TARAS SHEVCHENKO NATIONAL UNIVERSITY OF KYIV

INTERNATIONAL CONFERENCE

PROBABILITY, RELIABILITY AND STOCHASTIC OPTIMIZATION

APRIL 7–10, 2015, KYIV, UKRAINE

CONFERENCE MATERIALS

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PROBABILITY, RELIABILITY AND STOCHASTIC OPTIMIZATION

Dedicated to the 90th anniversary of V. S. Koroliuk,
80th anniversary of I. M. Kovalenko,
75th anniversary of P. S. Knopov and
75th anniversary of Yu. V. Kozachenko

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$x_n := x(\tau_n)$ [1,2] with semi-Markov kernel

$$G(u, dv, t; x) := G(u, dv; x)F_u(t), u \in R_d, dv \in \mathcal{R}^d, t \geq 0, x \in E,$$
$$G(u, dv; x) := \mathbb{P}\{\Delta\eta_{n+1} \in dv | \eta_n = u, x_n = x\}, \Delta\eta_{n+1} := \eta_{n+1} - \eta_n,$$
$$F_u(t) := \mathbb{P}\{\theta_{n+1} \leq t | \eta_n = u\} = P(\theta_u \leq t), t \geq 0;$$

$\gamma(t; x), t \geq 0, x \in E$ is a diffusion process with generator

$$\Gamma(x)\varphi(u) := a(u; x)\varphi'(u) + \int_{R^d} (\varphi(u+v) - \varphi(u) - v\varphi'(u))\Gamma(u, dv; x).$$

For semi-Markov random evolution consider two problems: weak convergence in the average scheme and in the diffusion approximation scheme

$$\xi^\varepsilon(t) = \xi(0) + \varepsilon^3[\int_0^{t/\varepsilon^3} \eta(ds; x(\varepsilon s)) + \int_0^{t/\varepsilon^3} \gamma(ds; x(\varepsilon s))] \quad (1)$$

$$\zeta^\varepsilon(t) = \xi(0) + \varepsilon^3[\int_0^{t/\varepsilon^6} \eta(ds; x(\varepsilon^2 s)) + \int_0^{t/\varepsilon^6} \gamma(ds; x(\varepsilon^2 s))]. \quad (2)$$

For processes (1) and (2) we found limiting processes for $\varepsilon \downarrow 0$.

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RENEWAL APPROXIMATION FOR THE ABSORPTION TIME IN NONINCREASING MARKOV CHAINS

Alexander Marynych

Let $\{\mathcal{M}_k : k \in \mathbb{N} \cup \{0\}\}$ be a Markov chain with state space $\mathbb{N}$. Assume that $\mathcal{M}_0 = n$ and $\{\mathcal{M}_k\}$ is eventually nonincreasing, i.e. there exists $a \in \mathbb{N}$ such that $\mathbb{P}\{\mathcal{M}_{k+1} \leq \mathcal{M}_k | \mathcal{M}_k > a\} = 1$ and $\mathbb{P}\{\mathcal{M}_k = \mathcal{M}_{k+1} | \mathcal{M}_k > a\} < 1$, $k \in \mathbb{N} \cup \{0\}$. Denote by I_n the first decrement of $\{\mathcal{M}_k\}$, namely $I_n := n - \mathcal{M}_1$.

We are interested in the asymptotic behaviour of the stopping time $X_n := \inf\{k \in \mathbb{N} \cup \{0\} : \mathcal{M}_k \leq a \text{ given } \mathcal{M}_0 = n\}$ as $n \to \infty$. In this talk we provide general results concerning the weak convergence of X_n if either one of the following stationarity conditions holds true: there exists proper and non-degenerate random variable ξ such that

$$I_n \xrightarrow{d} \xi \quad \text{as} \quad n \to \infty, \quad (1)$$

or there exists non-degenerate random variable η such that

$$\frac{I_n}{n} \xrightarrow{d} 1 - \eta \quad \text{as} \quad n \to \infty. \quad (2)$$

Our approach is based on the obseravtion that in both cases (1) and (2) the sequence $\{X_n\}$ can be approximated by a suitable renewal counting process and the error of such an approximation is estimated in terms of an appropriate probability distance.

This method has been used in [1] to derive a weak convergence result for the number of collisions in beta-coalescents and in [2] to prove a central limit theorem for the number of zero increments in the random walk with a barrier.

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RELIABILITY AND ACCURACY OF CALCULATION OF INTEGRALS DEPENDENT ON PARAMETER USING MONTE CARLO METHOD

Yu. Yu. Mlavets

We investigate how the Monte Carlo method can be used for calculation of integrals

$$I(t) = \int_{R^d} \cdots \int f(t, \vec{x}) p(\vec{x}) d\vec{x}, \ t \in \mathbf{T}.$$

Here $\int_{R^d} \cdots \int p(\vec{x}) d\vec{x} = 1$ and $p(\vec{x})$ is a probability density function of some random vector. There were found the conditions, under which these integrals can be calculated with given reliability and accuracy in the space $\mathbf{C}(\mathbf{T})$.

In order to obtain these results the methods of random processes from the space $\mathbf{F}_\psi(\Omega)$ have been used (see [1]). Space $\mathbf{F}_\psi(\Omega)$ consists of such random variables ξ that $\sup\limits_{u\geq 1} \frac{(E|\xi|^u)^{1/u}}{\psi(u)} < \infty$, where $\psi(u) > 0$, $u \geq 1$ is the monotonically increasing and continuous function for which $\psi(u) \to \infty$ as $u \to \infty$.

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ANALOGUE OF THE BERRY-ESSEEN THEOREM FOR FUNCTIONALS OF WEAKLY ERGODIC MARKOV PROCESS

G. M. Molyboga

Bounds for the rate of convergence in the central limit theorem for functionals of weakly ergodic Markov processes are obtained. The method of proof generalizes the one proposed in [1,2], aimed to prove diffusion approximation type theorems for systems with weakly ergodic Markov perturbations.

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THE INTEGRAL EQUATION FOR THE FOURIER-STIELTJES TRANSFORM OF THE SEMI-MARKOV RANDOM WALK WITH POSITIVE TENDENCY AND NEGATIVE JUMPS

T. H. Nasirova[1], S. Tunc Yavuz[2], U. C. Idrisova[3]

Let the sequence of i.i.d. random variables $\{\xi_k, \zeta_k\}_{k=\overline{1,\infty}}$ be defined on the probability space $(\Omega, F, P(\cdot))$. Random variables ξ_k and ζ_k are mutually independent. We construct the process

$$X(t) = z + t - \sum_{i=1}^{k} \zeta_i, \quad \text{if} \quad \sum_{i=0}^{k} \xi_i \leq t < \sum_{i=0}^{k+1} \xi_i, \quad k = 0, 1, 2, ...,$$

using random variables ξ_k and ζ_k. This process is called the semi-Markov random walk with positive tendency and negative jumps. Let

$$R(t, x\,|z) = P\{X(t) < x\,|X(0) = z\}, \quad x \in R,$$

$$\tilde{R}(\theta, x\,|z) = \int_{t=0}^{\infty} e^{-\theta t} R(t, x\,|z), \quad \theta > 0,$$

$$\tilde{\tilde{R}}(\theta, \gamma\,|z) = \int_{x=-\infty}^{\infty} e^{i\gamma x} d_x \tilde{R}(\theta, x\,|z), \quad \gamma \in R.$$

Theorem 1. *If $\{\xi_k, \zeta_k\}_{k=\overline{1,\infty}}$ is the sequence of i.i.d. positive random variables, then*

$$\tilde{\tilde{R}}(\theta, \gamma\,|z) = \int_{x=-\infty}^{\infty} e^{i\gamma x} d_y \tilde{R}(\theta, x\,|z), \quad \gamma \in R,$$

satisfies the following integral equation

$$\tilde{R}(\theta, x\,|z) = \varepsilon(x - z) \int_{t=0}^{x-z} e^{-\theta t} P\{\xi_1 > t\}\, dt -$$

$$- \int_{y=-\infty}^{z} \tilde{R}(\theta, x\,|y) \int_{t=0}^{\infty} e^{-\theta t} d_y P\{\zeta_1 < z + t - y\}\, dP\{\xi_1 < t\} -$$

$$- \int_{y=z}^{\infty} \tilde{R}(\theta, x\,|y) \int_{t=y-z}^{\infty} e^{-\theta t} d_y P\{\zeta_1 < z + t - y\}\, dP\{\xi_1 < t\}.$$

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