hbar(\omega + \Delta), \quad (26)$$

where the upper sign corresponds to the symmetric state Ψ_s of the atomic pair, while the lower sign refers to its antisymmetric state Ψ_a , ω is the frequency of a real photon, Δ is the resonance off-tuning. Let us assume that the quantum transition $\Psi_m^{(0)} \rightarrow \Psi_{s(a)}$ corresponds to the case of annihilation of a photon. Thus, we shall consider the transitions $\Psi_m^{(0)} \rightarrow \Psi_a$ and $\Psi_m^{(0)} \rightarrow \Psi_s$ separately from one another. For the symmetric channel $\Psi_m^{(0)} \rightarrow \Psi_s$ we have the following system of equations defining the probability amplitudes a_m and a_n under the condition $\Gamma_s t_1 \rightarrow 0$:

$$\left. \begin{aligned} i\hbar \frac{da_m}{dt_1} &= F_{mn} \exp[i(\omega_{mn} + \omega)t_1] a_n = F_{mn} \exp(i(\varepsilon_+ + i\gamma_n)t_1) a_n, \\ i\hbar \frac{da_n}{dt_1} &= F_{nm} \exp(-i(\varepsilon_+ + i\gamma_n)t_1) a_m, \end{aligned} \right\} \quad (27)$$

where $\varepsilon_+ = (\delta E_s - \hbar\Delta)/\hbar$; the matrix element of the transition is given by

$$F_{mn} = -\frac{i}{c} \sqrt{\frac{n\omega}{2}} \omega_0 \vec{A}_0 \vec{d}_{n0}^{\vec{e}ff}(R) \exp(i\vec{k}\vec{R}), \quad (28)$$

\vec{A}_0 is the amplitude of the vector potential, \vec{k} is the wave vector of the real photon which is absorbed at the location of the second atom with the radius vector \vec{R} . The matrix element \vec{d}_{n0} in (28) takes into account all terms of the function (5) of the initial state. The system of equations (27) corresponds to the two-level approximation for a system of atoms, such that the most significant contribution is from those terms of the Schrödinger wave equation in which the dependence on time is determined by a low frequency ($\omega_0 - \omega$). A similar system of equations will also take place for the antisymmetric channel if in (27) we replace ε_+ with ε_- , defined as $\varepsilon_- = -(\delta E_s + \hbar\Delta)/\hbar$. Let us make the substitutions below in Eqs. (27):

$$a_n \exp(i\varepsilon_+ t_1) = b_n \quad (29)$$

and eliminate a_m from the resulting equations. We then obtain the following equation:

$$\ddot{b}_n - i\varepsilon_+ \dot{b}_n + b_n |F_{mn}|^2 / \hbar^2 = 0, \quad (30)$$

with the coefficients being specified functions of the local time t_1 .

It is well known that the exchange by excitations between atoms through their resonance interaction is determined by the characteristic exchange time $\hbar/\delta E'_s$ [10]. If, at a certain point in time, a system consisting of two identical dipole atoms is in a state where one atom is excited, this excitation, as shown in [10], will be transferred to the other atom in a time $\tau' = \hbar/\delta E'_s$ through resonance interaction. With that, the time τ' required to transfer the excitation is considerably shorter than the lifetime $\Gamma_{s(a)}^{-1}$ of atoms in the symmetrical and (antisymmetrical) states, therefore, the energy $\delta E'_s$ (13) is independent of time. In this paper, we consider a situation where the time t_1 for which a system of two interacting atoms exists is also short compared to $\Gamma_{s(a)}^{-1}$. However, in contrast to [10], we take into account, besides the interaction of the atoms with one another, their interaction with the field of real photons.

Following [1], we obtain a wave function which corresponds to a symmetric channel for the interaction of a pair of atoms with the field of real photons:

$$\begin{aligned} \tilde{\Psi}_s(t_1) = & \frac{1}{\sqrt{2}} \exp\left(-\frac{\Gamma_s}{4}t_1 + i\frac{\varepsilon_+}{2}t_1\right) \left[\cos((\Omega_+ + i\beta_+)t_1) \right. \\ & - \frac{i(\varepsilon_+ + i\Gamma_s/2)}{2(\Omega_+ + i\beta_+)} \sin((\Omega_+ + i\beta_+)t_1) \left. \right] \tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle \\ & - \frac{iF_{nm}}{\hbar(\Omega_+ + i\beta_+)} \exp\left(-\frac{\Gamma_s}{4}t_1 - i\frac{\varepsilon_+}{2}t_1\right) \sin((\Omega_+ + i\beta_+)t_1) \\ & \times \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) + \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)] \exp(-i\delta E_s t_1/\hbar)|n_\omega - 1\rangle. \end{aligned} \quad (31)$$

For an antisymmetric interaction channel we replace ε_+ with ε_- and Ω_+ with Ω_- . Here we have

$$\varepsilon_\pm = (\pm\delta E_s - \hbar\Delta)/\hbar, \quad \Omega_\pm + i\beta_\pm = \left\{ \frac{|F_{nm}|^2}{\hbar^2} + \frac{(\varepsilon_\pm + i\Gamma_{s(a)}/2)^2}{4} \right\}^{1/2}. \quad (32)$$

The wave function of the final state of an atomic pair, $\tilde{\Psi}(t_1) = [\tilde{\Psi}_a(t_1) + \tilde{\Psi}_s(t_1)]/\sqrt{2}$, will then take the form

$$\tilde{\Psi}(t_1) = A_1\tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle + A_2\tilde{\varphi}_n(1)\tilde{\varphi}_0(2)|n_\omega - 1\rangle + A_3\tilde{\varphi}_0(1)\tilde{\varphi}_n(2)|n_\omega - 1\rangle, \quad (33)$$

where the probability amplitudes for possible states of the system are given by

$$\begin{aligned} A_1 = & \frac{1}{2} \left\{ \exp\left(-\frac{\Gamma_s}{4}t_1 + i\frac{\varepsilon_+}{2}t_1\right) \left[\cos((\Omega_+ + i\beta_+)t_1) - \frac{i(\varepsilon_+ + i\Gamma_s/2)}{2(\Omega_+ + i\beta_+)} \right. \right. \\ & \times \sin((\Omega_+ + i\beta_+)t_1) \left. \right] + \exp\left(-\frac{\Gamma_a}{4}t_1 + i\frac{\varepsilon_-}{2}t_1\right) \left[\cos((\Omega_- + i\beta_-)t_1) \right. \\ & \left. \left. - \frac{i(\varepsilon_- + i\Gamma_a/2)}{2(\Omega_- + i\beta_-)} \sin((\Omega_- + i\beta_-)t_1) \right] \right\}, \end{aligned} \quad (34)$$

$$\begin{aligned} A_2 = & -\frac{iF_{nm}}{2\hbar} \left[\frac{\sin((\Omega_+ + i\beta_+)t_1)}{\Omega_+ + i\beta_+} \exp\left(-\frac{\Gamma_s}{4}t_1 - i\frac{\varepsilon_+}{2}t_1\right) \exp(-i\delta E_s t_1/\hbar) \right. \\ & \left. + \frac{\sin((\Omega_- + i\beta_-)t_1)}{\Omega_- + i\beta_-} \exp\left(-\frac{\Gamma_a}{4}t_1 - i\frac{\varepsilon_-}{2}t_1\right) \exp(i\delta E_s t_1/\hbar) \right], \end{aligned} \quad (35)$$

$$\begin{aligned} A_3 = & \frac{iF_{nm}}{2\hbar} \left[-\frac{\sin((\Omega_+ + i\beta_+)t_1)}{\Omega_+ + i\beta_+} \exp\left(-\frac{\Gamma_s}{4}t_1 - i\frac{\varepsilon_+}{2}t_1\right) \exp(-i\delta E_s t_1/\hbar) \right. \\ & \left. + \frac{\sin((\Omega_- + i\beta_-)t_1)}{\Omega_- + i\beta_-} \exp\left(-\frac{\Gamma_a}{4}t_1 - i\frac{\varepsilon_-}{2}t_1\right) \exp(i\delta E_s t_1/\hbar) \right]. \end{aligned} \quad (36)$$

The probability amplitudes (13) are normalized by the condition $|A_1|^2 + |A_2|^2 + |A_3|^2 = 1$. As follows from (36), at $t_1 = 0$ a system of two atoms is in a state where both atoms have an energy E_0 , i.e., $A_1 = 1$, $A_2 = A_3 = 0$. Let us consider the behavior of the function $\tilde{\Psi}(t_1)$ at subsequent points in time.

5.1 True resonance

Large interatomic distances. When $\Delta = 0$, $\Gamma_s = \Gamma_a = \gamma_0$ and sufficiently large $R \gg \lambda_0$ value ε_{\pm} , Ω_{\pm} and β_{\pm} are as follows: $\varepsilon_+ = -\varepsilon_- = \delta E_s/\hbar$,

$$\Omega_{\pm} = \Omega = \frac{1}{\sqrt{2}} \left\{ \sqrt{\rho^2 + \frac{\gamma_0^2 (\delta E_s)^2}{16\hbar^2}} + \rho \right\}^{1/2}, \quad \rho = \frac{|F_{nm}|^2}{\hbar^2} + \frac{(\delta E_s)^2 - \hbar^2 \gamma_0^2/4}{4\hbar^2},$$

$$\beta_+ = -\beta_- = \beta = \frac{\text{sign } \delta E_s}{\sqrt{2}} \left\{ \sqrt{\rho^2 + \frac{\gamma_0^2 (\delta E_s)^2}{16\hbar^2}} - \rho \right\}^{1/2}.$$

The exact resonance ($\Delta = 0$) when the condition $R \gg \lambda_0$ amplitude (34)–(36) acquire the following values ($\tilde{\Omega}_{\pm} = \Omega \pm i\beta$):

$$A_1 = \frac{1}{2} \exp\left(-\frac{\gamma_0}{4} t_1\right) \left\{ \exp\left(\frac{i\delta E_s t_1}{2\hbar}\right) \left[\cos(\tilde{\Omega}_+ t_1) + \frac{\gamma_0/2 - i\delta E_s/\hbar}{2\tilde{\Omega}_+} \sin(\tilde{\Omega}_+ t_1) \right] \right. \\ \left. + \exp\left(-\frac{i\delta E_s t_1}{2\hbar}\right) \left[\cos(\tilde{\Omega}_- t_1) + \frac{\gamma_0/2 + i\delta E_s/\hbar}{2\tilde{\Omega}_-} \sin(\tilde{\Omega}_- t_1) \right] \right\}, \quad (37)$$

$$A_{2,3} = -\frac{iF_{nm}}{2\hbar} \exp\left(-\frac{\gamma_0}{4} t_1\right) \left[\frac{\sin(\tilde{\Omega}_+ t_1)}{\tilde{\Omega}_+} \exp\left(-\frac{3i\delta E_s t_1}{2\hbar}\right) \right. \\ \left. \pm \frac{\sin(\tilde{\Omega}_- t_1)}{\tilde{\Omega}_-} \exp\left(\frac{3i\delta E_s t_1}{2\hbar}\right) \right], \quad (38)$$

where the upper (lower) sign corresponds to A_2 (A_3). As it follows from (38), probability amplitude A_2 (A_3) reaches its maximum (minimum) value at time $t'_1 = \pi/2\Omega$.

Small interatomic distances. For the case of true resonance ($\Delta = 0$) for a time which is short compared to $\Gamma_{s(a)}^{-1}$, we obtain the following formulas for the probability amplitudes:

$$A_1 = \cos(\Omega_0 t_1) \cos(\delta E_s t_1/2\hbar) + \frac{\delta E_s}{2\hbar} \frac{\sin(\Omega_0 t_1)}{\Omega_0} \sin(\delta E_s t_1/2\hbar), \quad (39)$$

$$A_2 = -\frac{iF_{nm}}{\hbar} \frac{\sin(\Omega_0 t_1)}{\Omega_0} \cos(3\delta E_s t_1/2\hbar), \quad (40)$$

$$A_3 = -\frac{F_{nm}}{\hbar} \frac{\sin(\Omega_0 t_1)}{\Omega_0} \sin(3\delta E_s t_1/2\hbar), \quad (41)$$

where $\Omega_0 = \sqrt{[|F_{nm}|^2 + (\delta E_s)^2]/\hbar^2}$. The amplitude A_2 (A_3) reaches a maximum (minimum) at a time given by $t'_1 = 2\pi\hbar/3|\delta E_s|$.

5.2 Nonresonance absorption of a photon

Large interatomic distances. The expressions for the probability amplitudes (34)–(36) can be represented in the form:

$$A_1 = \exp\left(-\frac{\gamma_0}{4} t_1 - i\frac{\Delta}{2} t_1\right) \left[\cos((\Omega + i\beta)t_1) + \frac{(i\Delta + \gamma_0/2)}{2(\Omega + i\beta)} \sin((\Omega + i\beta)t_1) \right],$$

$$\begin{aligned}
A_2 &= -\frac{iF_{nm}}{\hbar(\Omega + i\beta)} \exp\left(-\frac{\gamma_0}{4}t_1 + i\frac{\Delta}{2}t_1\right) \sin((\Omega + i\beta)t_1), \quad A_3 = 0, \quad (42) \\
\Omega &= \frac{1}{\sqrt{2}} \left[\sqrt{\tilde{\rho}^2 + \frac{\gamma_0^2 \Delta^2}{16}} + \tilde{\rho} \right]^{1/2}, \quad \tilde{\rho} = \frac{|F_{nm}|^2}{\hbar^2} + \frac{\Delta^2 - \gamma_0^2/4}{4}, \\
\beta &= -\frac{\text{sign } \Delta}{\sqrt{2}} \left[\sqrt{\tilde{\rho}^2 + \frac{\gamma_0^2 \Delta^2}{16}} - \tilde{\rho} \right]^{1/2}.
\end{aligned}$$

The probability amplitude A_2 reaches its maximum value at time $t_1'' = \pi/2\Omega$.

Small interatomic distances. Let us consider the case where $\Delta \neq 0$, i.e., the frequency of the real photon does not coincide with the frequency of the transition $(E_m^{(0)} - E_n^{(0)})/\hbar$. Moreover, we assume that the condition $\Delta \gg \delta E_s/\hbar$ is fulfilled. For this case, according to (32), we have $\Omega_{\pm} = \Omega_0 = \sqrt{(\Delta^2/4) + |F_{mn}|^2/\hbar^2}$ and the amplitudes (36) become

$$\begin{aligned}
A_1 &= \exp\left(-i\frac{\Delta}{2}t_1\right) \left[\cos(\Omega_0 t_1) + \frac{i\Delta}{2\Omega_0} \sin(\Omega_0 t_1) \right], \\
A_2 &= -\frac{iF_{nm}}{\hbar\Omega_0} \exp\left(i\frac{\Delta}{2}t_1\right) \sin(\Omega_0 t_1), \quad A_3 = 0. \quad (43)
\end{aligned}$$

Thus, during nonresonance absorption of a photon two of the three possible states are realized. The state with the amplitude A_2 is realized as a result of the absorption of a photon at the position of the second atom (polarizer atom) and the transition of the first atom (observer atom) into the excited state. This state reaches a maximum at a time given by $t_1'' = \pi/2\Omega_0$.

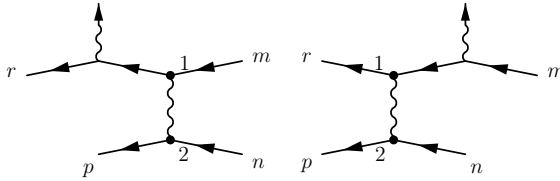


Figure 7: The Feynman diagrams of radiation interaction of two hydrogen-like atoms $A(1)$ and $A(2)$ with real photon absorption.

5.3 Resonance absorption of a photon.

Large interatomic distances

Given $\Delta = 0$ and $\delta E_s = 0$, the amplitudes (36) become

$$\begin{aligned}
A_1 &= \exp\left(-\frac{\gamma_0}{4}t_1\right) \left[\cos(\Omega t_1) + \frac{\gamma_0}{4\Omega} \sin(\Omega t_1) \right], \\
A_2 &= -\frac{iF_{nm}}{\hbar\Omega} \sin(\Omega t_1) \exp\left(-\frac{\gamma_0}{4}t_1\right), \quad A_3 = 0. \quad (44)
\end{aligned}$$

At the initial point in time $t_1 = 0$, as follows from (44), both atoms are in the ground state. However, even at an infinitely close subsequent point in time, the state $\tilde{\Phi}_n(1)\tilde{\Phi}_0(2)$ from the superposition (33) starts being realized. At the time $t_1''' = \pi\hbar/2|F_{nm}|$ the probabilities become $|A_2|^2 = 1$ and $|A_1|^2 = 0$, i.e., one quantum bit of information is transferred from one atom to another at an infinitely large interatomic distance in the time t_1''' . We refer to this effect as the quantum teleportation in a system of two resonance atoms. This effect is of quantum in nature, being related to the superposition principle of quantum mechanics, and corresponds to the quantum means of information transfer whose characteristic time is defined as $t_1''' = \hbar/|F_{nm}|$, in contrast to the classical means of information transfer with the characteristic time t_p being the time of flight of a photon through a distance R . In accordance with the sense of the obtained solution to Eq. (30), the condition $\Gamma_{s(a)}t_1''' \rightarrow 0$ should be fulfilled. This condition is fulfilled the more precisely, the greater the amplitude of the vector potential in (28). Using the notion of the Rabi frequency [14], we obtain $t_1''' = \hbar/\sqrt{2}|\vec{d}_{n0}|E_0$, where E_0 is the amplitude of the electric field of the light wave acting on the second atom.

Thus, say we have been able to create entangled states (6), (7) for two widely spaced atoms. Bouweester et al. [5] could do this using one photon source which distributes in a random manner photons randomly polarized in two different directions. Krenn and Zeilinger [6] used for this purpose an electron source. In our case, one possible way of creating entangled states may be the use of two beams of double-level atoms irradiated in a random manner with a light field.

In accordance with the sense of the obtained solution for a system of two resonance atoms in the field of actual photons, we represent the operation of the information system as follows: Let us assume that the system operation begins at a time $t_2 - R/c$ at the location of the second, polarizer atom when this atom is irradiated with the field of a light wave with a frequency $\omega = \omega_0$. At a local point in time, $t_1 = 0$, both atoms are in the ground state with an energy E_0 , and at a point in time t_1''' the observer atom 1 goes into an excited state with an energy $E_n = E_0 + \hbar\omega$. As this takes place, the atom 2 remains in the former state with the energy E_0 , and the average energy of the system, calculated with the help of the wave function (33), will be determined by the formula

$$\langle E \rangle = |A_1|^2(2E_0 + \hbar\omega n_\omega) + |A_2|^2(E_0 + E_n + \hbar\omega(n_\omega - 1)). \quad (45)$$

Here angle brackets denote the operation of averaging. The characteristic time of information transmission through quantum correlations is given by the expression

$$t_1''' = \left(\frac{|\vec{d}_{n0}|^2 \mathcal{E}_0^2}{\hbar^2} - \frac{\gamma_0^2}{16} \right)^{-1/2} \quad (46)$$

and do not depend on interatomic distances.

For an interatomic distance $R = 30$ km, the time it takes for a photon to cover this distance with the classical means of information transfer will be $t_p = 10^{-4}$ s. The time of information transfer due to the effect of quantum teleportation is given by $t_1''' = \hbar/\sqrt{2}|d_{n0}|E_0$. For $d_{n0} = 4.8 \times 10^{-21}$ units of the cgs electrostatic

system and $E_0 = 10^{-2}$ units of the cgs electrostatic system, we obtain $t_1''' = 1.5 \times 10^{-5}$ s. Thus, the velocity of the information transfer due to the effect of quantum teleportation, $v_{QT} = R/t_1'''$, is substantially higher than the velocity of light. Nevertheless, this does not imply any contradiction with the relativity principle, since $v_{QT} = R/t_1'''$ is a phase velocity.

In the course of quantum computing the qubits of the computer are exposed to the relaxation processes, which hamper quantum computing seriously: started computer process acquires the features of a random one during the time of decoherence. To increase this time one should use the antisymmetric state $\Psi_{s(a)}$ of pair of atoms, which is insensitive to the relaxation processes. Through only this undecaying channel the process of transfer of quantum information from a single two-level atom to another is realised. The existence of such a communication channel between a pair of closely spaced atoms with a field of real photons allows to realize approximately 10^{14} unitary transformations, if we arrange the atoms at distance $R = 1$ nm and use short optical pulses of a few femtoseconds.

References

- [1] K.A. Valiev, *Usp. Fiz. Nauk* **175**, 3 (2005).
- [2] O.N. Gadomsky and K.K. Altunin, *Russian Physics Journal* **43**, 893 (2000).
- [3] O.N. Gadomsky and Yu.Yu. Voronov, *Zh. Eksp. Teor. Fiz.* **121**, 1028 (2002).
- [4] O.N. Gadomsky and Yu.Ya. Kharitonov, *Quantum Electronics* **34**, 249 (2004).
- [5] D. Bouweester, J-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, *Nature* **390**, 575 (1997).
- [6] G. Krein and A. Zeilinger, *Phys. Rev. A* **54**, 1793 (1996).
- [7] L. Davidovich, N. Zaguzy, M. Brune, et al., *Phys. Rev. A* **50**, R895 (1994).
- [8] V.Yu. Lazur, S.I. Myhalyna, and O.K. Reity, *Phys. Rev. A* **81**, 062707 (2010).
- [9] V.Yu. Lazur, O.F. Pavlyk, and O.K. Reity, *Teor. Mat. Fiz.* **165**, 70 (2010).
- [10] A.S. Davydov, *Quantum Mechanics [in Russian]*, Fizmatgiz, Moscow (1963).
- [11] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Nonrelativistic Theory [in Russian]* (Course of Theoretical Physics, Vol. 3), Nauka, Moscow (1989); English transl. of prev. ed., Oxford Univ. Press, Oxford (1975).
- [12] O.N. Gadomsky, *Usp. Fiz. Nauk* **170**, 1145 (2000).
- [13] O.N. Gadomsky and Yu.Yu. Voronov, *Pisma Zh. Eksp. Teor. Fiz.*, **69**, 750 (1999).
- [14] I.I. Sobelman, *Atomic Spectra and Radiative Transitions*, Springer-Verlag, Berlin, Heidelberg (1992).