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Slightly generalized Maxwell system and longitudinal components of solution

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Abstract. We consider slightly generalized Maxwell equations with electric and magnetic currents and charges densities of the gradient type. Among other versions of the Maxwell system these equations differ by the extended symmetry properties. Such system of equations is invariant with respect to a 256-dimensional algebra, and this algebra is not yet the maximum of possible symmetry. The longitudinal components of both vectors of electric and magnetic field strengths, together with two corresponded scalar waves, are found as the exact solution of such generalized Maxwell equations. The longitudinal wave component of the electric field strength vector itself is found as an exact solution of the standard Maxwell equations with fixed current and charge of the gradient type. This wave is corresponded to the scalar wave component, which is propagated in the same direction. The analysis of found solutions demonstrates that longitudinal components are located near the corresponded current and charge densities, which are the sources of such fields. The relationship with modern experiments is considered briefly.

1. Introduction

The theoretical and experimental description of the longitudinal electromagnetic waves is a subject of the different level discussions. There is a lot of indications of the existence of such waves in different media. Note, e.g., the longitudinal waves in plasmons and in plasma. Nevertheless, these oscillations only sometimes are considered as the electromagnetic ones. On the other hand, it is well known fact that the Maxwell equations for the free electromagnetic field contain only the transverse solutions.

Recently in [1] a simple experimental method for generating an intense longitudinal electric field from transverse electromagnetic waves (laser pulses) with radially symmetric polarization has been presented. The laser-generated longitudinal electric field was observed in two dimensions and distinguished from the transverse component using the optical Kerr shutter method. Authors of [2] experimentally demonstrated that a radially polarized field can be focused to a spot size significantly smaller than for linear polarization. For strong focusing, a radially polarized field leads to a longitudinal electric field component at the focus which is sharp and centered at the optical axis. Experimental and theoretical investigations in [3] demonstrates that the longitudinal field component has the maximal amplitude where the "usual" field component is zero. The high-power industrial CO₂ laser is applied. Authors of [4] studied



that, when focused, a radially polarized beam has a net longitudinal field useful for particle acceleration and, perhaps, other unique applications.

The goal of this paper is to contribute in the problem by means of standard classical electrodynamics, e. g., an exact solutions of well defined physical and mathematical systems of equations. Presented below result that the longitudinal wave component of the electric field strength vector is the exact solution of the standard Maxwell equations with specific gradient-type case of electric current and charge densities can be verified by anyone after individual substitution into standard Maxwell system with corresponding sources.

Note that one can find some confusion in using the notions "longitudinal electric wave" and "longitudinal wave component of the electric field strength vector". It leads to the confusion in understanding.

The question about longitudinal electromagnetic waves is the old problem of classical electrodynamics. The first theoretical model has been considered in [5]. Longitudinal electromagnetic waves between parallel plates were investigated.

The hypothesis about the longitudinal electromagnetic waves, which presence in the mathematical formalism follows from the massless Dirac equation (on the basis of a link between the massless Dirac and slightly generalized Maxwell equations), was suggested in [6]. The start in [6] was related to the results [7]. Following [6, 7], in [8, 9] we presented our preliminary point of view on this problem. The general solution of such kind of the Maxwell equations has been found. The conclusion that the longitudinal electric waves can exist within the framework of standard Maxwell electrodynamics in the case, when the electric currents and charges have specific gradient-like form, has been suggested. Such longitudinal electric wave appears [8, 9] in the pair with scalar wave.

However, the problem on the longitudinal electromagnetic waves can be considered independently (without any relation to the massless Dirac equation and the Maxwell equations in the Dirac-like form).

Below (see also [10]) the longitudinal wave component of the electric field strength vector \vec{E} is found as the exact solution of the standard Maxwell equations with specific partial case of electric current and charge densities.

In the period after our publications [8, 9] an interest to the problem of longitudinal electromagnetic waves in electrodynamics has been arose. Other approaches to the problem should be briefly mentioned as well, see, e. g. [11, 12]. Nevertheless, the generalization of classical electrodynamics to admit a scalar field and longitudinal waves mentioned in [12] was known already from [8, 9].

Note that in [13] scalar potential wave together with a longitudinal electric field \vec{E} in the direction of propagation has been detected by a ball antenna. Nevertheless, the authors of [14–16] have been doubted in this result both from the theoretical [14, 15] and experimental [16] points of view. The authors of [16] tried to repeat the experiment [13] without any success. In [15] it is proved that the authors of [13] detected classical TEM waves emitted by currents flowing in the Earth and launched by the ball antenna used in the experiment.

Here the contemporary status of our original approach started from [6–9] is presented.

2. Exact solution of the Maxwell system with gradient-type electric sources

As ordinarily in field theory, the system of units $\hbar = c = 1$ is used. Further, in the Minkowski space-time $M(1, 3) = \{x \equiv (x^\mu) = (x^0 = t, \vec{x} \equiv (x^j)); \mu = \overline{0, 3}, j = 1, 2, 3\}$, the variable x^μ denote the Cartesian (covariant) coordinates of the points of the physical space-time in the arbitrary-fixed inertial reference frame.

Consider the Maxwell equations in the form

$$\partial_0 \vec{E} - \text{curl} \vec{H} = -\text{grad} E^0, \quad \partial_0 \vec{H} + \text{curl} \vec{E} = 0, \quad (1)$$

$$\operatorname{div} \vec{E} = -\partial_0 E^0, \quad \operatorname{div} \vec{H} = 0.$$

Compare with the standard Maxwell equations for the free field

$$\begin{aligned} \partial_0 \vec{E} - \operatorname{curl} \vec{H} &= 0, & \partial_0 \vec{H} + \operatorname{curl} \vec{E} &= 0, \\ \operatorname{div} \vec{E} &= 0, & \operatorname{div} \vec{H} &= 0, \end{aligned} \quad (2)$$

and with the standard Maxwell system with electric sources.

From the physical point of view, equations (1) are the partial case of the standard Maxwell equations (the partial case of a standard classical electrodynamics). In this case, the current $\vec{j}(x)$ and charge $\rho(x)$ densities have the form

$$\vec{j}(x) = -\operatorname{grad} E^0(x), \quad \rho(x) = -\partial_0 E^0(x). \quad (3)$$

The mathematical point of view shows that objects (1) and (2) are the different partial differential equations. Indeed, the general solution of the system (2) is well known and is given by the transverse electromagnetic waves only

$$\begin{aligned} \vec{E}(x) &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left\{ \left[c_k^1 \vec{e}_1 + c_k^2 \vec{e}_2 \right] e^{-ikx} + \left[c_k^{*1} \vec{e}_1^* + c_k^{*2} \vec{e}_2^* \right] e^{ikx} \right\}, \\ \vec{H}(x) &= \frac{i}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left\{ \left[c_k^1 \vec{e}_1 - c_k^2 \vec{e}_2 \right] e^{-ikx} - \left[c_k^{*1} \vec{e}_1^* - c_k^{*2} \vec{e}_2^* \right] e^{ikx} \right\}. \end{aligned} \quad (4)$$

Here $(\vec{E}(x), \vec{H}(x))$ are the real electric and magnetic field strengths, c_k^1, c_k^2 are the complex quantum-mechanical momentum-helicity amplitudes of a photon (the amplitudes of the transverse electromagnetic waves),

$$kx \equiv \omega t - \vec{k}\vec{x}, \quad \omega \equiv \sqrt{\vec{k}^2}, \quad (5)$$

and the 3-component basis vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$, which, without any loss of generality, can be taken as

$$\vec{e}_1 = \frac{1}{\omega \sqrt{2(k^1 k^1 + k^2 k^2)}} \begin{vmatrix} \omega k^2 - ik^1 k^3 \\ -\omega k^1 - ik^2 k^3 \\ i(k^1 k^1 + k^2 k^2) \end{vmatrix}, \quad \vec{e}_2 = \vec{e}_1^*, \quad \vec{e}_3 = \frac{\vec{k}}{\omega}, \quad (6)$$

are the eigen vectors of the quantum-mechanical helicity operator for the spin $s = 1$.

The general solution (4) is ordinarily found by the Fourier method, and the normalization factor $C \equiv \sqrt{\frac{\omega}{2(2\pi)^3}}$ in (4) is taken from the condition

$$P_0 = \frac{1}{2} \int d^3x \left(\vec{E}^2 + \vec{H}^2 \right) = \int d^3k \omega \left(|c_k^1|^2 + |c_k^2|^2 \right). \quad (7)$$

It is interesting to find the general solution of the system (1), which is expected to have another (maybe not only transverse) form. The Fourier method in the corresponding rigged Hilbert space leads to the general solution

$$\begin{aligned} \vec{E}(x) &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left\{ \left[c_k^1 \vec{e}_1 + c_k^2 \vec{e}_2 + \alpha_k \vec{e}_3 \right] e^{-ikx} + \left[c_k^{*1} \vec{e}_1^* + c_k^{*2} \vec{e}_2^* + \alpha_k^* \vec{e}_3^* \right] e^{ikx} \right\}, \\ \vec{H}(x) &= \frac{i}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left\{ \left[c_k^1 \vec{e}_1 - c_k^2 \vec{e}_2 \right] e^{-ikx} - \left[c_k^{*1} \vec{e}_1^* - c_k^{*2} \vec{e}_2^* \right] e^{ikx} \right\}, \end{aligned} \quad (8)$$

$$E^0(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left(\alpha_{\vec{k}} e^{-ikx} + \alpha_{\vec{k}}^* e^{ikx} \right).$$

It is easy to see that here the electric field strength $\vec{E}(x)$ contains (together with the ordinary transverse waves) the longitudinal wave as well. This longitudinal electric wave is determined by the amplitude $\alpha_{\vec{k}}$. The scalar function $E^0(x)$ specifies the electric current and charge densities in the Maxwell system (1). The scalar wave $E^0(x)$ is longitudinal as well and is determined by the amplitude $\alpha_{\vec{k}}$.

The validity of the solution (8) can be verified by the direct substitution of (8) into equations (1).

3. Exact solution of the Maxwell system with gradient-type electric and magnetic sources

Consider the Maxwell-like system of equations

$$\begin{aligned} \partial_0 \vec{E} - \text{curl} \vec{H} &= -\text{grad} E^0, & \partial_0 \vec{H} + \text{curl} \vec{E} &= -\text{grad} H^0, \\ \text{div} \vec{E} &= -\partial_0 E^0, & \text{div} \vec{H} &= -\partial_0 H^0. \end{aligned} \quad (9)$$

Equations (9) contain both electric and magnetic gradient-type current and charge densities. System (9) is directly related to the massless Dirac equation (see e.g. the consideration in [17–19] and the references therein).

The general solution of the system (9) is given by

$$\begin{aligned} \vec{E}(x) &= \int d^3k C \left\{ \left[c_k^1 \vec{e}_1 + c_k^2 \vec{e}_2 + (c_k^3 + c_k^4) \vec{e}_3 \right] e^{-ikx} + A \right\}, \\ A &\equiv \left[c_k^{*1} \vec{e}_1^* + c_k^{*2} \vec{e}_2^* + (c_k^{*3} + c_k^{*4}) \vec{e}_3^* \right] e^{ikx}, \\ \vec{H}(x) &= i \int d^3k C \left\{ \left[c_k^1 \vec{e}_1 - c_k^2 \vec{e}_2 + (c_k^3 - c_k^4) \vec{e}_3 \right] e^{-ikx} - B \right\}, \\ B &\equiv \left[c_k^{*1} \vec{e}_1^* - c_k^{*2} \vec{e}_2^* + (c_k^{*3} - c_k^{*4}) \vec{e}_3^* \right] e^{ikx}, \\ E^0(x) &= \int d^3k C \left[(c_k^3 + c_k^4) e^{-ikx} + (c_k^{*3} + c_k^{*4}) e^{ikx} \right], \\ H^0(x) &= i \int d^3k C \left[(c_k^3 - c_k^4) e^{-ikx} - (c_k^{*3} - c_k^{*4}) e^{ikx} \right], \end{aligned} \quad (10)$$

$C \equiv \sqrt{\frac{\omega}{2(2\pi)^3}}$ is the normalization factor. Here both the electric field strength $\vec{E}(x)$ and the magnetic field strength $\vec{H}(x)$ contain (together with ordinary transverse waves) the corresponding longitudinal waves as well. These longitudinal electric and longitudinal magnetic wave are given by the amplitudes $c_k^3 + c_k^4$ and $c_k^3 - c_k^4$, respectively. The scalar functions ($E^0(x)$, $H^0(x)$) specify the electromagnetic currents and charges densities in the Maxwell-like system of equations (9). The scalar waves ($E^0(x)$, $H^0(x)$) are longitudinal as well.

In our articles [17–19], the system of equations (9) is called as the slightly generalized Maxwell equations with gradient-type sources. Indeed, at the first step, the system (9) is the generalization of the Maxwell equations because it contains the condition $\text{div} \vec{H} \neq 0$ and the nonzero magnetic current density in the equation $\partial_0 \vec{H} + \text{curl} \vec{E} = -\text{grad} H^0$. Nevertheless, at the second step the system (9) is the simplification (specification) of the Maxwell equations. Its

electromagnetic currents and charge densities are the partial gradient-type forms of the general form of the electromagnetic sources.

Thus, the system of equations (9) due to the conditions $\partial_0 \vec{H} + \text{curl} \vec{E} \neq 0$, $\text{div} \vec{H} \neq 0$ is not the standard Maxwell electrodynamics. Therefore, it is better to start the experimental detection of the longitudinal electromagnetic waves in the experimental modeling of the situation given by the system (1), which is inside the standard Maxwell electrodynamics. This system of equations predicts an existence of the longitudinal component of the vector of electric field strength \vec{E} .

The arguments in the prospect of system (9) are not so evident. Nevertheless, let us mention (i) that system (9) has the maximally possible symmetry properties among the Maxwell and the Maxwell-like systems of equations. In [17] the 256 dimensional algebra of invariance of equations (9) was mentioned. The equations (9) have been considered in the terms of complex functions

$$\mathcal{E} \equiv E - iH = \begin{vmatrix} \vec{E} - i\vec{H} \\ E^0 - iH^0 \end{vmatrix}. \quad (11)$$

Taking into account the symmetries found recently in [20–26], the number 256 can be increased. Let us recall that the role of the symmetry principle in electrodynamics is known from the times of Maxwell and Heaviside.

(ii) The system of equation (9) has the property of the Fermi–Bose duality. It can describe both the massless spin 1/2 particle–antiparticle doublet of the fermions and the massless spin (1,0) doublet of bosons (photon and massless spinless boson). In the terms of complex 4-vector (11) equations (9) has the manifestly covariant form

$$\partial_\mu \mathcal{E}_\nu - \partial_\nu \mathcal{E}_\mu + i\varepsilon_{\mu\nu\rho\sigma} \partial^\rho \mathcal{E}^\sigma = 0, \quad \partial_\mu \mathcal{E}^\mu = 0, \quad (12)$$

and the form of massless Dirac equation

$$\tilde{\gamma}^\mu \partial_\mu \mathcal{E} = 0, \quad (13)$$

with specific Clifford–Dirac algebra γ matrix representation

$$\tilde{\gamma}^0 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} C, \quad \tilde{\gamma}^1 = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -1 & 0 & 0 & 0 \end{vmatrix} C, \quad (14)$$

$$\tilde{\gamma}^2 = \begin{vmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \\ -i & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix} C, \quad \tilde{\gamma}^3 = \begin{vmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{vmatrix} C,$$

where C is the operator of complex conjugation, $C\mathcal{E} \equiv \mathcal{E}^*$, $\tilde{\gamma}^\mu \tilde{\gamma}^\nu + \tilde{\gamma}^\nu \tilde{\gamma}^\mu = 2g^{\mu\nu}$.

(iii) The system of equations (9) considered in the specific medium [27, 28] can be applied to inneratomic phenomena and describe the hydrogen spectrum [17–19] (another approach to the hydrogen spectrum description on the basis of stationary Maxwell equations in this medium is given in [29, 30]). In [29, 30] and [17–19] the role of longitudinal components is evident.

Therefore, there is some sense in experimental modeling the situation given by the system (9) in the problem of the longitudinal electromagnetic waves investigation. Of course, the advantages of system (1), which is inside the standard electrodynamics, are evident.

Note that solution (8) can be found both by the direct application of the Fourier method and as a partial case of solution (10).

The validity of solution (10) can be verified by the direct substitution of (10) into equations (9).

4. The subsystems of the Maxwell-like equations with gradient-type electric and magnetic sources

Due to the above mentioned properties, the system of equations (1) is the most interesting subsystem of the Maxwell-like equations (9) (in the problem of longitudinal electromagnetic waves consideration). Equations (1) follow from equations (9) after substitution $H^0 = 0 \Rightarrow c_k^3 = c_k^4$. In this case, in solution (8) the amplitude of the longitudinal wave $\alpha_{\vec{k}} = 2c_k^3$. This partial case of (9) is presented in Sec. 2 in details.

Another interesting subsystem of (9) follows from (9) after the substitution $E^0(x) = 0$. In this case, the vector $\vec{H}(x)$ of the magnetic field strength contains the longitudinal wave component. Other properties of the general solution are similar to (8) and are to (8) in evident symmetry. Nevertheless, the interest to this case is lower then for $H^0 = 0$ case. The conditions $\text{div} \vec{H} \neq 0$ and $\partial_0 \vec{H} + \text{curl} \vec{E} \neq 0$ are not in the framework of the standard Maxwell electrodynamics.

5. Conclusion

The electric field strength vector $\vec{E}(x)$ that contains the longitudinal wave component is found as the exact solution of the standard Maxwell equations with the partial gradient case of electric current and charge densities (3). The explicit form of this solution is given in (8). Note that the set of general solutions (8) of such Maxwell equations contains the vector $\vec{E}(x)$ together with the scalar field $E^0(x)$, which is longitudinal as well. The validity of the solution (8) can be verified by the direct substitution of (8) into equations (1). Thus, the link between the longitudinal electric waves and the standard Maxwell electrodynamics is demonstrated. The author has a hope that corresponding experimental situation can be created by modeling the sources (3) and may be useful. Moreover, the experiments in [1–4] look like to be the physical reality of such longitudinal electric waves.

The analysis of (3) and (8) demonstrates that longitudinal components are located near the current and charge densities. Indeed, the current and charge densities (3) and the longitudinal components in the solution (8) are determined by the same amplitudes $\alpha_{\vec{k}}, \alpha_{\vec{k}}^*$. The best examples of corresponding physical reality are such big charges as the whole water area of closed sea, the Earth in general, their oscillations and corresponding longitudinal electric and scalar waves. The subclass of the functions $\vec{j}(x), \rho(x)$ from (3) is generated by the oscillating charges in plasma, plasmons and laser beams [1–4]. Longitudinal electric wave in [1–4] appears after the interaction of circularly polarized transverse electromagnetic waves. The authors are considered now the possibility of experiment in solid samples.

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References

- [1] Miyaji G, Miyanaga N, Tsubakimoto K, Sueda K and Ohbayashi K 2004 *Appl. Phys. Lett.* **84** 3855
- [2] Dorn R, Quabis S and Leuchs G 2003 *Phys. Rev. Lett.* **91** 233901
- [3] Niziev V G, Yakunin V P and Turkin N G 2009 *Quantum Electronics* **39** 505
- [4] Tidwell S C, Ford D H and Kimura W D 1990 *Appl. Opt.* **29** 2234
- [5] Ferraro V C A and Flint H T 1941 *Proc. Phys. Soc.* **53** 170
- [6] Khvorostenko N P 1992 *Rus. Phys. J.* **35** 223
- [7] Simulik V M 1991 *Theor. Math. Phys.* **87** 386
- [8] Krivsky I Yu and Simulik V M 1997 General solution of the Maxwell equations with gradient-like sources and longitudinal electromagnetic waves *Proceedings of the First International conference "Non-Euclidean geometry in modern physics. BGL-1."* Uzhgorod, Ukraine 193–199
- [9] Krivsky I Yu and Simulik V M 1998 *Uzhgorod University Sci. Herald, Ser. Phys.* **2** 121

- [10] Simulik V M 2016 Longitudinal electromagnetic waves in the framework of standard classical electrodynamics *arXiv*: 1606.01738 [physics. class-ph]
- [11] Barashenkov V S, Pestov A V and Yuriev M Z 2001 Transmission of information by longitudinal electromagnetic waves *Preprint of the Joint Institute for Nuclear Research. Dubna*. P2-2001-253 (in russian)
- [12] K.J. Van Vlaenderen K J and Waser A 2001 *Hadr. J.* **24** 609
- [13] Monstein C and Wesley J P 2002 *Europhys. Lett.* **59** 514
- [14] Bray J R and Britton M C 2004 *Europhys. Lett.* **66** 153
- [15] Rebilas K 2008 *Europhys. Lett.* **83** 60007
- [16] Butterworth E J, Allison C B, Cavazos D and Mullen F M 2013 *J. Sci. Explor.* **27** 13
- [17] Simulik V M and Krivsky I Yu 2002 *Ann. Fond. L. de Broglie* **27** 303
- [18] Simulik V M and Krivsky I Yu 2002 *Rep. Math. Phys.* **50** 315
- [19] Simulik V M and Krivsky I Yu 2003 *Electromagnetic Phenomena* **3** 103
- [20] Simulik V M and Krivsky I Yu 2009 Extended real Clifford–Dirac algebra and bosonic symmetries of the Dirac equation with nonzero mass *arXiv*: 0908.3106 [math-ph]
- [21] Simulik V M and Krivsky I Yu 2010 *Reports of the National Academy of Sciences of Ukraine* **5** 82 (in Ukrainian)
- [22] Krivsky I Yu and Simulik V M 2010 *Cond. Matt. Phys.* **13** 43101(1–15)
- [23] Simulik V M and Krivsky I Yu 2011 *Phys. Lett. A* **375** 2479
- [24] Simulik V M, Krivsky I Yu and Lamer I L 2012 *Cond. Matt. Phys.* **15** 43101(1–10)
- [25] Simulik V M, Krivsky I Yu and Lamer I L 2013 *TWMS J. App. Eng. Math.* **3** 46
- [26] Simulik V M, Krivsky I Yu and Lamer I L 2013 *Ukr. J. Phys.* **58** 523
- [27] Sallhofer H 1978 *Z. Naturforsch. A* **33** 1378
- [28] Sallhofer H 1986 *Z. Naturforsch. A* **41** 468
- [29] Sallhofer H 1990 *Z. Naturforsch. A* **45** 1361
- [30] Sallhofer H 1991 *Z. Naturforsch. A* **46** 869