УДК 519.21
DOI https://doi.org/10.24144/2616-7700.2020.2(37).122-129

R. Yamnenko[1], **N. Yurchenko**[2]

[1] Taras Shevchenko National University of Kyiv,
As. Professor at the Department of Probability Theory, Statistics and Actuarial Mathematics,
Ph. D. (Phys.- Math.)
yamnenko@univ.kiev.ua
ORCID: https://orcid.org/0000-0002-9612-7959

[2] Uzhhorod National University,
Department of Algebra, Associate Professor,
Ph. D. (Phys.- Math.)
nataliia.yurchenko@uzhnu.edu.ua
ORCID: https://orcid.org/0000-0002-2825-8180

ON AN ESTIMATE OF PROBABILITY OF EXCEEDING A LINE BY WEIGHTED AGGREGATE OF SUB-GAUSSIAN RANDOM PROCESS

Sub-gaussian random variables are majorized in distribution by Gaussian random variables, and thus are their natural generalization. This paper considers the problem of estimating the probability of exceeding a level given by a line ct, $c > 0$, by trajectories of the sum of sub-Gaussian random processes X_i, $i = \overline{1,n}$, defined on a compact set B with certain weighting functions $w_i(t)$. Namely, upper estimates of the following type $\mathbf{P}\left\{\sup_{t\in B}\left(\sum_{i=1}^{n} w_i\left(t\right)X_i(t)-ct\right) > x\right\}$, $\mathbf{P}\left\{\inf_{t\in B}\left(\sum_{i=1}^{n} w_i\left(t\right)X_i(t)-ct\right) < -x\right\}$ or $\mathbf{P}\left\{\sup_{t\in B}\left|\sum_{i=1}^{n} w_i\left(t\right)X_i(t)-ct\right| > x\right\}$ are derived. This problem can be applied directly in the queuing theory in estimating the finite size $x > 0$ buffer overflow probability with linear service intensity, as well as in insurance mathematics in estimating the bankruptcy probability for the corresponding risk process. Using the method of metric entropy, the previous results obtained in [1] for a more general class of Φ-sub-Gaussian random processes are generalized and improved. As an example, the derived estimate is applied to the average sum of sub-Gaussian Wiener random processes, i.e. random processes that have the same covariance function as the (Gaussian) Wiener process, but with sub-Gaussian trajectories.

Ключові слова: sub-Gaussian random process, supremum distribution, method of metric entropy, Wiener process.

1. Introduction. In this paper a weighted aggregate of independent sub-Gaussian random processes defined on a compact set are considered and the probability that such its trajectories exceeds some linear function is investigated. The problem of such type was previously studied for random processes from various Orlicz spaces (including sub-Gaussian) in works [1-3].

Definition 1. *([4]) A random variable ξ is called sub-Gaussian if there exists a number $a \in [0,\infty)$ such that the inequality*

$$\mathbf{E}\exp\{\lambda\xi\} \le \exp\left\{\frac{a^2\lambda^2}{2}\right\} \tag{1}$$

holds for all $\lambda \in \mathbb{R}$. The class of all sub-Gaussian random variables defined on a common probability space $(\Omega, \mathcal{F}, \mathbf{P})$ is denoted by $\mathrm{Sub}(\Omega)$.

Recall (see [4, Theorem 1.2]) that the space $\mathrm{Sub}(\Omega)$ is a Banach space with respect to the norm

$$\tau\left(\xi\right) = \inf\left\{a \ge 0 :\ \mathbf{E}\exp\left\{\lambda\xi\right\} \le \exp\left\{\frac{a^2\lambda^2}{2}\right\},\ \lambda \in \mathbb{R}\right\} \tag{2}$$

and (see [4, Lemma 1.1])

$$\mathbf{E}\exp\{\lambda\xi\} \le \exp\left\{\frac{\lambda^2 \tau(\xi)^2}{2}\right\}. \tag{3}$$

Other properties of sub-Gaussian random variables and processes can be found in the classical monograph of Buldygin V. and Kozachenko Yu. [4] and in the book [2], properties of more general class of φ-sub-Gaussian random variables and processes are reviewed in the book of Vasylyk O., Kozachenko Yu. and Yamnenko R. [5]. Here the results obtained in [1] are improved for the specific case of linear function.

2. Main result. Let (T,ρ) be a pseudometric (metric) separable space with psedometric (metric) ρ. Recall that $N_{(T,\rho)}(\epsilon) = N_T(\epsilon)$ is the metric massiveness of the space (T,ρ), i.e. the number of elements in the minimal ϵ-covering of the set T.

Consider a set of independent separable sub-Gaussian random processes $X_i = \{X_i(t), t \in T\}, i = \overline{1,n}$ satisfying the following assumption.

Assumption 1. *There exist such continuous monotone increasing functions $\sigma_i = \{\sigma_i(h), h > 0\}$ such that $\sigma_i(h) \to 0$ as $h \to 0$ and the following inequality holds true*

$$\sup_{\rho(t,s)\le h} \tau(X_i(t) - X_i(s)) \le \sigma_i(h), \quad i = \overline{1,n}.$$

Consider the problem of exceeding by mixture of processes X_i a line on a compact set $B \subset T$.

Put $\tau_i(t) = \tau(X_i(t))$, $\sigma(h) = \max_{i=\overline{1,n}} \sigma_i(h)$ and let $\beta > 0$ be such a number that $\beta \le \sigma(\inf_{s\in B}\sup_{t\in B}\rho(t,s))$.

Theorem 1. *Let $X_i = \{X_i(t), t \in T\}$ be separable sub-Gaussian random processes satisfying Assumption 1 with functions $\sigma_i(h) \le d_i\sigma(h)$, $0 < d_i \le 1, i = 1,\ldots,n$. Let $r = \{r(u) : u \ge 1\}$ be such a continuous function that $r(u) > 0$ as $u > 1$, $s(t) = r(e^t)$ is a convex function for $t \ge 0$ and the following entropy integral is finite*

$$\int_0^\beta r\left(N_B\left(\sigma^{(-1)}(u)\right)\right) \mathrm{d}u < \infty.$$

Then for all $p \in (0;1)$, $c > 0$ and $x > \max\left\{0, \sigma^{(-1)}(\beta p) - \min_{u\in B} u\right\}$ the following inequalities take places for the random mixture

$$X(t) = \sum_{i=1}^n w_i(t)\ X_i(t),$$

where $w_i(t) = \{w_i(t), t \in T\}$ are continuous nonnegative weighting functions

$$\mathbf{P}\left\{\sup_{t\in B}(X(t) - ct) > x\right\} \le Z(p,\beta,x),$$

$$\mathbf{P}\left\{\inf_{t\in B}(X(t) - ct) < -x\right\} \le Z(p,\beta,x),$$

$$\mathbf{P}\left\{\sup_{t\in B}|X(t) - ct| > x\right\} \le 2Z(p,\beta,x),$$

where

$$Z(p,\beta,x)=\inf_{p\in(0,1)} \quad r^{(-1)}\left(\frac{1}{\beta p}\int\limits_0^{\beta p} r\left(N_B\left(\sigma^{(-1)}(u)\right)\right)\mathrm{d}u\right)\times$$

$$\times\exp\left\{-\inf_{u\in B}\frac{c^2\left(x+u-\sigma^{(-1)}(\beta p)\right)^2(1-p)^2}{2\left((1-p)\sum\limits_{i=1}^{n}w_i^2(u)\,\tau_i^2(u)+\beta^2 p\sum\limits_{i=1}^{n}d_i^2\max_{v\in B}w_i^2(v)\right)}\right\}.$$

Proof. Let V_{ϵ_k} denote a set of the centers of closed balls with radii $\epsilon_k=\sigma^{(-1)}\left(\beta p^k\right)$, $p\in(0,1)\,, k=0,1,2,\ldots,$ which forms minimal covering of the compact space (B,ρ). Number of elements in the set V_{ϵ_k} is equal to $N_B(\epsilon_k)\,.$ It follows from [4, Lemma 1.3] and Assumption 1 that for any $\epsilon>0$

$$\mathbf{P}\left\{|X_i(t)-X_i(s)|>\epsilon\right\}$$

$$\leq 2\exp\left\{-\frac{\epsilon^2}{2\tau^2\left(X_i(t)-X_i(s)\right)}\right\}\leq 2\exp\left\{-\frac{\epsilon^2}{2\sigma_i\left(\rho(t,s)\right)}\right\}.$$

Therefore all processes $X_i(t)$ are continuous in probability and the mixture $X= =\sum_{i=1}^n w_i(t)X_i(t)$ is continuous in probability as well. Hence the set $V=\bigcup_{k=1}^{\infty}V_{\epsilon_k}$ is a set of separability of the process X and with probability one

$$\sup_{t\in\ B}(X(t)-ct)\ =\sup_{t\in\ V}(X(t)-ct) \tag{4}$$

Consider a mapping $\alpha_m=\{\alpha_m(t), m=0,1\ldots\}$ of the set V into the subset V_{ϵ_m}, where $\alpha_m(t)\in \mathrm{V}_{\epsilon_\mathrm{m}}$ is such a point that $\rho(t,\alpha_m(t))<\epsilon_m$. If $t\in V_{\epsilon_m}$ then $\alpha_m(t)=t$. If there exist several such points from the set V_{ϵ_m} that $\rho(t,\alpha_m(t))<\epsilon_m$ then we choose one of them and denote it $\alpha_m(t)$.

It follows from [4, Lemma 1.2], Chebyshev's inequality and Assumption 1 that

$$\mathbf{P}\left\{|X_i(t)-X_i(\alpha_m(t))|>p^{\frac{m}{2}}\right\}\leq\frac{\mathbf{E}(X_i(t)-X_i(\alpha_m(t)))^2}{p^m}$$

$$\leq\frac{\tau^2\left(X_i(t)-X_i(\alpha_m(t))\right)}{p^m}\leq\frac{\sigma^2(\epsilon_m)}{p^m}=\beta^2p^m.$$

This inequality implies that

$$\sum_{n=1}^{\infty}\mathbf{P}\left\{|X_i(t)-X_i(\alpha_m(t))|>p^{\frac{m}{2}}\right\}<\infty.$$

It follows from the Borel-Kantelli lemma that $X_i(t)-X_i(\alpha_m(t))\to 0$ as $m\to\infty$ with probability one. Since the set V is countable, then $[X(t)-ct]-[X(\alpha_m(t))- -c\alpha_m(t)]\to\ 0$ as $n\to\infty$ for all t simultaneously.

Let t be an arbitrary point from the set V. Denote by $t_m=\alpha_m(t)$, $t_{m-1}=\alpha_{m-1}(t_m)\,,\ldots,t_1=\alpha_1(t_2)$ for any $m\geq 1$. Since for all $m\geq 2$

$$X(t)-ct=X(t_1)-ct_1+c(t_1-t)+\sum_{k=2}^{m}\left(X(t_k)-X(t_{k-1})\right)$$

$$+X(t) - X(\alpha_m(t))$$

we have

$$\sup_{t\in V}(X(t)-ct) \le \max_{u\in V_{\epsilon_1}}(X(u)-cu) + c\epsilon_1 + \sum_{k=2}^{m}\max_{u\in V_{\epsilon_k}}(X(u)-X(\alpha_{k-1}(u))$$

$$+X(t)-X(\alpha_m(t)). \qquad (5)$$

It follows from (4) and (5) that with probability one

$$\sup_{t\in T}(X(t)-ct) \le c\epsilon_1 +$$

$$+\lim_{m\to\infty}\inf\left(\max_{u\in V_{\epsilon_1}}(X(u)-cu) + \sum_{k=2}^{m}\max_{u\in V_{\epsilon_k}}(X(u)-X(\alpha_{k-1}(u))\right). \qquad (6)$$

Let $\{q_k, k=1,2,\dots\}$ be such a sequence that $q_k>1$ and $\sum_{k=1}^{\infty} q_k^{-1} \le 1$. It follows from the Hölder's inequality, the Fatou's lemma and (6) that for all $\lambda>0$

$$\mathbf{E}\exp\left\{\lambda\sup_{t\in B}(X(t)-ct)\right\}$$

$$\le \mathbf{E}\lim_{m\to\infty}\inf\exp\left\{\lambda\left(c\epsilon_1 + \max_{u\in V_{\epsilon_1}}(X(u)-cu) + \sum_{k=2}^{m}\max_{u\in V_{\epsilon_k}}(X(u)-X(\alpha_{k-1}(u))\right)\right\}$$

$$\le \lim_{m\to\infty}\inf\mathbf{E}\exp\left\{\lambda\left(c\epsilon_1 + \max_{u\in V_{\epsilon_1}}(X(u)-cu) + \sum_{k=2}^{m}\max_{u\in V_{\epsilon_k}}(X(u)-X(\alpha_{k-1}(u))\right)\right\}$$

$$\le \lim_{m\to\infty}\inf\left(\mathrm{E}\exp\left\{q_1\lambda\max_{u\in V_{\epsilon_1}}(X(u)-cu)\right\}\right)^{\frac{1}{q_1}} \times$$

$$\times\prod_{k=2}^{m}\left(\mathbf{E}\exp\left\{q_k\lambda\max_{u\in V_{\epsilon_k}}(X(u)-X(\alpha_{k-1}(u))\right\}\right)^{\frac{1}{q_k}}\exp\{\lambda c\epsilon_1\}$$

$$\le \left(\mathbf{E}\exp\left\{q_1\lambda\max_{u\in V_{\epsilon_1}}(X(u)-cu)\right\}\right)^{\frac{1}{q_1}} \times$$

$$\times\prod_{k=2}^{\infty}\left(\mathbf{E}\exp\left\{q_k\lambda\max_{u\in V_{\epsilon_k}}(X(u)-X(\alpha_{k-1}(u))\right\}\right)^{\frac{1}{q_k}}\exp\{\lambda c\epsilon_1\}\ . \qquad (7)$$

Consider each of the factors in the right-hand side of inequality (6) separately. It follows from inequality (3) that for all $1\le i\le n$

$$\mathbf{E}\exp\{q_1\lambda w_i(u)X_i(u)\} \le \exp\left\{\frac{q_1^2\lambda^2 w_i^2(u)\tau_i^2(u)}{2}\right\}$$

and

$$\mathbf{E}\exp\{q_k\lambda(w_i(u)X_i(u) - w_i(\alpha_{k-1}(u))X_i(\alpha_{k-1}(u)))\} \le \exp\left\{\frac{q_k^2\lambda^2 w_i^2(u)\sigma_i^2(\epsilon_{k-1})}{2}\right\}.$$

Therefore,

$$\left(\mathbf{E}\exp\left\{q_1\lambda\max_{u\in V_{\epsilon_1}}\left(X\left(u\right)-cu\right)\right\}\right)^{\frac{1}{q_1}}\leq$$

$$\leq\left(\sum_{u\in V_{\epsilon_1}}\mathbf{E}\exp\left\{q_1\lambda\sum_{i=1}^{n}w_i\left(u\right)X_i\left(u\right)\right\}\exp\left\{-q_1\lambda cu\right\}\right)^{\frac{1}{q_1}}\leq$$

$$\leq\left(\sum_{u\in V_{\epsilon_1}}\prod_{i=1}^{n}\mathbf{E}\exp\left\{q_1\lambda w_i\left(u\right)X_i\left(u\right)\right\}\exp\left\{-q_1\lambda cu\right\}\right)^{\frac{1}{q_1}}\leq$$

$$\leq\left(N_B\left(\epsilon_1\right)\right)^{\frac{1}{q_1}}\exp\left\{\sup_{u\in B}\left(\frac{q_1\lambda^2}{2}\sum_{i=1}^{n}w_i^2(u)\tau_i^2\left(u\right)-\lambda cu\right)\right\}$$

and

$$\left(\mathbf{E}\exp\left\{q_k\lambda\max_{u\in V_{\epsilon_k}}(X(u)-X(\alpha_{k-1}(u))\right\}\right)^{\frac{1}{q_k}}\leq$$

$$\leq\left(N_B\left(\epsilon_k\right)\max_{u\in V_{\epsilon_k}}\mathbf{E}\exp\left\{q_k\lambda\sum_{i=1}^{n}w_i\left(u\right)\left(X_i(u)-X_i\left(\alpha_{k-1}\left(u\right)\right)\right)\right\}\right)^{\frac{1}{q_k}}\leq$$

$$\leq\left(N_B\left(\epsilon_k\right)\right)^{\frac{1}{q_k}}\left(\max_{u\in V_{\epsilon_k}}\exp\left\{\frac{q_k^2\lambda^2}{2}\sum_{i=1}^{n}w_i^2\left(u\right)\sigma_i^2\left(\epsilon_{k-1}\right)\right\}\right)^{\frac{1}{q_k}}$$

$$\leq\left(N_B\left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right)^{\frac{1}{q_k}}\exp\left\{\frac{q_k\lambda^2\beta^2p^{2(k-1)}}{2}\sum_{i=1}^{n}d_i^2\max_{u\in B}w_i^2\left(u\right)\right\}.$$

From inequality (7) after substitution of $q_k=p^{1-k}/(1-p)$, $k\geq1$, we have

$$\mathbf{E}\exp\left\{\lambda\sup_{t\in B}\left(X\left(t\right)-ct\right)\right\}\leq$$

$$\leq\prod_{k=1}^{\infty}\left(N_B\left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right)^{(1-p)p^{k-1}}\exp\left\{\sup_{\mathrm{u}\in\mathrm{B}}\left(\frac{\lambda^2}{2\left(1-\mathrm{p}\right)}\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{w}_{\mathrm{i}}^2\left(\mathrm{u}\right)\tau_{\mathrm{i}}^2\left(\mathrm{u}\right)-\lambda\mathrm{cu}\right)+\right.$$

$$\left.+\sum_{k=2}^{\infty}\frac{\lambda^2\beta^2p^{k-1}}{2\left(1-p\right)}\sum_{i=1}^{n}d_i^2\max_{u\in B}w_i^2\left(u\right)+\lambda c\sigma^{(-1)}\left(\beta p\right)\right\}=$$

$$=\prod_{k=1}^{\infty}\left(N_B\left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right)^{(1-p)p^{k-1}}\exp\left\{\sup_{\mathrm{u}\in\mathrm{B}}\left(\frac{\lambda^2}{2\left(1-\mathrm{p}\right)}\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{w}_{\mathrm{i}}^2\left(\mathrm{u}\right)\tau_{\mathrm{i}}^2\left(\mathrm{u}\right)-\lambda\mathrm{cu}\right)+\right.$$

$$\left.+\frac{\lambda^2\beta^2p}{2(1-p)^2}\sum_{i=1}^{n}d_i^2\max_{v\in B}w_i^2\left(v\right)+\lambda c\sigma^{(-1)}\left(\beta p\right)\right\}=$$

$$=\prod_{k=1}^{\infty}\left(N_B\left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right)^{(1-p)p^{k-1}}\exp\left\{\sup_{\mathrm{u}\in\mathrm{B}}\left(\frac{\lambda^2}{2\left(1-\mathrm{p}\right)}\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{w}_{\mathrm{i}}^2\left(\mathrm{u}\right)\tau_{\mathrm{i}}^2\left(\mathrm{u}\right)-\lambda\mathrm{cu}\right)+\right.$$

$$+\frac{\lambda^2\beta^2 p}{2(1-p)^2}\sum_{i=1}^{n} d_i^2 \max_{v\in B} w_i^2(v) \ + \lambda c\sigma^{(-1)}(\beta p)\Bigg\} \qquad (8)$$

From (8) and Chebyshev's inequality

$$\mathbf{P}\left\{\sup_{t\in B}(X(t)-f(t))>x\right\}$$

$$\leq \inf_{\Lambda>0}\prod_{k=1}^{\infty}\left(N_B\left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right)^{(1-p)p^{k-1}}\exp\left\{\sup_{\mathrm{u}\in\mathrm{B}}\left(\frac{\lambda^2}{2(1-\mathrm{p})}\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{w}_{\mathrm{i}}^2(\mathrm{u})\,\tau_{\mathrm{i}}^2(\mathrm{u})-\lambda\mathrm{cu}+\right.\right.$$

$$\left.\left.+\frac{\lambda^2\beta^2 p}{2(1-p)^2}\sum_{i=1}^{n} d_i^2 \max_{v\in B} w_i^2(v) \ + \lambda c\sigma^{(-1)}(\beta p)-\lambda x\right)\right\}. \qquad (9)$$

Polynomial $\lambda^2 A-\lambda B$ attaints its minimum at the point $\lambda=\frac{B}{2A}$, therefore

$$\inf_{\Lambda>0}\left[\lambda^2\left(\frac{1}{2(1-p)}\sum_{i=1}^{n} w_i^2(u)\,\tau_i^2(u)+\frac{\beta^2 p}{2(1-p)^2}\sum_{i=1}^{n} d_i^2\max_{v\in B} w_i^2(v)\right)-\lambda c\cdot\right.$$

$$\left.\cdot\left(u+x-\sigma^{(-1)}(\beta p)\right)\right]=-\frac{c^2\left(x+u-\sigma^{(-1)}(\beta p)\right)^2(1-p)^2}{2\left((1-p)\sum_{i=1}^{n} w_i^2(u)\,\tau_i^2(u)+\beta^2 p\sum_{i=1}^{n} d_i^2\max_{v\in B} w_i^2(v)\right)}$$

if $x+u>\sigma^{(-1)}(\beta p)$ and 0 otherwise due to the restriction $\lambda>0$.

Since

$$\prod_{k=1}^{\infty}\left(N_B\left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right)^{(1-p)p^{k-1}} =$$

$$= r^{(-1)}\left(r\left(\exp\left\{\sum_{k=1}^{\infty}(1-p)\,p^{k-1}\ln N_B\ \left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right\}\right)\right)\leq$$

$$\leq r^{(-1)}\left(\sum_{k=1}^{\infty}(1-p)\,p^{k-1} r\left(N_B\left(\sigma^{(-1)}\left(\beta p^k\right)\right)\right)\right)$$

$$\leq r^{(-1)}\left(\frac{1}{\beta p}\int_0^{\beta p} r\left(N_B\left(\sigma^{(-1)}(u)\right)\right)\mathrm{d}u\right), \qquad (10)$$

the assertion of theorem follows from (8) – (10).

Example 1. Consider a mixture of independent sub-Gaussian Wiener processes $W_i=\{W_i(t),\ t\in[a,b]\}$ with constant weighting functions $w_i(t)=\frac{1}{n}$. Recall that a sub-Gaussian Wiener process $W=\{W(t),\ t\in T\}$ is a sub-Gaussian random process with covariance function $R(t,s)=\min(t,s)\ ,\ t,s\in T$.

Let's assume that for $0<d_1\leq\cdots\leq d_n=1$ we have $\tau_i(u)=d_i u^{\frac{1}{2}}$ and $\sigma_i(u)=d_i u^{\frac{1}{2}}$. It is easy to see that $\sigma^{(-1)}(u)=u^2$ and

$$\inf_{u\in B}\frac{c^2\left(x+u-\sigma^{(-1)}(\beta p)\right)^2(1-p)^2}{2\left((1-p)\sum_{i=1}^{n} w_i^2(u)\,\tau_i^2(u)+\beta^2 p\sum_{i=1}^{n} d_i^2\max_{v\in B} w_i^2(v)\ \right)} =$$

$$=\inf_{u\in B}\frac{n^2c^2(1-p)\left(x+u-(\beta p)^2\right)^2}{2\sum\limits_{i=1}^{n} d_i^2\left(u+\frac{\beta^2 p}{1-p}\right)}=\frac{2n^2c^2(1-p)}{\sum\limits_{i=1}^{n} d_i^2}\left(x-(\beta p)^2-\frac{\beta^2 p}{1-p}\right).$$

Put $r(u) = u^{\alpha}, \alpha < \frac{1}{2}$. Then (see, e.g. [1])

$$\exp\left\{\frac{1}{\beta p}\int_0^{\beta p} r\left(N_B\left(\sigma^{(-1)}(u)\right)\right)\mathrm{d}u\right\} \le \frac{2\left((b-a)\right)}{(\beta p)^2}(1-2\alpha)^{-\frac{1}{2}} \to$$

$$\to 2(b-a)\left(\frac{e}{\beta p}\right)^2 \text{ as } \alpha \to 0.$$

So, it follows from the Theorem 1 that

$$\mathbf{P}\left\{\sup_{t\in B}(X(t)-f(t)) > x\right\} \le$$

$$\le (b-a)\left(\frac{e}{\beta p}\right)^2 \exp\left\{-\frac{2n^2c^2(1-p)}{\sum_{i=1}^n d_i^2}\left(x-\beta^2 p\left(p+\frac{1}{1-p}\right)\right)\right\}.$$

It is easy to see that the minimum of the right side of the above inequality is attained when $\beta^2 = \frac{1}{p\left(p+\frac{1}{1-p}\right)}$. Therefore, the following estimate holds true for $x > 1$

$$\mathbf{P}\left\{\sup_{t\in B}\left(\frac{1}{n}\sum_{i=1}^n W_i(t)-ct\right) > x\right\} \le$$

$$\le 2(b-a)e^2\left(1+\frac{1}{p(1-p)}\right)\exp\left\{-\frac{2n^2c^2(1-p)}{\sum_{i=1}^n d_i^2}(x-1)\right\}.$$

References

1. Kozachenko, Yu., Vasylyk, O., & Yamnenko, R. (2005). Upper estimate of overrunning by $\mathrm{Sub}_{\varphi}(\Omega)$ random process the level specified by continuous function. *Random Operators and Stochastic Equations, 13, 2*, 111–128.
2. Vasylyk, O., Kozachenko, Yu., & Yamnenko, R. (2008). φ-subgaussovi vypadkovi procesy: monographiya [φ- subgaussian stochastic processes: monograph]. *Kyiv: VPC "Kyivskyi universytet"*. [in Unkrainian]
3. Yamnenko, R., Kozachenko, Yu., & Bushmitch, D. (2014). Generalized sub-Gaussian fractional Brownian motion queueing model. *Queueing Systems, 77, 1*, 75–96.
4. Buldygin, V. V., & Kozachenko, Yu. V. (2000). Metric characterization of random variables and random processes. *AMS: Providence, Rhode Island.*
5. Kozachenko, Yu. V., Pogoriliak, O. O., Rozora, I.V., & Tegza, A. M. (2016). Simulation of Stochastic processes with given accuracy and reliability. *London: ISTE Press Ltd, Elsevier Ltd.*

Ямненко Р.Є., Юрченко Н.В. Про оцінку ймовірності перевищення лінії зваженою сумою субгауссових випадкових процесів

Субгауссові випадкові величини мажоруються за розподілом центрованими гауссовими випадковими величинами, а тому є їхнім природним узагальненням. У цій роботі розглядається задача оцінювання ймовірності перевищенням рівня, що заданий деякою прямою ct, $c > 0$, траєкторіями зваженої суми субгауссових випадкових процесів X_i, $i = \overline{1,n}$, визначених на компактній множині B, із певними ваговими функціями $w_i(t)$. А саме, будуються оцінки зверху імовірностей вигляду $\mathbf{P}\left\{\sup_{t\in B}\left(\sum_{i=1}^n w_i(t)X_i(t)-ct\right) > x\right\}$, $\mathbf{P}\left\{\inf_{t\in B}\left(\sum_{i=1}^n w_i(t)X_i(t)-ct\right) < -x\right\}$ чи $\mathbf{P}\left\{\sup_{t\in B}\left|\sum_{i=1}^n w_i(t)X_i(t)-ct\right| > x\right\}$. Така задача має безпосереднє застосування в теорії черг при оцінюванні ймовірності переповнення буфера $x > 0$ скінченного розміру у системі з одиничним сервером і лінійною інтенсивністю обслуговування, а також у страховій математиці при оцінюванні ймовірності банкрутства відповідного процесу ризику. Використовуючи метод метричної ентропії, узагальнено і покращено попередні результати, отримані автором у роботі [4] для більш загального класу Ф-субгауссових випадкових процесів. Як приклад, отриману оцінку застосовано до усередненої суми субгауссових вінерівських випадкових процесів – випадкових процесів, що мають таку саму коваріаційну функцію, як і (гауссівський) вінерівський процес, але із субгауссовими траєкторіями.

Ключові слова: субгауссовий випадковий процес, розподіл супремума, метод метричної ентропії, вінерівський процес.

Список використаної літератури

1. Kozachenko Yu., Vasylyk O., Yamnenko R. Upper estimate of overrunning by $\mathrm{Sub}_{\varphi}(\Omega)$ random process the level specified by continuous function. *Random Operators and Stochastic Equations.* 2005. Vol. 13, Iss. 2. P. 111–128.
2. Василик О., Козаченко Ю., Ямненко Р. φ-субгауссові випадкові процеси: монографія. Київ: ВПЦ "Київський університет", 2008. 231 с.
3. Yamnenko R., Kozachenko Yu., Bushmitch D. Generalized sub-Gaussian fractional Brownian motion queueing model. *Queueing Systems.* 2014. Vol. 77, Iss. 1. P. 75–96.
4. Buldygin V. V., Kozachenko Yu. V. Metric characterization of random variables and random processes. AMS: Providence, Rhode Island, 2000. 257 p.
5. Kozachenko Yu. V., Pogorilyak O. O., Rozora I. V., Tegza. A.M. Simulation of stochastic processes with given accuracy and reliability. London: ISTE Press Ltd, Elsevier Ltd., 2016. 346 p.

Одержано 25.09.2020