FURTHER DEVELOPMENT OF THE CLASSICAL ELECTRODYNAMICAL MODEL OF ATOM

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New results in construction of classical electrodynamical model of atom are presented. The classical electrodynamical equation which is the basic statement of the model is solved directly by means of separation of variables method. The Bohr's postulates are proved to be the consequences of the equation under considerations.

The foundations of the model under consideration were formulated in [1-4] and are based on weakly generalized Maxwell equations

$$\begin{aligned}&\mathrm{rot}\vec{H} - \partial_0 \varepsilon \vec{E} = \vec{j}_e, \ \mathrm{rot}\vec{E} + \partial_0 \mu \vec{H} = \vec{j}_{mag}, \\ &\mathrm{div}\varepsilon\vec{E} = \rho_e, \ \mathrm{div}\mu\vec{H} = \rho_{mag},\end{aligned} \quad (1)$$

where

$$\begin{aligned}&\vec{j}_e = \mathrm{grad}E^0, \ \vec{j}_{mag} = -\mathrm{grad}H^0, \\ &\rho_e = -\varepsilon\mu\partial_0 E^0 + \vec{E}\,\mathrm{grad}\,\varepsilon, \\ &\rho_{mag} = -\varepsilon\partial_0 H^0 + \vec{H}\,\mathrm{grad}\,\mu,\end{aligned} \quad (2)$$

$$\begin{aligned}&\varepsilon(\vec{x}) = 1 - \frac{\Phi(\vec{x}) + m_0}{\omega}, \\ &\mu(\vec{x}) = 1 - \frac{\Phi(\vec{x}) - m_0}{\omega}\end{aligned}, \quad (3)$$

and, moreover, where $(\vec{E},\vec{H})$ are the electric and magnetic field strengths, (2) are the corresponding densities of currents and charges, E^0, H^0 are two real scalar fields generating the gradient-like sources (2), and (3) are the electric and magnetic permeabilities of the medium in Sallhofer's form [5] ($\Phi \equiv -Ze^2/r$, the system of units $\hbar = c = 1$ being used).

One can easily see that Eqs. (1) are not ordinary equations known from the Maxwell theory. These Eqs. (1) have the additional terms which can be considered as the magnetic current and charge densities – in one possible interpretation, or the Eqs. (1) can be considered as the system of equations for electromagnetic $(\vec{E},\vec{H})$ and scalar E^0, H^0 fields – in another possible interpretation.

The reasons of our gently generalization of the classical Maxwell electrodynamics are the following.

1. The existence of direct relationship of Eqs. (1) with the Dirac equation for the massive particle in external electromagnetic field in the stationary case – these equations were shown [3] to be mathematically equivalent in this case.

2. The Eqs. (1) can be derived from the principle of maximally possible symmetry – these equations have both spin 1 and spin ½ Poincare symmetries and in the limit of vanishing the interaction with medium, where ε=μ=1, they represent [6, 7] the maximally symmetrical form of the Maxwell equations. This fact means first of all that from the group-theoretical point of view of Wigner, Bargmann – Wigner (and of modern field theory in general) Eqs. (1) can describe both bosons and fermions. As a consequence of this fact one can use these equations particularly for the description of the electron. On the other hand this fact means that inneratomic classical electrodynamics of electron needs further (than it was done by Maxwell) symmetrization of Weber–Faraday equations of classical electromagnetic theory which leads to the maximally symmetrical form (1). Below we demonstrate the possibilities of the Eqs. (1) in the description of testing example of the hydrogen atom.

In contradiction to [1–4] here Eqs. (1) are solved directly by means of separation of variables method. It is useful to rewrite these equations in the mathematically equivalent form where the sources are maximally simple:

$$\begin{aligned} &\mathrm{rot}\,\vec{H} - \varepsilon\partial_0\vec{E} = \vec{j}_e, \\ &\mathrm{rot}\,\vec{E} + \mu\partial_0\vec{H} = \vec{j}_{mag}, \\ &\mathrm{div}\,\vec{E} = \rho'_e, \\ &\mathrm{div}\,\vec{H} = \rho'_{mag}, \end{aligned} \qquad (4)$$

where

$$\begin{aligned} &\vec{j}_e = \mathrm{grad}E^0, \\ &\vec{j}_{mag} = -\mathrm{grad}H^0, \\ &\rho'_e = -\mu\partial_0 E^0, \\ &\rho'_{mag} = -\varepsilon\partial_0 H^0. \end{aligned} \qquad (5)$$

We suppose the harmonic time dependence and search for the solutions of Eqs. (4) in the harmonic form. It is useful to separate equations which were obtained into following subsystems:

$$\begin{cases} \omega\varepsilon E_B^3 - \partial_1 H_A^2 + \partial_2 H_A^1 + \partial_3 E_A^0 = 0, \\ \omega\varepsilon H_B^0 + \partial_1 H_A^1 + \partial_2 H_A^2 + \partial_3 H_A^3 = 0, \\ -\omega\mu E_A^0 + \partial_1 E_B^1 + \partial_2 E_B^2 + \partial_3 E_B^3 = 0, \\ \omega\mu H_A^3 - \partial_1 E_B^2 + \partial_2 E_B^1 - \partial_3 H_B^0 = 0. \end{cases}$$

$$\begin{cases} \omega\varepsilon E_B^1 - \partial_2 H_A^3 + \partial_3 H_A^2 + \partial_1 E_A^0 = 0, \\ \omega\varepsilon E_B^2 - \partial_3 H_A^1 + \partial_1 H_A^3 + \partial_2 E_A^0 = 0, \\ \omega\mu H_A^1 - \partial_2 E_B^3 + \partial_3 E_B^2 - \partial_1 H_B^0 = 0, \\ \omega\mu H_A^2 - \partial_3 E_B^1 + \partial_1 E_B^3 - \partial_2 H_B^0 = 0. \end{cases} \qquad (6)$$

We make the transition into the spherical coordinate system and for the first subsystem in (6) choose the d'Alembert Ansatz in the form

$$\begin{cases} \overline{E}_A^0 = \overline{C}_{E_4} R_{H_4} P_{l_{H_4}}^{\overline{m}_4} e^{-i\overline{m}_4\varphi}, \\ \overline{E}_B^k = \overline{C}_{E_k} R_{E_k} P_{l_{E_k}}^{\overline{m}_k} e^{-i\overline{m}_k\varphi}, \\ \overline{H}_B^0 = \overline{C}_{H_4} R_{E_4} P_{l_{E_4}}^{\overline{m}_4} e^{-i\overline{m}_4\varphi}, \\ \overline{H}_A^k = \overline{C}_{H_k} R_{H_k} P_{l_{H_k}}^{\overline{m}_k} e^{-i\overline{m}_k\varphi}, \end{cases} \qquad (7)$$

$$k = 1,2,3.$$

We use the following representation for ∂_1, ∂_2, ∂_3 operators in spherical coordinate system

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$$\begin{aligned} &\partial_1 CRP_l^m e^{\mp im\varphi} = \frac{e^{\mp im\varphi}C}{2l+1}\cos\varphi\left(R_{,l+1}P_{l-1}^{m+1} - R_{,-l}P_{l+1}^{m+1}\right) + e^{\mp i(m-1)\varphi}C\frac{m}{\sin\theta}P_l^m\frac{R}{r}, \\ &\partial_2 CRP_l^m e^{\mp im\varphi} = \frac{e^{\mp im\varphi}C}{2l+1}\sin\varphi\left(R_{,l+1}P_{l-1}^{m+1} - R_{,-l}P_{l+1}^{m+1}\right) \mp e^{\mp i(m-1)\varphi}C\frac{im}{\sin\theta}P_l^m\frac{R}{r}, \\ &\partial_3 CRP_l^m e^{\mp im\varphi} = \frac{e^{\mp im\varphi}C}{2l+1}\left(R_{,l+1}(l+m)P_{l-1}^m + R_{,-l}(l-m+1)P_{l+1}^m\right). \end{aligned} \qquad (8)$$

Substitutions (7) and (8) together with the assumptions

$$\begin{aligned} &R_{E_\alpha} = R_E, l_{E_\alpha} = l_E, R_{H_\alpha} = R_H, l_{H_\alpha} = l_H, \\ &\overline{m}_1 = \overline{m}_2 = \overline{m}_3 - 1 = \overline{m}_4 - 1 = m, \\ &\overline{C}_{H_1} = i\overline{C}_{H_2}, \overline{C}_{E_2} = -i\overline{C}_{E_1}, \overline{C}_{H_4} = -i\overline{C}_{E_3}, \overline{C}_{H_3} = -i\overline{C}_{E_4}, \\ &\overline{C}_{H_2}^l = \overline{C}_{E_4}^l\left(l_H^l + m + 1\right), \overline{C}_{E_3}^l = -\overline{C}_{E_4}^l \equiv \overline{C}^l, \\ &\overline{C}_{E_1}^l = \overline{C}_{E_3}^l\left(l_E^l - m\right), l_H^l = l_E^l - 1 = {.}^{l} \end{aligned} \qquad (9)$$

$$\bar{C}^{II}_{H_2} = -\bar{C}^{II}_{E_4}\left(l^{II}_{H} - m\right), \bar{C}^{II}_{E_3} = -\bar{C}^{II}_{E_4} \equiv \bar{C}^{II},$$
$$\bar{C}^{II}_{E_1} = -\bar{C}^{II}_{E_3}\left(l^{II}_{E} + m + 1\right), l^{II}_{H} = l^{II}_{E} + 1 \equiv l^{II}. \tag{9}$$

into the first subsystem in (6) guarantee the separation of variables in these equations and lead to the same radial equations as in the Dirac theory (for the second subsystem in (6) the procedure is similar)

$$\varepsilon\omega R^{I}_{E} - R^{I}_{H,-l} = 0, \quad \mu\omega R^{I}_{H} + R^{I}_{E,l+2} = 0, \tag{10}$$
$$\varepsilon\omega R^{II}_{E} - R^{II}_{H,l+1} = 0, \quad \mu\omega R^{II}_{H} + R^{II}_{E,-l+1} = 0; \quad R_{,a} \equiv \left(\frac{d}{dr} + \frac{a}{r}\right)R. \tag{11}$$

By substituting (9) into (7) one easily obtains the angular part of the hydrogen solutions for the $(\vec{E}, \vec{H}, E^0, H^0)$ field. Taking the real part of thus obtained expressions we finally get:

$$E^{-I0}_{A} = C^{-I}R^{I}_{H}P^{m+1}_{l^I}\cos(m+1)\varphi,$$
$$E^{-I1}_{B} = -C^{-I}(l^I - m + 1)R^{I}_{E}P^{m}_{l^I+1}\cos(m\varphi),$$
$$E^{-I2}_{B} = C^{-I}(l^I - m + 1)R^{I}_{E}P^{m}_{l^I+1}\sin(m\varphi),$$
$$E^{-I3}_{B} = -C^{-I}R^{I}_{E}P^{m+1}_{l^I+1}\cos(m+1)\varphi,$$
$$H^{-I0}_{B} = C^{-I}R^{I}_{E}P^{m+1}_{l^I+1}\sin(m+1)\varphi,$$
$$H^{-I1}_{A} = C^{-I}(l^I + m + 1)R^{I}_{H}P^{m}_{l^I}\sin(m\varphi),$$
$$H^{-I2}_{A} = C^{-I}(l^I + m + 1)R^{I}_{H}P^{m}_{l^I}\cos(m\varphi),$$
$$H^{-I0}_{B} = C^{-I}R^{I}_{E}P^{m+1}_{l^I+1}\sin(m+1)\varphi,$$

$$E^{-II0}_{A} = C^{-II}R^{II}_{H}P^{m+1}_{l^{II}}\cos(m+1)\varphi,$$
$$E^{-II1}_{B} = C^{-II}(l^{II} + m)R^{II}_{E}P^{m}_{l^{II}-1}\cos(m\varphi),$$
$$E^{-II2}_{B} = -C^{-II}(l^{II} + m)R^{II}_{E}P^{m}_{l^{II}-1}\sin(m\varphi),$$
$$E^{-II3}_{B} = -C^{-II}R^{II}_{E}P^{m+1}_{l^{II}-1}\cos(m+1)\varphi,$$
$$H^{-II0}_{B} = C^{-II}R^{II}_{E}P^{m+1}_{l^{II}-1}\sin(m+1)\varphi,$$
$$H^{-II1}_{A} = -C^{-II}(l^{II} - m)R^{II}_{H}P^{m}_{l^{II}}\sin(m\varphi),$$
$$H^{-II2}_{A} = -C^{-II}(l^{II} - m)R^{II}_{H}P^{m}_{l^{II}}\cos(m\varphi),$$
$$H^{-II3}_{A} = -C^{-II}R^{II}_{H}P^{m+1}_{l^{II}}\sin(m+1)\varphi,$$

$$E^{+I0}_{A} = C^{+I}(l^I + m + 1)R^{I}_{H}P^{m}_{l^I}\cos(m\varphi),$$
$$E^{+I1}_{B} = C^{+I}R^{I}_{E}P^{m+1}_{l^I+1}\cos(m+1)\varphi,$$
$$E^{+I2}_{B} = C^{+I}R^{I}_{E}P^{m+1}_{l^I+1}\sin(m+1)\varphi,$$
$$E^{+I3}_{B} = -C^{+I}(l^I - m + 1)R^{I}_{E}P^{m}_{l^I+1}\cos(m\varphi),$$
$$H^{+I0}_{B} = -C^{+I}(l^I - m + 1)R^{I}_{E}P^{m}_{l^I+1}\sin(m\varphi),$$
$$H^{+I1}_{A} = C^{+I}R^{I}_{H}P^{m+1}_{l^I}\sin(m+1)\varphi,$$
$$H^{+I2}_{A} = -C^{+I}R^{I}_{H}P^{m+1}_{l^I}\cos(m+1)\varphi,$$
$$H^{+I3}_{A} = C^{+I}(l^I + m + 1)R^{I}_{H}P^{m}_{l^I}\sin(m\varphi),$$

$$E^{+II0}_{A} = -C^{+II}(l^{II} - m)R^{II}_{H}P^{m}_{l^{II}}\cos(m\varphi),$$
$$E^{+II1}_{B} = C^{+II}R^{II}_{E}P^{m+1}_{l^{II}-1}\cos(m+1)\varphi,$$
$$E^{+II2}_{B} = C^{+II}R^{II}_{E}P^{m+1}_{l^{II}-1}\sin(m+1)\varphi,$$
$$E^{+II3}_{B} = C^{+II}(l^{II} + m)R^{II}_{E}P^{m}_{l^{II}-1}\cos(m\varphi),$$
$$H^{+II0}_{B} = C^{+II}(l^{II} + m)R^{II}_{E}P^{m}_{l^{II}-1}\sin(m\varphi),$$
$$H^{+II1}_{A} = C^{+II}R^{II}_{H}P^{m+1}_{l^{II}}\sin(m+1)\varphi,$$
$$H^{+II2}_{A} = -C^{+II}R^{II}_{H}P^{m+1}_{l^{II}}\cos(m+1)\varphi,$$
$$H^{+II3}_{A} = -C^{+II}(l^{II} - m)R^{II}_{H}P^{m}_{l^{II}}\sin(m\varphi). \tag{12}$$

Because of the fact that radial equations (10), (11) coincide with the R-equations of Dirac theory the procedure of their solution is the same as in well-known monographs on relativistic quantum mechanics and can be omitted. The final result is the Sommerfeld–Dirac formula for the hydrogen spectrum.

$$\omega^{hyd} = \frac{m_0 c^2}{\hbar\sqrt{1 + \frac{\alpha^2}{(n_r + \sqrt{k^2 - a^2})^2}}}, \tag{13}$$

Here we show on the basis of our model that the assertions known as a Bohr's postulates are the consequences of Eqs. (1) and of the classical interpretation of our model, i. e.

these assertions can be derived from the model, there is no necessity to postulate them from beyond the framework of classical physics as it was in Bohr's theory. To derive the first Bohr's postulate one can calculate the generalized Pointing vector for the hydrogen solutions (13) of Eqs. (1), i. e. for the system of stationary electromagnetic and scalar fields $(\vec{E},\vec{H},E^0,H^0)$

$$\bar{s}_{0j} \equiv \int d^3x(\vec{E}\times\vec{H} - \vec{E}E^0 - \vec{H}H^0)_j \,. \quad (14)$$

Such calculations lead to the result that not only the vector (14) is identically equal to zero but the Pointing vector itself and its scalar field term are also identically equal to zero:

$$\int d^3x \vec{E}\times\vec{H} \equiv 0, \qquad \int d^3x(\vec{E}E^0 + \vec{H}H^0) \equiv 0 \quad (15)$$

This means that in stationary states the hydrogen atom does not radiate any Pointing radiation neither due to the electromagnetic $(\vec{E},\vec{H})$ field, nor to the scalar (E^0,H^0) field. That is the mathematical proof of the first Bohr's postulate. The similar calculations of the energy for the same system

$$W \equiv \frac{1}{2}\int d^3x(\vec{E}^2 + \vec{H}^2 + E_0^2 + H_0^2) \quad (16)$$

give a constant W_{nl}, depending on *n, l* (or *n, j*) and independent of *m*. In our model this constant is to be identified with the parameter ω in Eqs. (1) which in the stationary states of $(\vec{E},\vec{H},E^0,H^0)$ field occurs to be equal just to the Sommerfeld–Dirac value ω_{nj}^{hyd} (13). By abandoning the $\hbar = c = 1$ system and putting arbitrary "A" in Eqs. (1) instead of *h* we obtain final ω_{nj}^{hyd} with "A" instead of *h*. Then the numerical value of *h* can be obtained by comparison of ω_{nj}^{hyd} containing "A" with the experiment. These facts complete the proof of the second Bohr's postulate.

This result means that together with Dirac or Schrodinger equations we have now the new equation which can be used for finding the solutions of atomic spectroscopy problems. In contradiction to the well-known equations of quantum mechanics our equation is the classical one. On this basis the classical electrodynamics model of atom is formulated.

References

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ПОДАЛЬШИЙ РОЗВИТОК КЛАСИЧНО ЕЛЕКТРОДИНАМІЧНОЇ МОДЕЛІ АТОМА

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Представлено нові результати в побудові класично електродинамічної моделі атома. Класично електродинамічне рівняння, що є основним твердженням моделі, розв'язано безпосередньо методом відокремлення змінних. Доведено, що постулати Бора є наслідками розглядуваного рівняння.