



Use of the Fractal Analysis of Non-stationary Time Series in Mobile Foreign Exchange Trading for M-Learning

A. Kuchansky¹, A. Biloshchytskyi¹, S. Bronin^{1(✉)}, S. Biloshchytska²,
and Yu. Andrashko³

¹ Taras Shevchenko National University of Kyiv, Kiev, Ukraine
kuczanski@gmail.com, sbronin@me.com

² Kyiv National University of Construction and Architecture, Kiev, Ukraine

³ State University “Uzhhorod National University”, Uzhhorod, Ukraine

Abstract. Mobile foreign exchange trading system for m-learning is proposed. It's used for time series analysis skills learning. Method of pre-forecasting fractal R/S analysis of non-stationary time series is integrated in system. This method includes: persistence, anti-persistence and random level determination based on the calculation of the Hurst exponent. To calculate the average value of the nonperiodic cycle of time series, as well as to establish the potential profitable of assets that are represented by financial time series. A criterion for determination of the average length of non-periodic cycles based on the smoothing of V-statistics with simple moving average and Kaufman's adaptive moving average is proposed. It has been confirmed that most financial time series are more or less persistent and endowed with long-term memory of their initial conditions using computer simulation. Time series of course pairs are close to random. Using fractal analysis in m-learning mobile foreign exchange trading systems for smartphones based on iOS or Android operating systems is suggested. The system is characterized by visualization and description of all stages, which has to be executed for time series analysis. Practical use of this system has shown high efficiency for time series analysis skills learning.

Keywords: M-learning · Mobile foreign exchange trading system · R/S-analysis · Hurst exponent · Time series

1 Introduction

1.1 Introduction to Time-Series Analysis for M-Learning Tasks

The most part of economic, physical, technical and natural processes are non-stationary. Time series representing these processes are the following components' complex: trend's function component, cyclic components with different periods, fluctuations, etc. While forecasting such time series certain difficulties may be faced. Thus, learning system development is up to date and top notch ad it allows to train financial market traders and analysts. Trained specialists will obtain skills in time series analysis, profitability of assets assessment, forecasting tools effectiveness, etc.

The important trader's skill is ability to use time series pre-forecasting analysis methods. It can be done on the basis of fractal analysis. Fractal analysis of time series was proposed by B. Mandelbrot and R. Hudson [1, 2] and developed by E. Peters and E. Feder [3, 4]. As an integral part of discrete nonlinear dynamics methods, it is designed to study nonlinearities in dynamics of time series, including financial ones.

Fractal analysis as an important element of pre-forecasting analysis can be integrated into a multi-tasking analytical decision support system for mobile foreign exchange trading. It may contain detailed market analysis, graphically outputs real-time information, analyzes expert forecasts and addition to training of traders, etc. This analytical system is presented as an application for a mobile phone supported by iOS and Android. The main functions of such systems are: broadcast video market experts, trade signals execution, display of current exchange rates, recommendation of trading strategies, information on events, news and forecasts and also trader training to work on the foreign exchange market. An important part of such a mobile application is the fractal time series analysis, which provides extensive information about the statistical characteristics of the process. The disadvantage of fractal market analysis application is the presence of empirical parameters and the need for the analysis of some statistical characteristics visually.

Authors propose a method for pre-forecasting fractal analysis of financial time series where disadvantages are automated. Investors and analysts in the financial markets may apply above mentioned method for identification in the long-term time series, determining the average length of non-periodic cycles. This method is integrated to the mobile foreign exchange trading system. The educational functionality of this system allows to train traders effectively in time series analysis skills.

1.2 Review

M-learning it's a new stage of e-learning systems' development and recently is often enough used for different areas specialists' trainings. Integration of m-learning systems and analysis as well as processing systems of financial indicators' time series enhances the training of traders and financial analysts and provide the possibility to obtain skills in real time mode. A general architecture for m-learning systems is described in [5].

The method described in this research is based on the algorithm of the Hurst exponent calculation. There are several methods for calculating the Hurst exponent, where the main is the R/S analysis [3] and the detrended fluctuation analysis (DFA) [6–8]. Approaches applied to the calculation of the Hurst exponent based on the R/S-analysis procedure are described in [3, 4, 9] in details. The empirical rules and guidelines for R/S analysis, as well as the visual analysis of V-statistics for determining the average length of the non-periodic cycle, are described in [3]. The effect of long-term memory in time series is described in [1, 3, 9].

Application of forecasting methods and pre-forecasting analysis of time series in various applications are described in [10–30]. In particular the forecasting power of the method of cluster analysis using the same behavior of the time series of the stock market, as well as the use of this method for efficient forecasting of stock prices, were investigated in [10]. The method for modeling samples is proposed for the task of short-term forecasting of time series is proposed in [11]. The method of selective

comparison with the sample for the problem of constructing combined forecasting models for signs of increments of time series with an unstable nature of oscillations, considering the identification of similarities or indexation in [12]. The method of forecasting the increments of time series in conditions of uncertainty using the trend model of moving averages is described in [13]. The method of the nearest neighbor for this problem is considered in [14, 15]. The method for indexing time series based on the identification of similarities between them was described in [16]. The clustering method for finding similarities in multidimensional spaces that are represented by multidimensional time series is described in [17].

Application of time series analysis methods is also used in education in order to predict the potential of scientific directions development [18]. The values of time series in this case are the ratio of results evaluations of scientist's research activities for different periods of time. The method can be used for identifying promising areas of research formed in a scientific environment. The method of classifying scholars in research, based on the identification of similarities in time series, is described in [19]. The method of constructing evaluations of the results of scientist's scientific research activity based on the analysis of publications citations is described in [20]. The articles [21, 22] describes a parametric model for assessing and predicting the quality of educational institutions, which uses the approaches to the comparison of assessments.

The task of identifying similarities and analyzing time series is also used to identify incomplete duplicates in textual information [23]. In [24], a conceptual model of the system for finding incomplete duplicates using the identification of similarities in electronic documents is described. In [25] describes the approaches of intellectual decision-making methods in business using classical time series forecasting models. Adaptive models of short-term forecasting of time series, methods of constructing combined methods are described in [26]. In [27] we consider models and methods of prediction of time series using intelligent data analysis: neural network, genetic algorithms, fuzzy analysis, etc.

Both classical forecasting models and forecasting models using similarity identification are often used as components of complex forecasting, modeling and decision-making systems. In [28] an analysis of the design features of information-analytical systems for forecasting of time series with the use of expert evaluation was carried out. In [29], in particular, the method of fuzzy cluster analysis, which uses the identification of similarities, is considered. The use of expert estimation to forecast time series in information systems is also described in [30]. [31] deals with the management of project configuration management in the development of distributed systems, which can use the identification of similarities between projects on the set of indicators that these projects characterize [32, 33].

1.3 Aims and Objectives

The aim of the research is to develop and describe the method of pre-forecasting fractal analysis of time series for use in mobile foreign exchange trading and traders m-learning. Also, to establish criteria in this method for estimating the average length of non-periodic cycles, identifying series with long-term memory, etc. Visualization of the

fractal analysis methods' work results should be laid in mobile foreign exchange trading systems for m-learning. It's needed for traders and financial analysts' effective preparation.

2 Fractal Analysis of Time-Series Data Sets

2.1 Fractal R/S Analysis

According to the principles of fractal analysis, time series have a fractal dimension $1 < D < 2$, endowed with properties of scale invariance (self-similarity) and memory of their initial conditions. It is believed that the time series that reflect the development of economic processes have a fractal structure. The fractal dimension indicates the degree of "spine" of the time series. In practice, the fractal dimension is replaced by the Hurst exponent, on the basis of which the degree of smoothness of the time series is determined [3, 34]. The H exponent is determined on the basis of the fractal dimension by the formula $H = 2 - D$, where is $0 \leq H \leq 1$. Hurst exponent characterizes the ratio of the component of the trend function to white noise and can be used for the classification of time series: the establishment of non-random time series with a stable trend and random rows (including non-Gaussian ones). The calculation of the Hurst exponent can be based on the R/S analysis procedure. In [35] justified the applicability of this method for the study of financial and economic processes.

There are three classifications for time series depending on the value of the Hurst exponent. If $0 < H < 0.5$ or $0 < H < E$, then there isn't correlation between retrospective and predictive values in the time series. This time series is antipersistence. If $E < H < 1$, then the time series is characterized by persistence behavior. If $H = E$ or $H = 0.5$, then the time series is random and has no memory of its initial conditions. The value of E is calculated according to the formula described in the work [36]:

$$E\left(\frac{R_\tau}{S_\tau}\right) = \sqrt{\frac{2}{\pi(\tau - 1)}} \sum_{i=1}^{\tau-1} \sqrt{\frac{\tau - i}{i}},$$

for $\tau > 300$.

In this article, for the calculation of the Hurst exponent we will apply the methodology proposed in [3]. Let a series of n observations are given $Z = \{z_i\}_{i=1}^n$. For each of the initial segments of this time series $\{z_i\}_{i=1}^\tau$ with length $\tau = 3, 4, \dots, n$ calculate the mean values by the formula $\bar{z}_\tau = \frac{1}{\tau} \sum_{i=1}^\tau z_i$, accumulated deviations find the formula $x_{\tau,t} = \sum_{i=1}^\tau (z_i - \bar{z}_\tau)$, for $\tau = 3, 4, \dots, n$, where $R_\tau = \max_{1 \leq t \leq \tau} x_{\tau,t} - \min_{1 \leq t \leq \tau} x_{\tau,t}$. Then the standard deviation for each of the segments determined by $S_\tau = \sqrt{\frac{1}{\tau} \sum_{i=1}^\tau (z_i - \bar{z}_\tau)^2}$, for $\tau = 3, 4, \dots, n$. The velocity of the accumulated deviation is normalized by dividing by the mean square deviation for each segment τ and the dependency schedule $\lg\left(\frac{R_\tau}{S_\tau}\right)$ by $\lg(\tau)$ is built also known as R/S trajectory. Then the linear regression equation is constructed based on the least squares' method; the coefficient with an independent variable will be the Hurst exponent ($\alpha = const$):

$$\lg\left(\frac{R_\tau}{S_\tau}\right) = \lg(\alpha) + H \lg(\tau).$$

The Hurst exponent can also be considered as a function of τ [3]: $H(\tau) = \frac{\lg(\frac{R_\tau}{S_\tau})}{\lg(\frac{\tau}{2})}$.

Behavior constructed on the basis of this function of the H trajectory or function dependence $H(\tau)$ of $\lg(\frac{\tau}{2})$, as well as R/S trajectories, can be used to detect such properties of the time series as: intervals of long-term and short-term dependence; the presence of cyclic components and the average length of the nonperiodic cycle.

Typically, a visual analysis of trends in the V-statistics is used to find the length of a cycle. It consists in identifying the points of trend change that can signal the end of the cycle, as well as the intervals of growth, stabilization and decline of the curve, which, with increasing number of observations, determines the attraction of the process to a persistent or random. The growth of V-statistics with an increase in the number of observations indicates the persistence of the current section of the series, and stabilization - on the predominance of white noise. V-statistics is calculated by: $V_\tau = \frac{R_\tau}{\sqrt{\tau}S_\tau}$, where R_τ is variance and S_τ is standard deviation for $\tau = 3, 4, \dots, n$. It is confirmed in [3] that the moment of change of the trend of the V-statistics graph, which is expressed by the dependence V_τ of $\lg(\tau)$ indicates the length of both the non-periodic cycle.

2.2 Method of Pre-forecasting Fractal Analysis of Time Series

The method of pre-forecasting analysis of non-stationary time series involves the calculation of the Hurst exponent based on the R/S analysis of the time series; definition of the average length of the nonperiodic cycle and the identification of “long memory” in the time series; selection of assets for trading.

Here is a description of these algorithms, illustrating calculations for a specific task. Analysis stages description and visualization in system is necessary component of traders’ learning of time series analysis skills. Let’s set the time series of prices for petrol for the period from 2016 to 2019, daily data on closing prices, the length of the series is 775. Let’s denote it by $\bar{Z} = \{\bar{z}_i\}_{i=1}^{n+1} = \{\bar{z}_{(t_1)}, \bar{z}_{(t_2)}, \dots, \bar{z}_{(t_n)}\}$, t_i – discrete moments of time. Perform the following steps:

Step 1. We visualize the given time series, that is, we will build a price chart.

Step 2. We implement the procedure of R/S analysis. Calculate the Hurst exponent.

To find the Hurst exponent we will consider the time series $Z = \{z_i\}_{i=1}^n$ where $z_i = \frac{\lg(\bar{z}_i)}{\lg(\bar{z}_{i-1})}$ for $i = 1, 2, \dots, n$. This requirement, described in [3] (see Fig. 1).

The next step is to calculate the scale R_τ and standard deviation S_τ for each of the segments $\{z_i\}_{i=1}^\tau$ of length $\tau = 3, 4, \dots, n$. Let’s introduce the designation $X = (x_3, x_4, \dots, x_n)$ where $x_\tau = \lg(\tau)$ and $Y = (y_3, y_4, \dots, y_n)$ where $y_\tau = \lg\left(\frac{R_\tau}{S_\tau}\right)$ for $\tau = 3, 4, \dots, n$ and suppose that between the factors X and Y there is a linear dependence, i.e. $Y = a + bX$. Identify the values of the coefficients a and b from the condition of

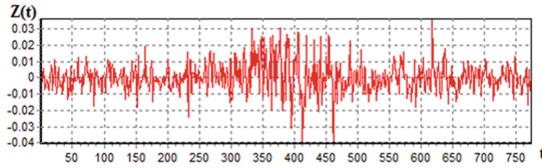


Fig. 1. Time series of prices for petrol (NYMEX) after conversion.

minimizing the function $\sum_{\tau=3}^n (y_{\tau} - (a + bx_{\tau}))^2 \rightarrow \min$ by the least squares method [27]. As a result, for the time series Z received the following estimates: $\hat{a} = -0.549$, $\hat{b} = 0.783$. The estimation of the coefficient b will be the Hurst exponent of the time series Z, i.e. $H = 0.783$. Determination factor is $R^2 = 0.9198$.

For a series Z, we construct on one graph R/S- and H-trajectories and the regression line $Y = -0.549 + 0.783X$ (see Fig. 2).

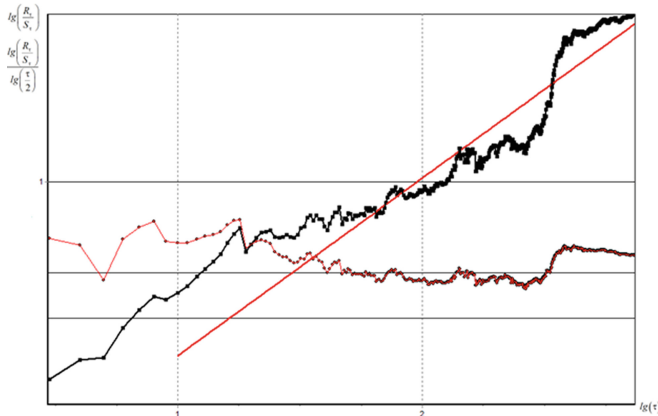


Fig. 2. R/S (black) and H (red) trajectory for the time series Z.

Step 3. Check the hypothesis about the significance of the H exponent for the Z series. For the series Z, the indicator $E\left(\frac{R}{S_{\tau}}\right) = 0.5452$. So, the Hurst exponent for this series $H = 0.783$, then the hypothesis about his accident is rejected. The value of the Hurst exponent indicates that the input time series Z is persistent, and the process described by this time series is characterized by the presence of long-term memory and has a trend-resistant non-periodic cycle.

2.3 Criteria for Determining the Length of a Non-periodic Cycle

The criterion is based on smoothing the V-statistics curve and identifying the moments of changing the initial trend of the curve, taking into account the smoothed values.

Smooth a series V_τ for $\tau = 3, 4, \dots, n$ with the help of a simple moving average with period p according to the formula:

$$s_{\tau+p} = \frac{1}{p} \sum_{j=0}^{p-1} V_{\tau+p-j}, \tau = 3, 4, \dots, n - p$$

and the Kaufman’s adaptive moving average by the formula:

$$a_t = c_t V_t + (1 - c_t) a_{t-1},$$

where $c_t = (E_t(f - s) + s)^2$, $E_t = \frac{V_t - V_{t-r}}{\sum_{i=0}^{r-1} |V_{t-i} - V_{t-i-1}|}$ are the coefficient of efficiency as the ratio of the total price movement to the sum of the absolute values of the noise market movement for the period r , $t = \overline{\tau + r}, n$, f , s are fast and slow smoothing factors, $f = \frac{2}{p_1 + 1}$, $s = \frac{2}{p_2 + 1}$ and $p_1 < p_2$ (see Fig. 3).

The length of the non-periodic cycle is k if the conditions are fulfilled at the time k :

- from the moment k the Kaufman index falls for at least two subsequent points, that is $a_k > a_{k+1} > a_{k+2}$. it is significant that the Kaufman index should increase monotonously to the k -th point, which is explained by the behavior of the V -statistics;
- the moving average flow at this moment does not exceed the value of the Kaufman index, i.e. $s_k < a_k$, $s_{k+1} < a_{k+1}$.
- there is a sharp change in the trend of V -growth statistics on the fall: $V_{k-1} < V_k$, $V_k > V_{k+1}$, at the same time, the value of V -statistics at the k^{th} point reaches the local maximum, exceeding the values of the simple moving average and the Kaufman index: $V_k > a_k > s_k$.

The choice of a Kaufman flow filter is due to the adaptive nature of its coefficients. Thus, the system, subject to the use of this criterion, can calculate the value of a non-periodic cycle without human intervention, that is, in automatic mode.

It should be noted that the abscissa axis for the plot on which the V -statistics and the mean averages are constructed is the logarithmic value of τ , that is, after obtaining the moment for which the specified conditions are fulfilled: the point from the abscissa axis x_k you need to use the formula $k = 10^{\lg(x_k)}$.

According to the graph V -statistics and moving averages for the studied series, one can see that these conditions are fulfilled for $x_k = 1.255$ that is, the value of a non-periodic cycle for a given series $10^{\lg(1.255)} \approx 18$. It should be noted that the verification of conditions must be carried out starting from the point $k = 10$, $x_k = 1$, as well as the construction of a regression line to determine the Hurst exponent. This empirical rule is formulated in [3].

Let’s consider other definition method of the length of a non-periodic cycle. It is known that the moment of a fracture or abrupt change in the initial trend of the H trajectory, as a rule, from the rising to a decreasing, provided that the R/S trajectory has previously changed its initial trend, indicates the length of the cycle.

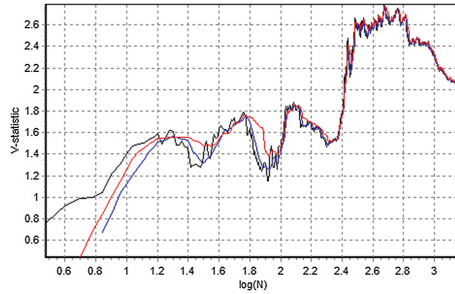


Fig. 3. Plot V-statistics and moving averages, $p = 7$, $r = 4$, $p_1 = 3$, $p_2 = 10$

Let's denote through $F(Z)$ a family of series of fixed length m , each of which is constructed from the input time series Z by a flow window method, that are the series $\{z_i\}_{i=1}^m$, $\{z_i\}_{i=2}^{m+1}$, \dots , $\{z_i\}_{i=n-m+1}^n$. For each of these series, apply the R/S analysis procedure, construct the corresponding H and R/S trajectories, and define the length of the cycles k_j , $j = \overline{1, n - m + 1}$ from condition that the point k_j match the length of the cycle for time series $\{z_i\}_{i=j}^{m+j-1}$ if H is trajectory at the point $k_j + 1$ or $k_j + 2$ crosses the R/S trajectory, while both trajectories change the previous downward trend starting from the point k_j so $H_{k_j+1} < H_{k_j}$ i $(R/S)_{k_j+1} < (R/S)_{k_j}$, $(R/S)_{k_j+2} > H_{k_j+2}$ and the R/S trajectory is not in the anti-persistence zone, i.e. $(R/S)_{k_j} > 0.5$.

Next, we will construct a histogram of the length distribution of cycles k_j for families of time series $\{z_i\}_{i=j}^{m+j-1}$, $j = \overline{1, n - m + 1}$. On the basis of the histogram, you can estimate the average length of the cycle.

By the histogram for the input row (see Fig. 4), the median is 18, and the average value is 18.81. Since the data on time series reflects the daily prices of series petrol, it is possible to conclude from the analysis that the average value of the cycle (quasicycle) is almost a month (18 working days).

Each of the described criteria shows the high efficiency of setting the value of cycles when working in automatic mode.

Next step is to analyze the behavior of the Hurst exponent in dynamics. In order to provide a more detailed time series analysis, it is proposed to investigate the behavior of the Hurst exponent in dynamics. The results of this study can be used to break down a number of sites by their level of persistence. This allows you to follow the current development of the process and forecast it for the future. The flux index is a function which is constructed from the Hurst exponent for the family of series $F(Z)$, which are formed from the studied time series Z by the flow window method. For the time series that is being investigated, the graph of the change in the indicator shows that the time series is persistent both in general and on local segments ($m = 500$).

Next step is determination of the potential profitable of the asset represented by the financial time series. The purpose of the R/S-based formulation of the R/S-analysis of asset selection criteria presented by time series for trading may also be from the point

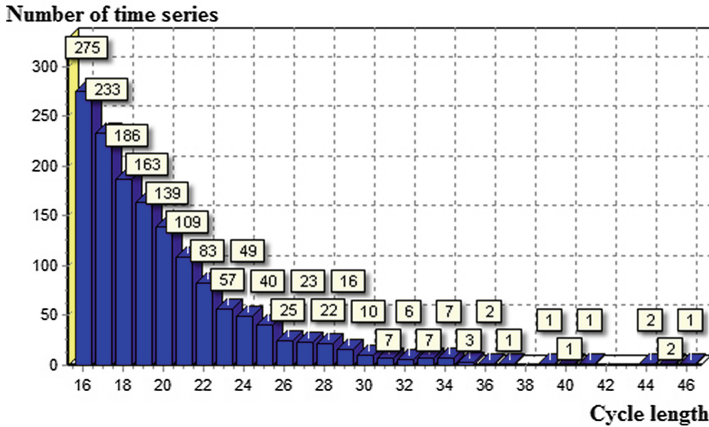


Fig. 4. Histogram of the distribution of the lengths of the cycles for the family of series F(Z)

of view of risk assessment of financial investment in asset data. This task can be used to construct asset selection criteria for an investment portfolio.

Let's put the task of assessing the risk of financial investment in assets of a certain type. The task of assessing the effectiveness of investing is a complex procedure for the sequential analysis of various economic and mathematical indicators, the calculation of which depends on the type of investment. We will consider investments that represent the flow of payments of primary payments and subsequent revenues when investing in commodity assets and performing speculative operations on the market. The task which is presented to the subject of risk, in this case, the investor, consists in choosing from a set of possible assets such, investing financial resources in which would provide the maximum economic effect in accordance with the requirements of the subject, taking into account financial risks. Under financial risks we will understand the risks associated with transactions with financial assets. Estimation of alternative investment options according to the requirements of the subject of risk can be realized both on the basis of analytical and calculation methods, and with the help of fractal analysis of the market.

Let each asset meet retrospective information in the form of time series of prices for this asset. The procedure for assessing the effectiveness of investing consists in the implementation of the main stages of the complex fractal analysis described above: the implementation of the procedure of successive R/S-analysis, the definition of the Hurst exponent, the construction of V-statistics, identification of the process with long-term memory, determination of the average length of the cycle. The final step is to check whether the criteria for selecting assets for trading is satisfied by the characteristics found. An asset represented by a financial time series, is potential profitable, if:

- the financial time series is near long-term memory;
- the behavior of the Hurst exponent in the dynamics demonstrates stable persistence in the area of the series, which precedes the input of the investor on the market;

- at high values of the Hurst exponent, and subject to the presence of short cycles, it can be argued that the market is increasing, and vice versa, if the Hurst exponent is low, then in the case of long cycles, it can be argued that the market is falling.

The ability to identify these characteristics is an important trader's skill. Therefore, this mobile foreign exchange trading system provides the opportunity to train traders with this skill. Traders need to analyze the time series stored in the system base. Then they get feedback about identification results of the described characteristics.

3 Conclusions

The article proposes a method for forecasting fractal analysis of time series, formulates the criteria for determining the average length of non-periodic cycles and other fractal characteristics. The results of the analysis can be used by investors and analysts to select the most potential profitable, assets represented by financial time series. This method is integrated to mobile foreign exchange trading systems. The system is characterized by the detailed visualization of all stages of time series' pre-forecasting analysis in the charts' form. The special feature of the developed mobile foreign exchange trading system is integrated m-learning tools for training traders' skills of analysis & processing of financial indicators' time series. The usage of system allows to enhance the training of traders' and financial analysts' effectiveness with the following features:

1. Development of the necessary skills for work with time series based on real data and in real time mode.
2. Detailed visualization and description of all stages of time series' analysis. Also, potential profitableness justification considering the calculated indicators.

References

1. Mandelbrot, B.: Statistical methodology for non-periodic cycles: from the covariance to R/S analysis. *Ann. Econ. Soc. Measur.* **1**, 259–290 (1972)
2. Mandelbrot, B.B., Hudson, R.L.: *The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin and Reward*. Basic Books, New York (2004)
3. Peters, E.E.: *Fractal Market Analysis: Applying Chaos Theory to Investment and Economics*. Wiley, New York (1994)
4. Feder, J.: *Fractals*. Springer US, New York (1988). <https://doi.org/10.1007/978-1-4899-2124-6>
5. Georgiev, T., Georgieva, E., Smrikarov, A.: M-learning – a new stage of e-learning. In: *International Conference on Computer Systems and Technologies*, p. 28 (2004)
6. Peng, C.-K., Buldyrev, S.V., Havtin, S., Simons, M., Stanley, H.E., Goldberger, A.L.: Mosaic organization of DNA nucleotides. *Phys. Rev. E* **49**(2), 1685–1689 (1994)
7. Peng, C.-K., Havlin, S., Stanley, H.E., Goldberger, A.L.: Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series. *Chaos* **5**, 82–87 (1995). <https://doi.org/10.1063/1.166141>

8. Kantelhardt, J.W.: Fractal and multifractal time series. In: Meyers, R. (eds.) *Mathematics of Complexity and Dynamical Systems*, pp. 463–487. Springer, New York (2012). https://doi.org/10.1007/978-1-4614-1806-1_30
9. Kyrychenko, L., Deineko, Z.: Estimating the self-similarity of a stochastic time series using the wavelet analysis method. *Radioelectron. Comput. Syst.* **4**(38), 99–105 (2009)
10. Parzen, E.: Long memory of statistical time series modeling. In: *NBER/NSF Time Series Conference*, p. 10. Texas A&M University (2004)
11. Nayak, R., te Braak, P.: Temporal pattern matching for the prediction of stock prices. In: *2nd International Workshop on Integrating Artificial Intelligence and Data Mining (AIDM 2007)*, Australia, pp. 95–103 (2007)
12. Singh, S.: Pattern modeling in time-series forecasting. *Cybern. Syst.* **31**(1), 49–65 (2000). <https://doi.org/10.1080/019697200124919>
13. Kuchansky, A., Biloshchytskyi, A.: Selective pattern matching method for time-series forecasting. *Eastern-Eur. J. Enterpr. Technol.* **6**(4(78)), 13–18 (2015). <https://doi.org/10.15587/1729-4061.2015.54812>
14. Perlin, M.S.: Nearest neighbor method. *Rev. Eletrôn. de Administr.* **13**(2), 15 (2007)
15. Fernández-Rodríguez, F., Sosvilla-Rivero, S., Andrada-Félix, J.: Nearest-neighbour predictions in foreign exchange markets. FEDEA Working Paper No. 2002-05 (2002). <https://doi.org/10.2139/ssrn.300404>
16. Kahveci, T., Singh, A.: Variable length queries for time series data. In: *Proceedings of the 17th International Conference on Data Engineering*, pp. 273–282 (2001). <https://doi.org/10.1109/icde.2001.914838>
17. Li, C., Chang, E., Garcia-Molina, H., Wiederhold, G.: Clustering for approximate similarity search in high-dimensional spaces. *IEEE Trans. Knowl. Data Eng.* **14**(4), 792–808 (2002). <https://doi.org/10.1109/tkde.2002.1019214>
18. Biloshchytskyi, A., Kuchansky, A., Andrashko, Yu., Biloshchytska, S., Dubnytska, A., Vatskel, V.: The method of the scientific directions potential forecasting in infocommunication systems of an assessment of the research activity results. In: *2017 IEEE International Conference « Problems of Infocommunications. Science and Technology » (PIC S&T)*, pp. 69–72 (2017). <https://doi.org/10.1109/infocommst.2017.8246352>
19. Biloshchytskyi, A., et al.: A method for the identification of scientists' research areas based on a cluster analysis of scientific publications. *Eastern-Eur. J. Enterpr. Technol.* **5**(2(89)), 4–10 (2017). <https://doi.org/10.15587/1729-4061.2017.112323>
20. Biloshchytskyi, A., Kuchansky, A., Andrashko, Yu., Biloshchytska, S., Kuzka, O., Terentyev, O.: Evaluation methods of the results of scientific research activity of scientists based on the analysis of publication citations. *Eastern-Eur. J. Enterpr. Technol.* **3**(2(87)), 4–10 (2017). <https://doi.org/10.15587/1729-4061.2017.103651>
21. Otradskaia, T., Gogunskii, V., Antoshchuk, S., Kolesnikov, O.: Development of parametric model of prediction and evaluation of the quality level of educational institutions. *Eastern-Eur. J. Enterpr. Technol.* **5**(3(83)), 12–21 (2016). <https://doi.org/10.15587/1729-4061.2016.80790>
22. Biloshchytskyi, A., et al.: A method to evaluate the scientific activity quality of HEIs based on a scientometric subjects presentation model. *Eastern-Eur. J. Enterpr. Technol.* **6**(2(90)), 16–22 (2017). <https://doi.org/10.15587/1729-4061.2017.118377>
23. Lizunov, P., Biloshchytskyi, A., Kuchansky, A., Biloshchytska, S., Chala, L.: Detection of near duplicates in tables based on the locality-sensitive hashing method and the nearest neighbor method. *Eastern-Eur. J. Enterpr. Technol.* **6**(4(84)), 4–10 (2016). <https://doi.org/10.15587/1729-4061.2016.86243>

24. Biloshchytskyi, A., Kuchansky, A., Biloshchytska, S., Dubnytska, A.: Conceptual model of automatic system of near duplicates detection in electronic documents. In: 2017 IEEE International Conference « The Experience of Designing and Application of CAD Systems in Microelectronics » (CADSM), pp. 381–384 (2017). <https://doi.org/10.1109/cadsm.2017.7916155>
25. Vercellis, C.: *Business Intelligence: Data Mining and Optimization for Decision Making*, p. 417. Wiley, Cornwall (2009). <https://doi.org/10.1002/9780470753866>
26. Lukashin, Yu.: *Adaptive Methods of Near-Term Time Series Forecasting*, p. 416. Finance and Statistics, Moscow (2003)
27. Snytyuk, V.E.: *Forecasting Models Methods Algorithms*, p. 364. Maklout, Kyiv (2008)
28. Mulesa, O., Geche, F., Batyuk, A., Buchok, V.: Development of combined information technology for time series prediction. In: Shakhovska, N., Stepashko, V. (eds.) CSIT 2017. AISC, vol. 689, pp. 361–373. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-70581-1_26
29. Mulesa, O., Geche, F., Batyuk, A.: Information technology for determining structure of social group based on fuzzy c-means. In: 2015 Xth International Scientific and Technical Conference “Computer Sciences and Information Technologies” (CSIT), pp. 60–62 (2015). <https://doi.org/10.1109/stc-csit.2015.7325431>
30. Mulesa, O., Geche, F.: Designing fuzzy expert methods of numeric evaluation of an object for the problems of forecasting. *Eastern-Eur. J. Enterpr. Technol.* **3**(4(81)), 37–43 (2016). <https://doi.org/10.15587/1729-4061.2016.70515>
31. Morozov, V., Kalnichenko, O., Liubyma, I.: Managing projects configuration in development distributed information systems. In: 2017 2nd IEEE International Conference on Advances Information and Communication Technologies (AICT), pp. 154–157 (2017). <https://doi.org/10.1109/aiact.2017.8020088>
32. Kuchansky, A., Biloshchytskyi, A., Andrashko, Yu., Biloshchytska, S., Shabala, Ye., Myronov, O.: Development of adaptive combined models for predicting time series based on similarity identification. *Eastern-Eur. J. Enterpr. Technol.* **1**(4(91)), 32–42 (2018). <https://doi.org/10.15587/1729-4061.2018.121620>
33. Kuchansky, A., et al.: Combined models for forecasting the air pollution level in infocommunication systems for the environment state monitoring. In: 2018 IEEE International Conferences on Intelligent Data Acquisition and Advanced Computing Systems, pp. 125–130 (2018). <https://doi.org/10.1109/idaacs-sws.2018.8525608>
34. Hurst, H.E.: Long-term storage capacity of reservoirs. *Trans. Am. Soc. Civ. Eng.* **116**, 770–799 (1951)
35. Mandelbrot, B.B.: When can price be arbitrated efficiently? A limit to the validity of the random walk and martingale. *Models Rev. Econ. Stat.* **53**(3), 225–236 (1971)
36. Anis, A., Lloyd, E.: The expected value of the adjusted rescaled Hurst Range of independent normal summands. *Biometrika* **63**, 111–116 (1976)