THE CURVATURE IN HIGH ENERGY pp AND $\bar{p}p$ ELASTIC SCATTERING

J.E. Kontros, K.J. Kontros, A.I. Lengyel

Institute of Electron Physics, Universitetska St. 21, Uzhgorod, 88016, Ukraine

E-mail: iep@iep.uzhgorod.ua

The values of the slope parameter $B(s,t)$, curvature $C(s,t)$ and $\rho(s,0)$ have been calculated within the wide s- and t-ranges with the allowance made for the diffraction cone curvature. The calculated t-dependence of $B(t)$ is tested via the local slopes obtained by the overlapping bins procedure. The slope parameter $B(s,t=0)$ increases as $\ln s$ while the curvature parameter $C(s,0)$ is predicted to be decreasing depending on s. By analysing the t-dependence of partial $\rho(t)$ the reduced value of $\rho(0)=0.189$ has been obtained for UA4 data.

Introduction

In the elastic hadron's scattering the long distances appear to be essential and, therefore, the perturbative methods are thus far unusable here. In this connection the knowledge of such fundamental parameters of the elastic scattering as the total cross section

$$\sigma_{tot}(s) = 4\pi \operatorname{Im} T(s,0), \tag{1}$$

the slope

$$B(s,t) = \frac{d}{dt}\ln\left(\frac{d\sigma(s,t)}{dt}\right), \tag{2}$$

the curvature

$$C(s,t) = \frac{1}{2}\frac{d}{dt}B(s,t), \tag{3}$$

and the phase of the Coulomb and nuclear forward amplitudes

$$\rho(s,0) = \frac{\operatorname{Re} T(s,0)}{\operatorname{Im} T(s,0)} \tag{4}$$

is decisive in further development of the Pomeron theories. Recently measured unexpected values of these parameters [1,2] have initiated the revision of the methods of their determination [3-7]. This concerns primarily the first measurement of the ρ value in the *UA4* experiment at $\sqrt{s} = 546 GeV$ [1], where the value of $\rho=0.24$ was obtained contradicting both the predictions of the most of models and experimental data for adjacent energy points. The second experiment, *UA4/2*, ($\sqrt{s} = 541 GeV$ [8]) gave $\rho=0.135$ which agrees completely both with most of theories and experiment. However, the problem of disagreement of these two results remains open.

Other new interesting phenomenon lies in that the $\bar{p}p$-scattering curvature vanishes at Tevatron energies $\sqrt{s} = 1.8\, TeV$ [2], whereas it has been considered as the condition of the transition to the "asymptopia" [9]. The total cross sections measured in various series of the Tevatron experiments also contradict each other (72.8 ± 3.1 [10] and 80.03 ± 2.24 [11]). In this case the first value agrees with the $\ln s$-physics, while the second one indicates that the "asymptopia" (i.e. the Froissart boundary) is attained. Taking into account that at high energies the values of σ_{tot}, B and ρ were determined in the same ISR experiments and keeping in mind mentioned above, the development of the up to date criteria of the unified approach for their determination seems to be natural. It has been shown in [3] that, starting with the available experimental data, we are up against a considerable uncertainty in the determination of σ_{tot}, B and ρ and the further experiments at lower momentum transfer values appear to be necessary. A modified method of determination of the slope, curvature and phase on the basis of the experimental data has been suggested in [4]. A critical analysis of the data at $\sqrt{s} = 19.4$, 546, 1800 GeV has been performed allowing one to find that the earlier values of the slope B are systematically underestimated, whereas the aforementioned ρ from the UA4 experiment takes the value of $\rho=0.24\pm0.04$. We have also used in [5] (as well as in [4]) the t-domain, which goes beyond the region of the interference of the Coulomb and nuclear amplitudes. Furthermore, we have applied the physical threshold properties of the scattering amplitude expressed in the nonlinear pomeron trajectory [12] and have obtained: $\rho=0.185\pm0.028$ [5]. In this relation the idea arises to revise available elastic $\bar{p}p$ and pp scattering data for high energies within a wide interval of t starting from the method suggested in [5]. Due to the unified approach in calculating the aforementioned characteristics B, C and ρ we shall predict their energy behaviour. This would be very desirable in view of the future experimental projects PP2PP [13] at RHIC and TOTEM [14] at LHC.

For this purposes we should find the scattering amplitude model, which reflects adequately the fine structure of diffraction cone peculiarities. Some models able to account for the curvature of the cone were compared in [4] by the best fit of the differential cross-section. However, the results did not differ appreciably in χ^2. It is hard to estimate blindly the quality of the diffraction cone curvature model. However, with the help of the overlapping bins method (OBM) procedure [6,14] one may construct the "experimental" slopes set which properly account for the mentioned peculiarities.

In this paper, we shall analyze the available data on the $\bar{p}p$- and pp-scattering with the help of the OBM and choose the model describing the observable curvature of the diffraction cone. Next, we shall study the t-behaviour of B and C within the framework of the phenomenological model which considers naturally the curvature as the revelation of the threshold behaviour of the scattering amplitude in the t-channel in the first cone region and at all available energies ($\sqrt{s} > 10 GeV$) and, thus, recalculate the $\rho(s,0)$, $B(s,0)$ and $C(s,0)$ values.

Finally, we shall try to elucidate the causes of the discrepancy in measured ρ values for UA4 [1] and UA4/2 [8] experiments and develop the recommendations for measuring this value keeping in mind the future experiments.

The overlapping bins procedure for obtaining the local slope

To check the expected behaviour of the slope (and curvature) over t we shall operate with its "experimental" value. This value can be obtained by the model-independent method in a form of an array of local b_i slopes defined as:

$$\left(\frac{d\sigma}{dt}\right)_i = \pi\left|a_i e^{b_i t/2}(\rho_i + i) + F_c\right|^2, \qquad (5)$$

for different t-bins from the available diffraction cross-section. In the Coulomb interference region the first term in (5) represents the nuclear contribution and the second term F_c is the standard Coulomb amplitude, which can be calculated from [15] with a good approximation (see below). Thus, one extra parameter is added, but one may fix it (for example, to its experimental value) at $t=0$ to keep the fitting procedure meaningful over a limited number of points.

Each bin contains a reasonable number of experimental differential cross section points (usually 10-20). Contrary to the procedure traditional in the studies of the break [17], the bins are shifted with respect to each other by one or more measuring channel in such a manner that they overlap (for more detail see [6]). Within the bin used for fitting the data $\left(\frac{d\sigma}{dt}\right)_i$, a_i and b_i are free parameters. Consequently, and that is the goal of using the overlapping bins, we obtained a number of "experimental" values $b_i\pm\Delta b_i$ close to the original number of true experimental points for $\left(\frac{d\sigma}{dt}\right)$ (Figure 1). The error bars Δb_i

shown in the figure represent the fitting uncertainty.

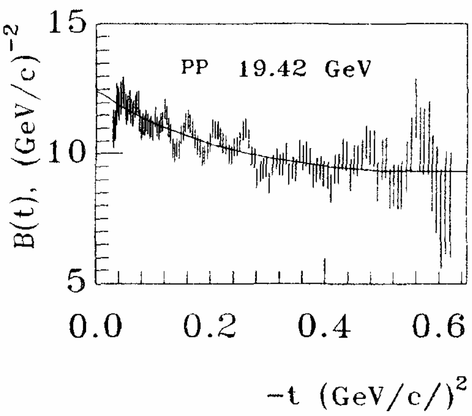

Figure 1. Calculated B(t) for pp at $\sqrt{s} = 19.4 GeV$ [17].

Solid line - calculated by the formula (7) with the fitted parameters.

Using this procedure, we observe that the slope plotted versus *-t* (see Figure 1) for the ISR energies when $|t|$ increases, the "experimental" *B* behaves itself from zero up to $|t|$~0.5GeV^2 as a decreasing sequence of local values distributed along a smooth curve and oscillating around it.

Calculation of the energy and *t*-behaviour of *B(s,t)*, *C(s,t)* and *ρ(s,0)*

Here we shall divert the reader's attention from our attempt to account for the oscillations observed in the slope and confine ourselves to the consideration of only the smooth component of the diffraction cone fine structure, i.e. the curvature. To do this we shall define the differential cross section at the fixed energy as:

$$\left(\frac{d\sigma}{dt}\right) = \pi\left|\frac{\sigma_{tot}}{4\pi}(\rho + i)e^{\beta(t)} + F_c\right|^2, \qquad (6)$$

where

$$\beta(t) = \frac{1}{2}\left|bt + \gamma\left(\sqrt{t_0} - \sqrt{t_0 - t}\right)\right| \qquad (7)$$

The slope and the curvature are determined by the nuclear part of (6) :

$$B(s,t) = b(s) + \frac{\gamma(s)}{2\sqrt{t_0 - t}}, \qquad (8)$$

$$C(s,t) = \frac{\gamma(s)}{8\sqrt{(t_0 - t)^3}}. \qquad (9)$$

In these calculations, we used the differential cross-section data for the diffraction cone region $|t|<1(GeV/c)^2$ comprising rather large number of experimental points in order to determine the *t*-dependence of the slope and curvature [1,2,8,16-25]. The last point of the calculating interval on the large $|t|$ side was chosen close to the beginning of the dip or shoulder near the $|t|\sim 1(GeV/c)^2$. It is convenient to take it with the help of the "experimental" points of the slopes obtained by OBM as the deepest point of the local bins set. This criterion of the diffraction cone end corresponds to the common concept of the "first diffraction cone".

Calculations were done by formulae (2)-(3) with free parameters *b*, γ, σ_{tot} and ρ.

In our calculations, we have restricted ourselves to the data at the energies not lower than those of the ISR (see Tables 1 and 2) where the contribution of secondary Reggeons decreases rapidly assuming that all mentioned above concerns the Pomeron.

Let us start the successive and simultaneous calculation of the parameters *B*, *C* and ρ including the data from the interference region into the computation procedure. In our calculations we have fixed the experimental σ_{tot}.

The most impressive result of our calculations are the new values of ρ at UA4 energy [1] (see Table 2) which go beyond the limits of experimental error for the first case, while for the second one, though it is overestimated, they lie within these limits. The results of calculations are presented in Tables 1 and 2. The energy behaviour of the slope and curvature parameters are:

$$B = 9.0 \pm 1.4 + (0.53 \pm 0.12) \ln s \qquad (10)$$

One can notice that the calculated *B(s)* lie, as earlier, along the line $B(s) = B_0 + 2\alpha' \ln s$, the value of the pomeron trajectory slope agrees with known data on α'.

As for the energy dependence of the curvature parameter, $C_0(s)$ decreases (Fig.2), agree with predictions [26] based on the model that describes the whole family of experimental elastic scattering data [27]. The behaviour $C_0(s)$ indicates the decay with s and $C_0(s)$ changes its sign at $\sqrt{s} = 4.5\,TeV$ [26].

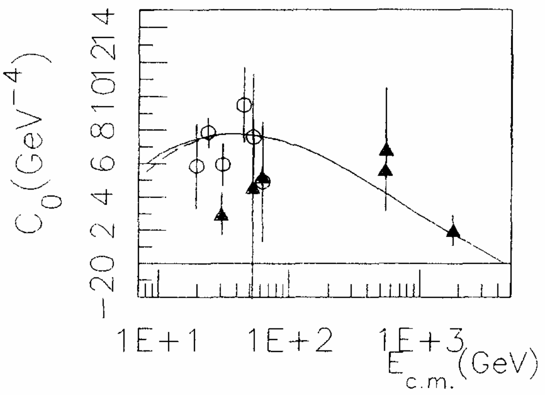

Figure 2. Calculated data for the curvature C(s,0), formula (8). Open circles pp-scattering, solid triangles $\bar{p}p$ -scattering. Solid curve for $\bar{p}p$ data obtained in [26]; dashed line is the same for pp-data.

Oscillation in the slope *B(t)* and the behavior of the partial *ρ(t)*

As follows from above discussion, each ρ corresponds to its own pattern of local slopes (see figure 1) for which the slope parameter b (or b and γ) fitted together with ρ defines the average local slope curve. In other words, the search of ρ is reduced to the quest of the best line over the local slopes. The shape of the local slope set (e.g. for the case of UA4, Figure 3(a)) is of explicit periodic structure. Therefore, the slope parameter that corresponds to his average curve and, hence, ρ (i.e. its partial value) will fluctuate in the same manner if one sequentially reduces the data under fitting from the low $|t|$ side, as is seen clearly in Figure 4(a). Evidently, an arbitrary addition or removal of the experimental points within the interval of $|t|$ comparable to the half-period of said oscillations may result in an abrupt change of the parameters sought. To illustrate this we shall add to the UA4 data [1] all experimental points of the UA4/2 data set [8] which lie beyond the domain of their overlapping (from the low $|t|$ side), i.e. from the $0.88{\cdot}10^{-4} \leq |t| \leq 2.25{\cdot}10^{-3}$ interval. This procedure results, thus, in a decrease of the value of the fitted ρ $(\rho=0.143)$, since the average value of b is increasing in this case. A sequential reduction of the added data leads to the gradual increase of ρ up to the value that corresponds to the best fit of the UA4 data $(\rho=0.233)$. To prevent the undesirable dependence of the calculations on the measurement edge in the interference region, the latter should be expanded as much as possible towards the first cone region, i.e., by adding the corresponding experimental points. In that case the solution is stable against the oscillations (Figures 3(a) and 3(b)). Therefore, we have also added to the UA4 data the data of the measurements in the first cone and calculated parameters ρ and b by reducing gradually the fitted set in the interference region. Now the pattern of the partial parameters and is similar to that for the case of the UA4/1 data (compare figures 3(a), (b) and 4(a), (b)).

As follows from that analysis the overestimated ρ in UA4 [1] results from not sufficiently wide interval of measurements by $|t|$ both from the low $|t|$ $(|t|>|t|_{int}$, see Table 1 in [9]) and large $|t|$ sides. The behaviour of the partial ρ should be the same as in Figures 6(b) and 7(b) shows, i.e. the smooth transition from the boundary value $\rho(t=0)$ to the $\rho=0$ is observed outside the interference region.

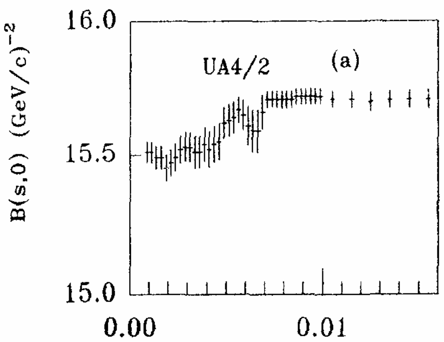

Figure 3a. Partial B vs t for UA4/2 [8] data.

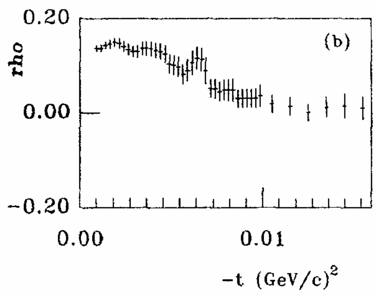

Figure 3b. Partial ρ vs t for UA4/2 [8] data.

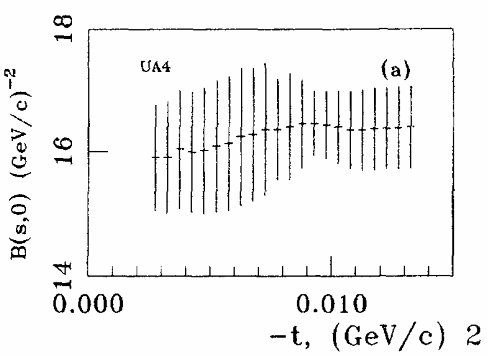

Figure 4a. The same as Fig. 3a for UA4 data [1] complemented by UA4 data of diffraction cone [19].

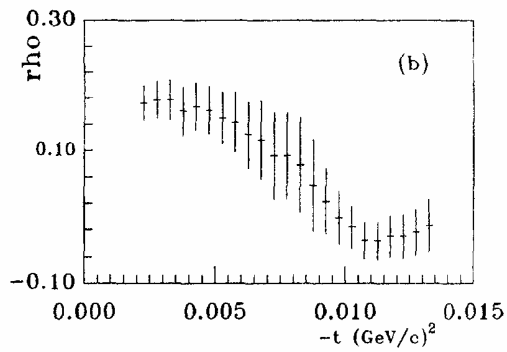

Figure 4b. The same as Fig. 3b for UA4 data [1] complemented by UA4 data of diffraction cone [19].

Conclusion

We have studied the phenomenology of the $\bar{p}p$ and pp elastic small-$|t|$ scattering within a wide energy range by using a model in which the analytical properties of the scattering amplitude are accounted by the threshold singularity in the cross-channel. It has been shown that such features reflect adequately the smooth part of the t-dependence of the slope in a form of a concave curve found with the help of the model-independent procedure of overlapping t-bins. The values of the slope and curvature not exceed noticeably those calculated earlier in terms of the similar models [4] mainly due to that we have used the fitted averaged threshold in the cross channel (see Tables 1 and 2). The slope $B(s,0)$ calculated with the allowance made for the curvature retains its ln s-rise. The $C(s,0)$ values for both processes of $\bar{p}p$- and pp-scattering decrease with energy. It has been shown that $\rho(s=0)$ calculated with the inclusion of the curvature lies in most cases within the limits of experimental error. For the critical UA4 and UA4/2 experiments and this value reaches 0.189 and 0.146, respectively, if one take into account the correct t-behaviour of the partial ρ. To obtain the correct values of $\rho(s=0)$, the region of differential cross-section measurements must involve as large as possible part of the interference region and a sufficiently large region of the first cone to ensure the reliable definition of the behaviour of the nuclear amplitude.

Table 1. Results of $\rho(s,0)$ calculation for pp-scattering

$\sqrt{s}$ (GeV)	σ_{tot} (mb)	ρ_{exp}	$\Delta\rho_{exp}$	ρ_{th}	χ^2/N	$\|t\|$ $(GeV)^2$	m_0	Ref.
19.4	38.98	-0.034	0.009	-0.038	238/203	$6.6\cdot10^{-4}$–0.0315 0.0206–0.66	0.297	[21] [18]
23.5	38.93	0.022	0.014	0.036	166/117	$6.6\cdot10^{-4}$–0.0316 0.042–0.238	0.380	[21] [17]
30.6	40.16	0.034	0.008	0.040	199/115	$1.06\cdot10^{-3}$–0.1 0.016–0.456	0.333	[19] [17]
44.7	41.7	0.062	0.011	0.060	308/165	$9.9\cdot10^{-4}$–0.0521 0.0538–0.2887	0.357	[19] [17]
52.6	42.67	0.078	0.010	0.072	250/157	$1.26\cdot10^{-3}$–0.100 0.0308–0.0721 0.076–0.448		[19] [24] [17]
62.5	43.1	0.080 0.095	0.010 0.011	0.095	137/84	$1.67\cdot10^{-3}$–0.03596 0.037–0.099 0.13–0.85	[22] [21]	[21] [20] [22]

Table 2. Results of $\rho(s,0)$ calculation for the $\bar{p}p$ -scattering

$\sqrt{s}$ (GeV)	σ_{tot} (mb)	ρ_{exp}	$\Delta\rho_{exp}$	ρ_{th}	χ^2/N	$\|t\|$ $(GeV)^2$	m_0	Ref.
30.4	42.00	0.055	0.029	0.055	56/51	$6.7\cdot10^{-4}$–0.0156 0.05–0.85	0.210	[26] [22]
52.6	43.65	0.106	0.016	0.090	84/55	$9.7\cdot10^{-4}$–0.039 0.11–0.85	0.355	[26] [22]
62.5	43.9	0.120	0.030	0.124	41/40	$6.32\cdot10^{-4}$–0.038 0.13–0.85	0.380	[26] [22]
541	62.2	0.135	0.016	0.146	210/185	$7.58\cdot10^{-4}$–0.12 0.0325–0.495	0.420	[8] [20]
546	62.2	0.24	0.04	0.189	162/152	$2.25\cdot10^{-3}$–0.035 0.0325–0.495	0.384	[1] [20]
1800	72.8	0.126	0.067	0.140	30/51		0.210	[2]

1. D. Bernard et al. UA4 Collaboration, Phys. Lett. B **198,** 583 (1987)
2. N.A. Amos et al. Antiproton-Proton Elastic Scattering at s=1.8 TeV from t=0.034 to 0.65 (GeV)^2 (Fermilab Pub.-90/96-E), (1990)
3. O.V. Selyugin, J. Nucl. Phys. **55,** 841-853 (1992)
4. J. Pumplin, Phys. Lett. B **267,** 517-525 (1992)
5. J.E. Kontros and A.I. Lengyel, Ukr. J. Phys. **38,** 329-332 (1993)
6. J. Kontros and A. Lengyel, in Strong interactions at long distances 1995 ed. L. L. Jenkovszky (Palm Harbor: Hadronic Press FL), 67 (1994)
7. O.V. Selyugin, Phys. Lett. B **333,** 345 (1994)
8. C. Augier et al. UA4/2 Collaboration, Phys. Lett. **B** 316 (1993)
9. M.M. Block and R.N. Cahn, Rev. Mod. Phys. **57,** 563-598 (1985)
10. N.A. Amos et al. E710 Collaboration, Phys.Rev.Lett. **68**, 2433 (1991)
11. F. Abe et al. CDF Collaboration, Phys.Rev. **D 50,** 5550 (1994)
12. G. Cohen-Tannoudji, V.V. Iljin, L.L. Jenkovszky, R.S. Tutik, Lett. Nuovo Cim. **5,** 957-962 (1972)
13. W. Guryn, in Frontiers in Strong Interactions, VII Blois Workshop on Elastic and Diffractive Scattering, Chateau de Blois, France, June 1995, edited by P. Chiappetta, M. Haguenauer and J.Tran Thanh Van (Editions Frontieres), p.419 (1996)Buenard, ibid p.437
14. J.E. Kontros, A.I. Lengyel, Ukr. J. Phys. **40,** 263-271 (1995)
15. G.B. West, D.R. Yenni, Phys. Rev. **172,** 1413 (1968)

16. G. Barbiellini et al. Phys. Lett. B **39**, 663 (1972)
17. A. Schiz et al. Phys. Rev. **D24,** 26 (1981)
18. B. Schubert, Tables of pp-Elastic Scattering, Berlin: Springer, (1979)
19. M. Bozzo et al. UA4 Collaboration, Phys. Lett. B **147,** 385-391 (1984)
20. A.A. Kuznetsov et al. J.Nucl. Phys. **33**, 142 (1981)
21. A. Breakstone et al. Nucl. Phys. B **248,** 253 (1984)
22. A. Bohm et al. Phys. Lett. B **49,** 491 (1974)
23. L. Baksay et al. Nucl. Phys. B **141,** 1 (1978)
24. A. Breakstone et al. Phys. Rev. Lett. **54,** 2180 (1985)
25. N.A. Amos et al. Nucl. Phys. B **262,** 689 (1985)
26. P. Desgrolard, J. Kontros, A.I. Lengyel, E.S. Martynov, Nuovo Cim. **110A**, 615 (1997)
27. P. Desgrolard, M. Giffon, E. Predazzi, Z. Phys. C. **63,** 241 (1994).

КРИВИЗНА У ВИСОКОЕНЕРГЕТИЧНОМУ pp ТА $\bar{p}p$ РОЗСІЯННІ

Є.Е. Контрош, К.Є. Контрош, О.І. Лендєл

Інститут електронної фізики НАН України, 88016, м. Ужгород, вул. Університетська, 21

Параметри нахилу $B(s,t)$, кривизни $C(s,t)$ та фази амплітуди розсіяння $\rho(s,0)$ обчислені у широкій області змінних s і t з урахуванням кривизни дифракційного конусу. Знайдена t-залежність $B(s,t)$ порівнюється з сукупністю локальних нахилів, одержаних за допомогою процедури прекриваючих бінів. Виявлено, що кривизна міняє знак при енергії кілька ТеВ.

Єне Ендрійович Контрош – ст..н.с. ІЕФ НАН України

Народився в 1941 р. Закінчив УжДУ в 1964 р., кандидат фізико-математичних наук з 1971 р.

Корнел Єневич Контрош – аспірант ІЕФ НАН України

Народився в 1976 р. Закінчив УжДУ в 1999 р.,

Олександр Іванович Лендьел – ст..н.с. ІЕФ НАН України

Народився в 1939 р. Закінчив УжДУ в 1961 р., кандидат фізико-математичних наук з 1973 р.