UDC 517.9

M.J. RONTO[1], Y.V. VARHA[2]
[1]University of Miskolc, Hungary,
[2]Uzhgorod National University, Ukraine,
matronto@uni-miskolc.hu, mailto:jana.varha@mail.ru, jana.varha@mail.ru

CONSTRUCTIVE EXISTENCE ANALYSIS OF TWO SOLUTIONS OF SOME NON-LINEAR INTEGRAL BVPS

We consider the following non-linear integral boundary value problem

$$\frac{dx}{dt} = f(t,x),\ t \in [a,b]\ , \tag{1}$$

$$\int_a^b [g(s,x(s)) + h(s, f(s,x(s)))]ds = d\ . \tag{2}$$

Here we suppose that the functions $f:[a,b]\times D \to R^n$, $g:[a,b]\times D \to R^n$ and $h:[a,b]\times D \to R^n$ satisfy the Caratheodory and the Lipschitz condition in the domain D and d is a given vector. Let D_a and D_b be a convex subsets of R^n where one looks for respectively the initial value $x(a)$ and the value $x(b)$ of the solution of the boundary value problem (1), (2).

The problem is to find and establish the existence of an absolutely continuous solution $x:[a,b] \to D$ of the problem (1), (2) with initial value $x(a) \in D_a$.

We note, that the domain D will be defined by using convex linear combinations of subsets D_a and D_b**.** We introduce the vectors of parameters $z = col(z_1, z_2, \ldots z_n) = x(a)$, $\eta = col(\eta_1, \eta_2, \ldots \eta_n) = x(b)$ and now**,** instead of integral problem (1), (2) we will consider the following "model-type" two-point boundary value problem with separated parameterized conditions: $\frac{dx}{dt} = f(t,x)$, $t \in [a,b]$, $x(a) = z$, $x(b) = \eta$.

We connect the introduced model type problem with the special parameterized sequence of function $x_m(t,z,\eta)_{m=0}^{\infty}$, satisfying the boundary conditions $x(a) = z$, $x(b) = \eta$ for all $z, \eta \in R^n$. We prove the uniform convergence of the sequence of functions : $x_\infty(t,z,\eta) = \lim_{m\to\infty} x_m(t,z,\eta)$ .The limit function $x_\infty(t,z^*,\eta^*)$ will be a solution of the original integral boundary value problem if and only if the pair of parameters (z^*,η^*) satisfies the following system of $2n$ algebraic determining equations:

$$[\eta - z] - \int_a^b f(s, x_\infty(s,z,\eta))ds = 0\,,\ \int_a^b [g(s, x_\infty(s,z,\eta)) + h(s, f(s, x_\infty(s,z,\eta)))]ds - d = 0\,.$$

The existence of two solutions was proved by studying the approximate determining system:

$$[\eta - z] - \int_a^b f(s, x_m(s,z,\eta))ds = 0\,,\ \int_a^b [g(s, x_m(s,z,\eta)) + h(s, f(s, x_m(s,z,\eta)))]ds - d = 0\,.$$

1. Rontó A., Rontó M., and Varha Y. "A new approach to non-local boundary value problems for ordinary differential systems," Applied Mathematics and Computation. – 2015. – Vol. 250. – P. 689–700.
2. Rontó M., Varha Y., and Marynets K. "Further results on the investigation of solutions of integral boundary value problems," Tatra Mt. Math. Publ., to be published.