UDC 004.85

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Learning of Bithreshold Neuron is NP-Complete

Introduction. Neural network technologies are widely used in computational intelligence and have numerous application (see [1, 2]). The design of fast learning algorithms is very actual task [1]. The one of the most important question in the learning theory is the question of the complexity of the learning. The general notions and definition of the algorithm hardness can be found in [3]. Anthony [4] considered in detail the hardness of learning the Boolean functions. Blum and Rivest [5] proved that training a 3-node neural networks is NP-complete task. We demonstrate that the same is true for the learning of bithreshold neurons. Thus, if P ≠ NP conjecture is true that there is no polynomial time algorithm for learning bithreshold neurons.

The bithreshold neuron is the computation unit having *n* inputs $x_1,\ldots,x_n$ and one output *y* [6]. This unit is capable of taking on a number of states, each described by a vector $\mathbf{w}=(w_1,\ldots,w_n)\in \mathrm{R}^n$ which is called as weight vector and two additional parameters $t_1, t_2\,(t_1<t_2)$ known as threshold. The output of the bithreshold neuron is defined by following equation

$$y=\begin{cases}0, & \text{if } t_1<(\mathbf{w},\mathbf{x})<t_2,\\ 1, & \text{otherwise,}\end{cases}$$

where $(\mathbf{w}, \mathbf{x})$ is the inner product of the vectors **w** and **x**.

It is evident that the bithreshold activation function is generalization of Heviside step functions [2]. The triplet $(\mathbf{w},t_1,t_2)$ is called the structure of the bithreshold neuron.

Two subset A^+ and A^- of the R^n space are bithreshold separable if there exists such bithreshold neuron of the structure $(\mathbf{w},t_1,t_2)$ that

$$\forall \mathbf{x}\in A^-\ t_1<(\mathbf{w},\mathbf{x})<t_2$$

and

$$\forall \mathbf{x}\in A^+\ (\mathbf{w},\mathbf{x})\le t_1 \text{ or } (\mathbf{w},\mathbf{x})\ge t_2\ .$$

In this case the partition (A^+,A^-) is "bithreshold" and the corresponding bithreshold neuron compute it (it should be noted that in the last definition the order of sets A^+ and A^- is important).

A Boolean function $f:\mathrm{Z}_2^n\to \mathrm{Z}_2$ is a Boolean bithreshold function if it is computable by a bithreshold threshold unit. This means that two sets $\{\mathbf{x}\in \mathrm{Z}_2^n \mid f(\mathbf{x})=1\}$ and $\{\mathbf{x}\in \mathrm{Z}_2^n \mid f(\mathbf{x})=0\}$ are bithreshold separable [7].

We study the problem of learning the bithreshold neuron as the task of changing weights and threshold in response to some training examples. The goal of learning is the search of the structure of bithreshold neuron which can compute the desirable partition $\left(A^{+}, A^{-}\right)$. Now we can present our main results.

Theorem 1. *Let f be a Boolean function defined by its disjunctive normal form formula. Then the task of the learning bithreshold neurons realizing function f is NP-complete.*

Theorem 2. *The task of verifying the bithreshold separability of two finite set* A^{+} *i* A^{-} *is NP-complete even if* $A^{+} \cup A^{-} \subset \{a, b\}^{n}$, *where* $a \in \mathrm{R}, b \in \mathrm{R}$ $(a \neq b)$ *and the absolute values of the neuron weights can have only two different values.*

Conclusions. In the contrast to the learning of ordinary threshold neuron even different "weak" forms of the task of the learning one bithreshold neuron are NP-hard.

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