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ON PROPERTIES OF THE HASSE DIAGRAM OF NONSERIAL POSETS WITH POSITIVE QUADRATIC TITS FORM

Posets with positive quadratic Tits form (classified by the authors earlier) can be serial and nonserial. Serial poset is a such one, for which there is an overposet with positive Tits form of the order to be equal to an arbitrary predetermined natural number (exceeded the order of the initial poset). Otherwise it is called nonserial. In this paper we consider the Hasse diagrams of nonserial posets and study their combinatorial properties.

Частково впорядковані множини з додатною квадратичною формою Тітса (які описані авторами раніше) бувають серійними і несерійними. Серійною називається така множина, для якої існує частково впорядкована надмножина з додатною формою Тітса, порядок якої дорівнює довільному наперед заданому натуральному числу (більшому за порядок початкової множини). В іншому разі множина називається несерійною. У цій статті ми розглядаємо діаграми Хассе несерійних множин і вивчаємо їх комбінаторні властивості.

1. Introduction. The Hasse diagram $H(S)$ of a finite poset S is a type of diagram that represents S in the plane. Namely, one represents each element of S as a vertex and each pair of elements x, y of S, such that y covers x (i. e. $x < y$ and there is no z satisfying $x < z < y$), as an edge that goes upward from x to y. For a class of finite posets $\mathcal{X}$ we denote by $\mathrm{VA}(\mathcal{X})$ the set of pairs (s, k) of non-negative integer numbers with s and k being respectively the number of vertices and edges of $H(X)$ for X running $\mathcal{X}$.

In this paper we study properties of the Hasse diagram of some class of posets connected with the quadratic Tits form. All posets are assumed to be finite.

Let S be a poset without an element denoted by 0. The Tits quadratic form of S is by definition the form $q_S : \mathbb{Z}^{S\cup 0} \to \mathbb{Z}$ defined by the equality

$$q_S(z) = z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j,\, i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i.$$

Posets with positive quadratic Tits form can be divided into two classes: serial and nonserial, or, for greater certainty, P-serial and P-nonserial [1]. A poset S is called serial if for any $N > |S|$ there exists a poset T_N of order N with positive quadratic Tits form, which contains S, and nonserial otherwise. The set of all P-nonserial posets will be denoted by $\mathcal{P}_{ns}$.

The aim of this paper is to prove the following theorem.

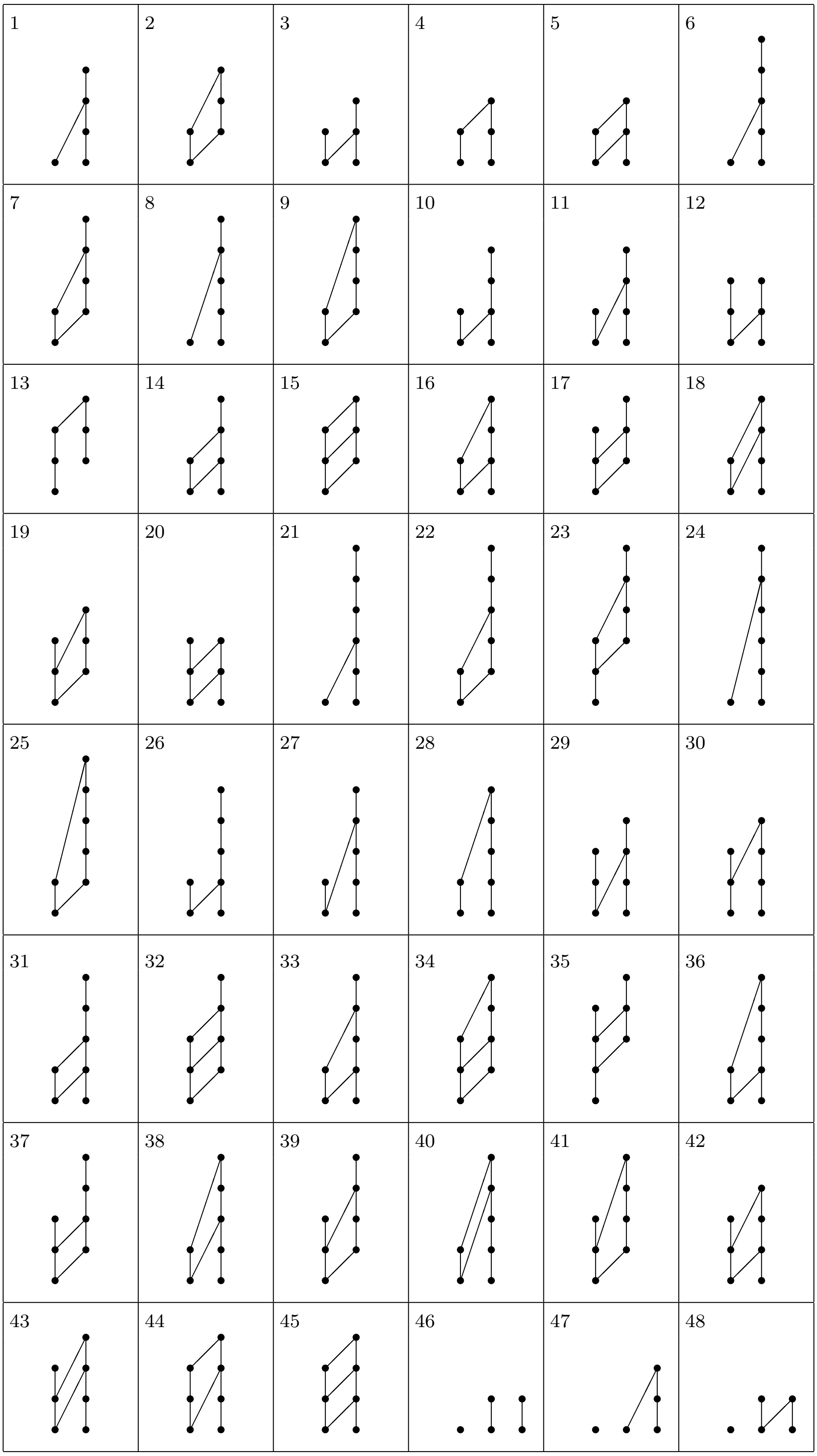
Theorem 1. *$VA(\mathcal{P}_{ns})$ consists of the following pairs:*
$(5,2)$, $(5,3)$, $(5,4)$, $(5,5)$,
$(6,3)$, $(6,4)$, $(6,5)$, $(6,6)$, $(6,7)$,
$(7,4)$, $(7,5)$, $(7,6)$, $(7,7)$, $(7,8)$.

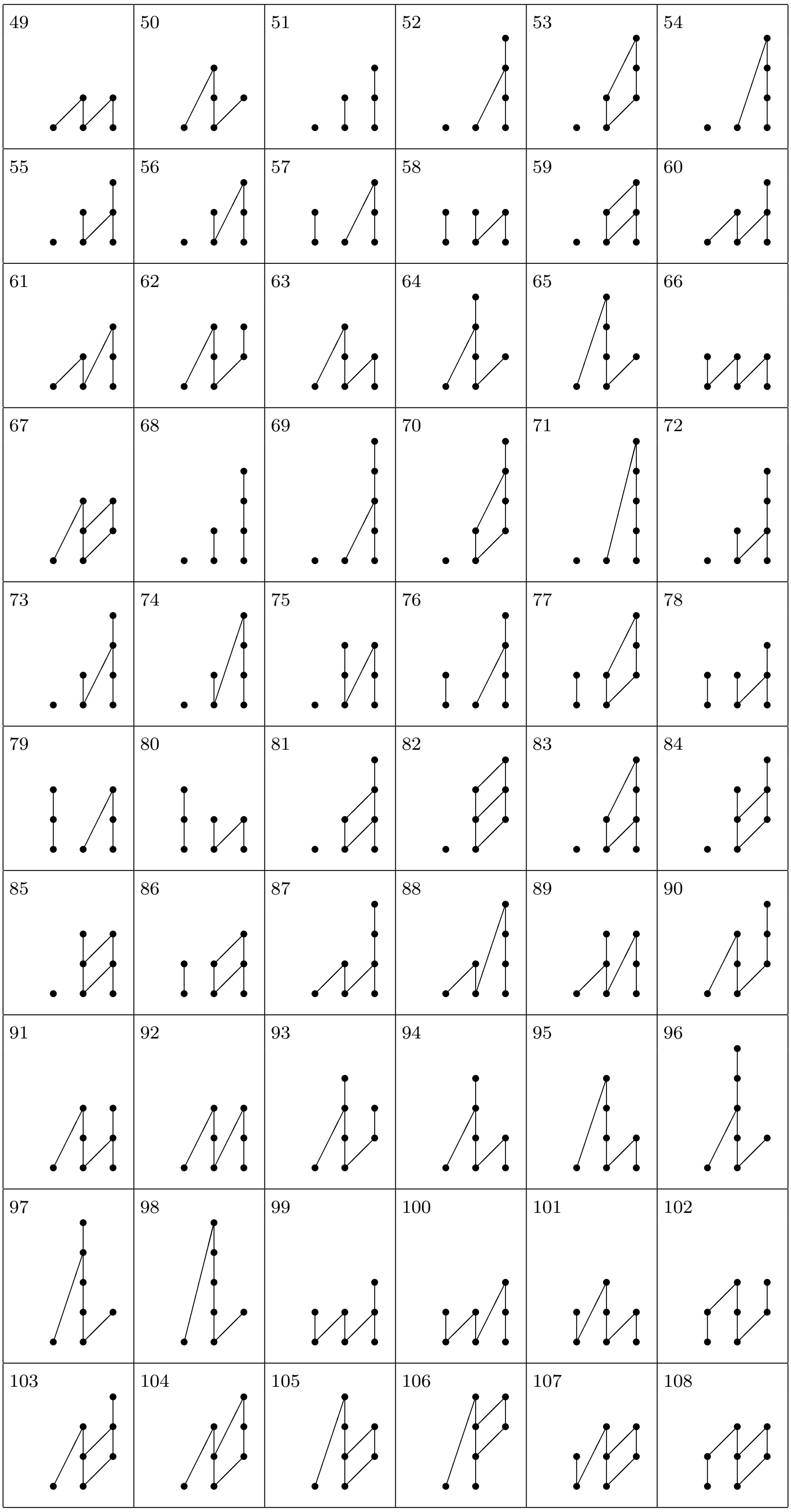
From this theorem we have the following corollaries.

Corollary 1. *If $(s,i), (s,j) \in VA(\mathcal{P}_{ns})$ and $i < k < j$, then $(s,k) \in VA(\mathcal{P}_{ns})$.*

Corollary 2. *If $(s,k) \in VA(\mathcal{P}_{ns})$, then $k \le s+1$.*

2. *P*-nonserial posets. The P-nonserial posets were classified in [1]. They are given (up to isomorphism and anti-isomorphism) by the following table.




49 50 51 52 53 54
55 56 57 58 59 60
61 62 63 64 65 66
67 68 69 70 71 72
73 74 75 76 77 78
79 80 81 82 83 84
85 86 87 88 89 90
91 92 93 94 95 96
97 98 99 100 101 102
103 104 105 106 107 108

3. Proof of Theorem 1. The theorem follows from the following table in which one indicates the P-noncritical posets numbers, their numbers of vertices and edges.

№	vertices	edges	№	vertices	edges	№	vertices	edges
1	5	4	37	7	7	73	7	5
2	5	5	38	7	7	74	7	5
3	5	4	39	7	7	75	7	5
4	5	4	40	7	7	76	7	5
5	5	5	41	7	7	77	7	6
6	6	5	42	7	7	78	7	5
7	6	6	43	7	7	79	7	5
8	6	5	44	7	7	80	7	5
9	6	6	45	7	8	81	7	6
10	6	5	46	5	2	82	7	7
11	6	5	47	5	3	83	7	6
12	6	5	48	5	3	84	7	6
13	6	5	49	5	4	85	7	6
14	6	6	50	5	4	86	7	6
15	6	7	51	6	3	87	7	6
16	6	6	52	6	4	88	7	6
17	6	6	53	6	5	89	7	6
18	6	6	54	6	4	90	7	6
19	6	6	55	6	4	91	7	6
20	6	6	56	6	4	92	7	6
21	7	6	57	6	4	93	7	6
22	7	7	58	6	4	94	7	6
23	7	7	59	6	5	95	7	6
24	7	6	60	6	5	96	7	6
25	7	7	61	6	5	97	7	6
26	7	6	62	6	5	98	7	6
27	7	6	63	6	5	99	7	6
28	7	6	64	6	5	100	7	6
29	7	6	65	6	5	101	7	6
30	7	6	66	6	5	102	7	6
31	7	7	67	6	6	103	7	7
32	7	8	68	7	4	104	7	7
33	7	7	69	7	5	105	7	7
34	7	8	70	7	6	106	7	7
35	7	7	71	7	5	107	7	7
36	7	7	72	7	5	108	7	7

References

1. Bondarenko V. M., Stepochkina M. V. (Min, max)-equivalence of partially ordered sets and the Tits quadratic form // Problems of Analysis and Algebra: Institute of Mathematics of Ukrainian NAS, Kiev. – 2005. – **2**, N3. – P. 18-58 (in Russian).

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