Complexity of Learning of the Artificial Bithreshold Neuron

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Abstract* – We study the difficulty of learning problems for bithreshld neurons. We prove if** $\mathrm{P} \neq \mathrm{NP}$ ***conjecture is true that there is no polynomial time algorithm for learning bithreshold neurons.

Key words – neuron, bithreshold neuron, neural networks, learning, complexity, NP-complexity.

I. Introduction

Artificial neural networks are intensively used for solving different theoretical and practical problems (see [1, 2]). That's why the design of fast learning methods is very actual task. The one of the most important question in the learning theory is the question of the complexity of the learning algorithms. The general notions and definition of the algorithm complexity can be found in [3]. Anthony [4] considered in detail the difficulty of learning the Boolean functions. He described the polynomial learning algorithm due to Makino. Further Blum and Rivest [5] proved that training a 3-node neural network is NP-complete task. The same questions concerning bithreshold neurons and their learning was open.

II. Complexity of learning of the bithreshold neurons

The bithreshold neuron is the computation unit having n inputs $x_1,\dots,x_n$ and one output y. This unit is capable of taking on a number of states, each described by a vector $\mathbf{w}=(w_1,\dots,w_n)\in \mathrm{R}^n$ which is called as weight vector and two additional parameters $t_1, t_2\,(t_1<t_2)$ known as threshold. The output of the bithreshold neuron is defined by following equation

$$y=\begin{cases} a, & \text{if } t_1<(\mathbf{w},\mathbf{x})<t_2, \\ b, & \text{otherwise,} \end{cases}$$

where $(\mathbf{w}, \mathbf{x})$ is the inner product of the vectors $\mathbf{w}$ and $\mathbf{x}$.

In practice one supposes that output set $\{a,b\}$ is equal to well-known binary set Z_2 or bipolar set E_2 where $Z_2=\{0,1\}$ and $E_2=\{-1,1\}$. It is evident the bithreshold activation function is the generalization of Heviside step functions [2]. The triplet $(\mathbf{w},t_1,t_2)$ is called the structure of the bithreshold neuron.

Two subset A^+ and A^- of the R^n space are bithreshold separable if exsists such bithreshold neuron of the structure $(\mathbf{w},t_1,t_2)$ that

$$\forall \mathbf{x}\in A^- \;\; t_1<(\mathbf{w},\mathbf{x})<t_2$$

and

$$\forall \mathbf{x}\in A^+ \;\; (\mathbf{w},\mathbf{x})\le t_1 \text{ or } (\mathbf{w},\mathbf{x})\ge t_2 \;.$$

In this case the partition (A^-,A^-) is "bithreshold" and the corresponding bithreshold neuron compute it (it should be noted that in the last definition the order of sets A^+ and A^- is important).

A Boolean function $f:Z_2^n\to Z_2$ is a Boolean bithreshld function if it is computable by a bithreshold threshold unit. This means that two sets $\{\mathbf{x}\in Z_2^n \mid f(\mathbf{x})=1\}$ and $\{\mathbf{x}\in Z_2^n \mid f(\mathbf{x})=0\}$ are bithreshold separable.

We stude the problem of learning the bithreshold neuron as the task of changing weights and threshold in response to some training examples. The goal of learning is the finding of the structure of bithreshold neuron which can compute the diserable partition (A^+,A^-). Now we can present our main results.

Theorem 1. Suppose that f is a Boolean function defined by its disjunctive normal form formula. Then the task of the learning bithreshold neurons computing function f is NP-complete.

Theorem 2. The task of verifying the bithreshold separabiity of two finite set A^- i A^- is NP-complete even if $A^+\cup A^-\subset\{a,b\}^n$, where $a\in\mathrm{R}, b\in\mathrm{R}\;(a\neq b)$ and the absolute values of the neuron weights can have only two different values.

Conclusion

We showed that in the contrast to the learning of ordinary threshold neuron even different "weak" forms of the task of the learning one bithreshold neuron are of NP-complexity.

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