On indecomposable objects of the category of monomial matrices over a commutative local ring

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Let K be a commutative local ring with radical $R \neq 0$ and let $\mathrm{Mmat}\,(K)$ denote the category of monomial matrices over K, i.e. the category with the objects to be the monomial matrices and the morphisms $A \to B$ to be the matrices X such that $AX = XB$ (by a monomial matrix we mean a quadratic matrix, in each row and each column of which there is exactly one non-zero element).

To each monomial $n \times n$ matrix $M = (m_{ij})$ over K there corresponds the directed graph $\Gamma(M)$ with n vertices numbered from 1 to n and arrows $i \to j$ for all $m_{ij} \neq 0$. Obviously, $\Gamma(M)$ is the disjoint union of $s > 0$ cycles, each of which has the same direction of arrows. If $s = 1$, the matrix M is called cyclic. A cyclic matrix of the form

$$M = \begin{pmatrix} 0 & \dots & 0 & m_{1n} \\ m_{21} & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & m_{n,n-1} & 0 \end{pmatrix}$$

is called canonical cyclic. The sequence $v = v(M) = (m_{21}, \dots, m_{n-1,n}, m_{1n})$ is called the defining sequence of M. If all elements m_{ij} have the form $t^{s_{ij}}$ ($t \in K$), where $s_{ij} \geq 0$, the matrix M is called canonical t-cyclic; obviously, then $t^{s_i} \neq 0$ for all i.

Matrices of the above form were studied in many works (see, e.g., [1]–[4]). Note that in details all the above definitions are discussed in [4].

We assume that $t \in R$. The sequence $v(M)$ is called periodic with a period $0 < p < n$ if $p|n$ and $x_{s+p} = x_s$ for any $1 \leq s \leq n - p$, and non-periodic if otherwise.

Theorem 1. *Any canonical t-cyclic matrix over K with non-periodic defining sequence is an indecomposable object of the category* $\mathrm{Mmat}\,(K)$.

The results are obtained together with V. M. Bondarenko.

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