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Spectroscopy of Heavy-Light Mesons in the Framework of the Relativistic Potential Model

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Abstract

The relativistic potential quark model of heavy-light mesons describing the motion of a light antiquark by the Dirac equation with a scalar-vector coupling is constructed. In the quasiclassical approximation we have obtained the simple asymptotic formulae for energy and mass spectra of D-, D_s-, B- and B_s-mesons ensuring an appropriate accuracy of calculations even for states with the radial quantum number $n_r \sim 1$. Our results fully agree with experimental data.

Introduction

As numerous experiments demonstrate most of the known particles have an inner structure, i.e. they are composite objects. First of all it concerns hadrons (both mesons and baryons), which according to modern vision are the bound states of color quarks and gluons.

The description of mass spectra and decay probabilities of composite objects precludes the construction of the consistent theory of bound states. Such theory should be founded on basic principles of the local quantum field theory (LQFT) and should use its formalism [1]. However direct calculations of the specified characteristics of composite systems in the framework of LQFT are hardly always possible, because in LQFT the only known way of calculation is based on the perturbation theory, while the nature of creation of bound states of interacting particles, unconditionally, should be determined by non-perturbative effects.

The most effective way to go beyond the framework of the perturbation theory when constructing the theory of bound states is usage of dynamic equations. Even if it is possible to construct cores (potentials) of dynamic equations only in the lowest orders of the perturbation theory, the elaboration of methods of their precise or approximative solving (but without the perturbation theory) allows to take into account the contribution of nonperturbative interaction effects at calculations of observed characteristics of bound states. In the nonrelativistic case the similar theory is constructed with the help of the dynamic Schrödinger equations on the language of classic potential. However at large bound energies the suitable theory should be essentially relativistic. In this regard about a half century ago the way of solving the given problem, based on using of the dynamic equations in the local quantum field theory, was scheduled. Examples of such equations are the Bethe-Salpeter equation [2], the quasipotential equation [3] and other ones [4].

The effective Dirac equation method plays an important role in the modern relativistic theory of bound states. This method allows to pass consistently from the two-particle theory to the external field approximation [5]. As it follows from the results in [5], [6], this possibility is realized and has practical advantages in the case of hydrogen-like atoms and $Q\bar{q}$ systems. However, in the majority of problems, where the external field concept [6] is physically justified, attempts to find exact solutions of the Dirac equation with more or less realistic interaction potentials still encounter insurmountable difficulties. Either numerical or asymptotic methods are most often used to calculate the solutions. In many theoretical and applied problems precisely the possibility of obtaining an asymptotic solution permits analyzing the problem most completely. Therefore, it hardly needs saying how important it is to create and develop asymptotic methods for solving the Dirac equation.

The construction of quasi-classical solutions of the spinor equation with scalar-vector coupling was recently given in [7, 8]. The scheme of quasi-classical quantization, offered in [8], allows to clear up connection of quasiclassical asymptotics in spectral problems for the Dirac equation in external scalar and vector fields with Lorentz structure of the interaction potentials corresponding to them.

In this paper, within the WKB method, the behavior of a spin-1/2 relativistic particle is studied in the presence of scalar-vector external fields with potentials of a confining type. More definitively, when creating the relativistic version of potential model which is taking into account the Lorentz structure of interquark interaction potentials, we used the Cornell model offered in [9] in which the effective color Coulomb attraction on small distances r and string interaction at large r are taken into account.

Quasiclassical description of energy spectrum of heavy-light quark-antiquark systems

The problem of the motion description of a relativistic particle of a spin 1/2 in a central field composed of scalar and vector external fields is reduced via the separation of variables to solving the system of the radial Dirac equations ($c = 1$)

$$\left.\begin{aligned} &\hbar\frac{dF}{dr} + \frac{\tilde{k}}{r}F - \left[(E - V(r)) + (m + S(r))\right]G = 0, \\ &\hbar\frac{dG}{dr} - \frac{\tilde{k}}{r}G + \left[(E - V(r)) - (m + S(r))\right]F = 0. \end{aligned}\right\} \tag{1}$$

where $F(r) = rf(r)$, $G(r) = rg(r)$, $f(r)$ and $g(r)$ are radial functions for upper and lower components of the Dirac bispinor [10], E and m are the total energy and the rest mass of a particle, $S(r)$ is the scalar Lorentz potential, and the potential $V(r)$ is the zero (time) component of the four-vector $A_\mu = (A_0, \mathbf{A})$: $\mathbf{A} = 0$, $V(r) = -eA_0(r)$, $e > 0$; $\tilde{k} = \hbar k$, the quantum number

$$k = \begin{cases} -(l+1) & \text{for} \quad j = l + 1/2 \quad (l = 0, 1, \ldots), \\ l & \text{for} \quad j = l - 1/2 \quad (l = 1, 2, \ldots), \end{cases}$$

j is the total angular momentum, l is the orbital angular momentum.

The theory of a quasiclassical approximation ($\hbar \to 0$) of the Dirac equation with a scalar-vector coupling began systematically to be elaborated in [8]. The formal asymptotic expansions in powers of $\hbar$ in the initial Dirac system (1) for radial functions $F(r)$ and $G(r)$ lead to the hierarchy of matrix equations, which are solved consistently by known technique of the left and right eigenvectors of a homogeneous system. The wave functions for the barrier-type effective potential (EP) in classically allowed and forbidden regions were constructed

$$U(r, E) = \frac{E}{m}V + S + \frac{S^2 - V^2}{2m} + \frac{k^2}{2m r^2}. \tag{2}$$

Also neglecting the barrier penetrability in the region $r_1 < r < r_2$ (see fig. 1), we have obtained the quantization condition

$$\int_{r_0}^{r_1} \left(p + \frac{k w}{p r}\right) dr = \left(n_r + \frac{1}{2}\right)\pi, \qquad w = \frac{1}{2}\left(\frac{V' - S'}{m + S + E - V} - \frac{1}{r}\right), \tag{3}$$

where $p(r) = \left[(E - V(r))^2 - (m + S(r))^2 - (k/r)^2\right]^{1/2}$ is the quasi-classical momentum for the radial motion of a particle, $n_r = 0, 1, 2, \ldots$ is the radial quantum number.

To apply a potential approach for the description of the property of heavy-light mesons it is important to construct an interaction potential between quark and antiquark. As is known in QCD [7, 11],

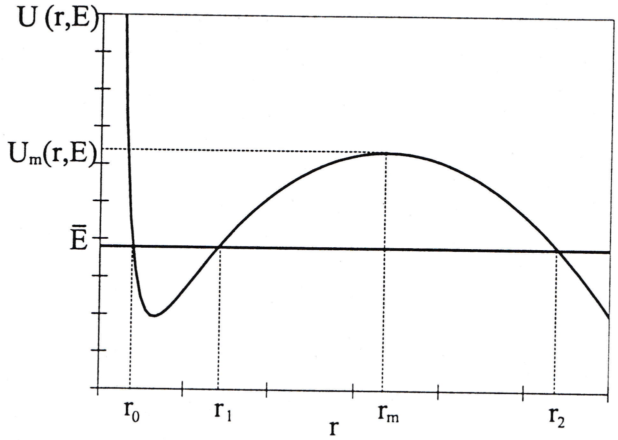

Figure 1: Form of the barrier-type effective potential $U(r,E)$; $\bar{E} = (E^2 - m^2)/2m$ is the effective energy; r_0, r_1, and r_2 are roots of equation $p(r) = 0$.

the asymptotic freedom at small distances implies that the main contribution to the $Q\bar{q}$ interaction is given by the usual Coulomb potential of the one-gluon exchange, $V(r) = -4\alpha_s/3r$, where α_s is the strong coupling constant. As the distance increases, the main contribution is given by the scalar confining interaction (confinement) whose "exact" form has not yet been found within QCD. The confining potential can be of complicated Lorentz structure. The lattice calculations [12] based on the first QCD principles distinguish the linear confinement as the most argued one. Thus, for the long-range part $v(r)$ of interquark interaction we take the linear potential $v(r) = \sigma r + V_0$ reproducing results of lattice QCD-calculations fairly well [13]. Hence we assume that $Q\bar{q}$ interaction consists of: the one-gluon exchange potential $V_{Coul}(r) = -\xi/r$ ($\xi = 4/3\,\alpha_s$); the long-range linear scalar confining potential $S_{l.r.}(r) = (1-\lambda)\,v(r)$, $v(r) = \sigma r + V_0$; the long-range linear vector potential $V_{l.r.}(r) = \lambda\, v(r)$.

In this case the vector and scalar parts of interquark interaction potential equal

$$V(r) = V_{Coul}(r) + V_{l.r.}(r) = -\xi/r + \lambda(\sigma\, r + V_0), \tag{4}$$

$$S(r) = S_{l.r.}(r) = (1-\lambda)(\sigma\, r + V_0), \tag{5}$$

where ξ is the electrostatic coupling constant, λ is the coefficient of mixing of the scalar $S_{l.r.}(r)$ and vector $V_{l.r.}(r)$ long-range potentials ($0 \leqslant \lambda \leqslant 1$), σ is the string tension, V_0 is the real constant.

In our model (4), (5) the asymptotic behavior of the EP $U\,(r,E)$ is of the form

$$U\,(r,E) = \begin{cases} \dfrac{(1-2\lambda)\sigma^2}{2m}\, r^2 + \ldots, & r \to \infty, \quad \lambda \neq \dfrac{1}{2}, \\[2ex] \dfrac{E+m}{2m}\,\sigma\, r + \ldots, & r \to \infty, \quad \lambda = \dfrac{1}{2}, \\[2ex] \dfrac{\gamma^2}{2m\, r^2}, \quad r \to 0, & \gamma^2 = k^2 - \xi^2. \end{cases} \tag{6}$$

It can be seen that regardless of the sign of the parameter σ EP $U(r,E)$ of the considered model (4), (5) is (at rather large distances) an attractive potential for $\lambda > 1/2$ and repulsing for $\lambda < 1/2$. Thus at $\lambda < 1/2$ EP $U(r,E)$ of model (4), (5) is an unboundedly increasing (with increase r) confining

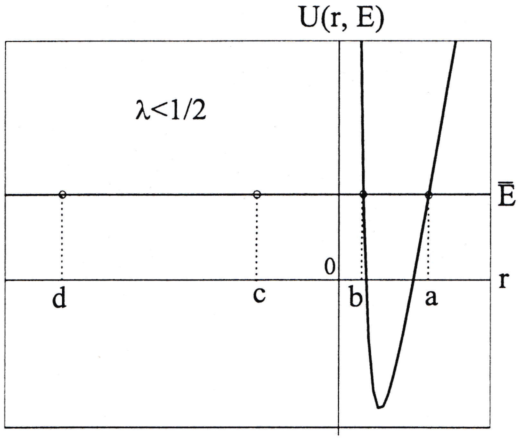

Figure 2: The effective potential $U(r,E)$ with potentials (4), (5) in the case $0 \leqslant \lambda < 1/2$, $\sigma > 0$; a, b, c and d are roots of quasimomentum $p(r) = 0$.

potential in which there is only discrete spectrum of energy levels. The provisional form of EP $U(r,E)$ at $\lambda < 1/2$ is shown in fig. 2.

At $\lambda > 1/2$ EP $U(r,E)$ of model (4), (5) looks like the well which is carved out from exterior range by wide (at $|\sigma| \ll 1$) potential barrier (see fig. 1). Due to the barrier the solution of the Dirac equation in an external field (4), (5) has a divergent wave asymptotics at the infinity. This corresponds to a particle being in a quasistationary state and eventually escaping from a decaying system [14].

The physical distinction between two considered cases ($\lambda < 1/2$ and $\lambda > 1/2$) can be explained as follows. The existence of term $(S^2 - V^2)/2m$ in the formula (2) for EP $U(r,E)$, which is characteristic for a relativistic problem, demonstrates that in the case of repulsing ($\lambda < 1/2$) in a long-range part $v(r)$ of interaction (4), (5) the relative weight $(1-\lambda)$ of a Lorentz scalar $S_{l.r.}(r)$ prevails over the relative weight λ of Lorentz vector $V_{l.r.}(r)$, and in the case of attraction ($\lambda > 1/2$) we have the contrary result. It means that the effective confining interaction ($\lambda < 1/2$) is predominantly scalar, and vector potential $V(r)$, which has anticonfining action regardless of the sign of the parameter σ, only corrects EP $U(r,E)$ at large distances.

At $\lambda = 1/2$ and positive values of σ we obtain the linear confining potential which has a discrete spectrum only. However, at $\lambda = 1/2$ and rather small negative values of σ EP $U(r,E)$ of model (4), (5) has a wide barrier. Due to this there is a capability of decay of a level by tunneling through the potential barrier, i.e. the bound level transforms in the quasistationary exponentially decaying state with complex energy $E = E_r - i\Gamma/2$.

Thus, for the specification statement $Q\bar{q}$ interaction by potentials (4), (5) we choose the range of values $0 \leqslant \lambda < 1/2$ of mixing coefficient λ. Since one fails to obtain an exact solution of the Dirac system (1) with potentials (4) and (5), we apply the method of a quasi-classical approximation which has, in the case of scalar and vector fields of the Coulomb-type and oscillator-type, a high accuracy even for small quantum numbers [8].

After substituting the potentials (4), (5) in the quantization rule (3) and integrating we arrive at the transcendental equation:

$$\frac{-2\sqrt{1-2\lambda}}{\sqrt{(a-c)(b-d)}}\left[\frac{|\sigma|(b-c)^2}{\Re}\left[N_1F(\chi)+N_2E(\chi)+N_3\Pi(\nu,\chi)+N_4\Pi\left(\frac{c}{b}\nu,\chi\right)\right]\right.$$
$$\left.+\frac{k}{2(1-2\lambda)|\sigma|}\left[(b-c)\left(N_5\Pi(\nu_+,\chi)+N_6\Pi(\nu_-,\chi)\right)+N_7F(\chi)\right]\right]=\left(n_r+\frac{1}{2}\right)\pi, \qquad (7)$$

where $F(\chi)$, $E(\chi)$, and $\Pi(\nu,\chi)$ are the elliptic integrals of 1-st, 2-nd, and 3-rd order, respectively. The expressions for ν, χ, $\nu_\pm$, $\Re$ and N_i $(i = 1, ..., 7)$ are rather cumbersome and given in the Appendix.

The tuning points $d < c < b < a$ satisfy the quartic equation $r^4 + f r^3 + g r^2 + h r + l = 0$, where

$$f = \frac{2\left(\widetilde{m}(1-\lambda) + \widetilde{E}\lambda\right)}{(1-2\lambda)\,\sigma}, \quad g = -\frac{\widetilde{E}^2 - \widetilde{m}^2 - 2\xi\sigma\lambda}{(1-2\lambda)\,\sigma^2}, \quad h = -\frac{2\widetilde{E}\xi}{(1-2\lambda)\,\sigma^2},$$

$$l = \frac{\gamma^2}{(1-2\lambda)\,\sigma^2}, \quad \widetilde{E} = E - \lambda V_0, \quad \widetilde{m} = m + (1-\lambda)V_0.$$

"Weak coupling" approximation

Of course, in general, it is impossible to obtain "exact" solution of the equation (7), however, situation is simplified in the process of increase of energy E or in the approximation of a "weak" (as contrasted to the Coulomb field) long-range field. The first case corresponds to not too large (i.e. "intermediate") values of parameters ξ and σ ($\sigma \lesssim 0.2\,\mathrm{GeV}^2$ and $0.3 < \xi < 0.8$) at which the condition $\widetilde{E}^2 \gg \sigma\gamma$ is well satisfied for all possible values of level energy $E_{n_r k}$ of heavy-light mesons, and the second case is realized at $\sigma \ll \xi\widetilde{m}^2$. For our purposes (i.e. for physics of heavy-light mesons) only the first case is interesting, whereas the second one most often meets in approximated calculations of those properties of lowest hadron states, which directly do not depend on existence or absence of confinement.

The simple and often effective way of obtaining asymptotic expansions of integral (3) consists in expanding the quasimomentum $p(r)$ on small argument – perturbation and in further termwise integration of the obtained series. Thus, depending on value of $\widetilde{E}$ with respect to the level $\widetilde{E} = \widetilde{m}$ we shall consider some most typical situations.

A. Assume that $\widetilde{E} < \widetilde{m}$ and $\sigma \ll \xi\widetilde{m}^2$, then expanding the expressions for tuning points in small parameter $\sigma/\xi\widetilde{m}^2 \ll 1$ and leaving only two terms in these series we obtain

$$a, b \approx \frac{\widetilde{E}\xi \pm \theta}{\mu^2}\left[1 - \frac{\widetilde{E}\xi \pm \theta}{\mu^4}\left(\eta_1 \pm \frac{\widetilde{m}\xi\eta_2}{\mu}\right)\sigma\right], \tag{8}$$

$$c \approx -\frac{\widetilde{m} - \widetilde{E}}{\sigma} - \frac{\xi}{\widetilde{m} - \widetilde{E}}, \qquad d \approx -\frac{\widetilde{m} + \widetilde{E}}{\sigma(1-2\lambda)} + \frac{\xi}{\widetilde{m} + \widetilde{E}}, \tag{9}$$

where $\theta = \sqrt{(\widetilde{E}\,k)^2 - (\widetilde{m}\,\gamma)^2}$, $\mu = \sqrt{\widetilde{m}^2 - \widetilde{E}^2}$, $\eta_1 = (1-\lambda)\widetilde{m} + \lambda\widetilde{E}$, $\eta_2 = \lambda\widetilde{m} + (1-\lambda)\widetilde{E}$. It follows from the formulae (8), (9) that at small positive values of σ the turning points c and d are rather far from the points a, b and tend to a minus infinity when $\sigma \to 0$. Expand the quasimomentum $p(r)$ in the range of the potential well $b < r < a$ in powers of $r/|c| \ll 1$ and $r/|d| \ll 1$. Then after termwise calculation in (3) tabular integrals with the precision $O((\sigma/\xi\widetilde{m}^2)^2)$ we obtain the result for the energy of levels:

$$E_{n_r k} = \widetilde{E}_0 + \lambda V_0 + \frac{\sigma}{2\xi\widetilde{m}^2}\left[\left(\frac{\xi^2\widetilde{m}^2}{\mu_0^2} - k^2\right)\eta_{10} + \left(\frac{2\xi^2\widetilde{m}\widetilde{E}_0}{\mu_0^2} - k\right)\eta_{20}\right], \quad \widetilde{E}_0 = \frac{\widetilde{m}}{\sqrt{1 + \xi^2/\left(n_r' + \gamma\right)^2}}, \tag{10}$$

where η_{10} and η_{20} are obtained accordingly from expressions for η_1, η_2 by substitution $\widetilde{E} \to \widetilde{E}_0$.

B. In practically important region $\widetilde{E} > \widetilde{m}$ and $\sigma > 0$, representing substantial interest for physics of heavy-light mesons, the small dimensionless parameter $\sigma\gamma/\widetilde{E}^2$ arises in the spectral problem. At $\sigma\gamma/\widetilde{E}^2 \ll 1$ the approximated expressions for turning points are of the form:

$$a \approx \frac{\widetilde{E} - \widetilde{m}}{\sigma} + \frac{\xi}{\widetilde{E} - \widetilde{m}}, \qquad b, c \approx \frac{-\widetilde{E}\xi \pm \theta}{\widetilde{E}^2 - \widetilde{m}^2}, \qquad d \approx -\frac{\widetilde{E} + \widetilde{m}}{\sigma(1-2\lambda)} + \frac{\xi}{\widetilde{E} + \widetilde{m}}.$$

Find the point $\widetilde{r}$ dividing the integration range $b \leqslant r \leqslant a$ into the segment $b \leqslant r \leqslant \widetilde{r}$, where the Coulomb potential dominates, and the segment $\widetilde{r} \leqslant r \leqslant a$, where the long-range linear potential $v(r)$ is dominating. In range $b \leqslant r \leqslant \widetilde{r}$ we calculate the quantization integral (3), expanding the quasimomentum $p(r)$ in a series in increasing powers of arguments $r/a \ll 1$ and $r/|d| \ll 1$, and at $\widetilde{r} \leqslant r \leqslant a$ the expansion of $p(r)$ is carried out in small parameters $b/r \ll 1$ and $|c|/r \ll 1$. As a result, we obtain the transcendental equation with respect to the energy E:

$$\frac{\eta_1\sqrt{\widetilde{E}^2-\widetilde{m}^2}}{2\sigma(2\lambda-1)} - \frac{\arccos(\eta_1/\eta_2)}{\sqrt{1-2\lambda}}\left(\frac{\eta_2^2}{2\sigma(2\lambda-1)}+\lambda\,\xi\right) - \gamma\arccos\left(\frac{-\widetilde{E}\xi}{\theta}\right)$$
$$-\frac{\widetilde{E}\xi}{\sqrt{\widetilde{E}^2-\widetilde{m}^2}}\ln\left(\frac{\sigma\,\eta_2\theta}{4\,e\,(\widetilde{E}^2-\widetilde{m}^2)^2}\right) - \frac{\operatorname{sgn} k}{2}\arccos\left(\frac{-\widetilde{m}\xi}{\theta}\right) = \left(n_r+\frac{1}{2}\right)\pi. \quad (11)$$

Though the equation (11) is much easier than the "exact" quasiclassical equation (7) for level energy, it is necessary to use numerical calculations to solve it. Below we consider some limit cases, when the equation (11) is simplified and can be analytically studied.

At values of arguments $\sigma \lesssim 0.2\,\mathrm{GeV}^2$ and $0.3 < \xi < 0.8$ for all possible values of levels energy $E_{n_r k}$ of heavy-light mesons the condition $\widetilde{E} \gg \widetilde{m}$ is well satisfied. If we expand the left-hand part of (11) in $\widetilde{m}/\widetilde{E} \ll 1$ to within the terms which are proportional to third power, then for the energy $E_{n_r k}$ we arrive at the transcendental equation

$$[(1-\lambda)A-\lambda]\,\widetilde{E}^2 + 2\widetilde{m}\widetilde{E}(1-\lambda)(\lambda A-1) - 2\pi\sigma(1-2\lambda)N - \lambda\widetilde{m}^2 + \lambda[\lambda\widetilde{m}^2 - 2\sigma\xi(1-\lambda)]A = 0, \quad (12)$$

$$A = (1-2\lambda)^{-1/2}\arccos\left(\frac{\lambda}{1-\lambda}\right),\ N = n_r + \frac{1}{2} + \frac{\operatorname{sgn} k}{4} + \frac{1}{\pi}\left(\gamma\arccos\left(-\frac{\xi}{|k|}\right) - \xi\right) + \frac{\xi}{\pi}\ln\frac{\sigma|k|(1-\lambda)}{4\widetilde{E}^2}.$$

Solving this equation by the method of series iterations, as a first approximation we obtain the required expression for eigenvalues of the energy $E_{n_r k}$ (to within $O(\sigma\gamma/\widetilde{E}^2)$):

$$E_{n_r k} = \zeta^{-1}\left\{B + \left(B^2 + \zeta\left[2\sigma(1-2\lambda)\left(3\xi + \lambda\xi A + \pi N_0\right) + \lambda\widetilde{m}^2(1-\lambda A)\right]\right)^{1/2}\right\} + \lambda V_0, \quad (13)$$

where $\zeta = (1-\lambda)^2 A - \lambda - 2\sigma\xi(1-2\lambda)/(\widetilde{E}^{(0)})^2$, $B = (1-\lambda)(1-\lambda A)\widetilde{m} - 4\sigma\xi(1-2\lambda)/\widetilde{E}^{(0)}$, $\widetilde{E}^{(0)} = E^{(0)} - \lambda V_0$, $N_0 = N(\widetilde{E} = \widetilde{E}^{(0)})$, $E^{(0)} \approx E_{n_r k}(\xi = 0)$ is the zero approximation.

Energy and mass spectra of heavy-light mesons

In tab. 1 the comparison of calculation results of levels energy $E_{n_r k}^{\mathrm{WKB}}$ and $E_{n_r k}^{\mathrm{WKB(as)}}$ in the basis of the transcendental equation (7) and the asymptotic formula (13), with precise values $E_{n_r k}$ obtained by numerical solving of the Dirac equation, are shown for $n_r = 0, 1, 2$ and $k = \pm 1, \pm 2$. We selected such values α_s, λ, V_0, $m_{u,d}$ and m_s, which are used in QCD at the state description of $B(b\overline{u}$ or $b\overline{d})$-, $B_s(b\overline{s})$-, $D(c\overline{u}$ or $c\overline{d})$- and $D_s\,(c\overline{s})$-mesons.

As it can be seen from table 1 the errors in calculating the quasiclassical values $E_{n_r k}^{\mathrm{WKB}}$ and $E_{n_r k}^{\mathrm{WKB(as)}}$ do not exceed 1% and 2%, respectively (with the exception of states energy with a radial quantum number $n_r = 0$, where the error of both formulae $\sim 8\%$). Thus, the precision of determination of $E_{n_r k}$ by the quasiclassical formula (13) is such that for practical purposes there is usually no sense to update the first approximation.

In the leading order on $1/m_Q$ the mass spectrum of meson states with one heavy quark is given by the expression [15]

$$M_{theor}(Q\bar{q}) = E_{n_r k} + \sqrt{E_{n_r k}^2 - m_q^2 + m_Q^2}, \quad (14)$$

Table 1: $\alpha_s = 0.3$, $\lambda = 0.3$, $V_0 = -0.45\,\text{GeV}$, $m_{u,d} = 0.33\,\text{GeV}$, $m_s = 0.5\,\text{GeV}$.

		$b\bar{u},\ b\bar{d}$			$b\bar{s}$		
$^{2s+1}N_j$	(n_r, k)	$E^{(n)}_{n_r k}$	$E^{WKB}_{n_r k}$	$E^{WKB(as)}_{n_r k}$	$E^{(n)}_{n_r k}$	$E^{WKB}_{n_r k}$	$E^{WKB(as)}_{n_r k}$
$^2S_{1/2}$	(0, -1)	0.4327	0.4408	0.4729	0.5248	0.5322	0.5623
	(1, -1)	0.8796	0.8838	0.8943	0.9750	0.9791	0.9912
	(2, -1)	1.1978	1.2009	1.2066	1.2946	1.2976	1.3049
$^2P_{3/2}$	(0, -2)	0.7355	0.7373	0.7504	0.8376	0.8392	0.8460
	(1, -2)	1.0880	1.0892	1.0947	1.1879	1.1890	1.1927
	(2, -2)	1.3658	1.3667	1.3699	1.4650	1.4659	1.4685
$^2P_{1/2}$	(0, 1)	0.7249	0.7293	0.7030	0.8235	0.8278	0.7985
	(1, 1)	1.0701	1.0733	1.0594	1.1696	1.1728	1.1572
	(2, 1)	1.3470	1.3496	1.3405	1.4466	1.4492	1.4390
$^2D_{3/2}$	(0, 2)	0.9661	0.9671	0.9343	1.0655	1.0665	1.0315
	(1, 2)	1.2588	1.2596	1.2385	1.3583	1.3591	1.3369
	(2, 2)	1.5058	1.5066	1.4914	1.6052	1.6059	1.5901

where m_Q and m_q are masses of a heavy quark Q and light antiquark $\bar{q}$ composing a meson $Q\bar{q}$. Thus, the problem of obtaining a mass spectrum of $Q\bar{q}$ mesons is reduced to successive calculations of energy eigenvalues of the Dirac equation (1) in the composite field (4), (5), the source of which, in this case, is a heavy quark Q.

Above we did not take into account hyperfine structure of levels (HFS) and, consequently, the offered potential model is able to foretell only the position of the center of weight of the HFS multiplet, formed by sublevels with the different moments $\vec{J} = \vec{j} + \vec{S}_Q$. In real $Q\bar{q}$ systems the degeneration of the doublet states, corresponding to the different moments $J = j \pm 1/2$ at given j, is removed first of all by $\vec{s}_q\,\vec{S}_Q$-interaction. Therefore, to have the capability to compare our theoretical predictions with the experimental data, in tab. 2-3 we calculate the observed values of weight center of masses of HFS multiplets by the known formula

$$M_{exp} = \left(\sum_J (2J+1)\cdot M_J\right) \Bigg/ \left(\sum_J (2J+1)\right), \tag{15}$$

where M_J is the state mass with the total angular moment J.

Based on these observations, we attempted to describe the mass spectra of the lowest states of heavy-light $B(b\bar{u}\,\text{or}\,b\bar{d})$-, $B_s(b\bar{s})$-, $D(c\bar{u}\,\text{or}\,c\bar{d})$-, $D_s(c\bar{s})$-mesons, considering σ and λ as universal values, and value of arguments α_s and V_0 as constants in each set of heavy-light mesons that is weak varying only at transition from one set to another.

The comparison of calculations results by the formulae (7) and (14) with the experimental data [16] demonstrates that the best agreement is reached for $\lambda = 0.3$ and following set of parameters: $\sigma = 0.18\,\text{GeV}^2$, $\alpha_s(c\bar{u}$ or $c\bar{d}) = 0.386$, $\alpha_s(b\bar{u}$ or $b\bar{d}) = 0.3$, $\alpha_s(s\bar{u}$ or $s\bar{d}) = 0.421$, $V_0(c\bar{u}$ or $c\bar{d}) = -375\,\text{MeV}$, $V_0(b\bar{u}$ or $b\bar{d}) = -450\,\text{MeV}$, $V_0(s\bar{u}$ or $s\bar{d}) = -536\,\text{MeV}$, $m_{u,d} = 330\,\text{MeV}$, $m_c = 1550\,\text{MeV}$, $m_s = 500\,\text{MeV}$, $m_b = 4880\,\text{MeV}$.

The mass spectra of D-, D_s-mesons calculated in this approximation are shown in tab. 2. Difference between the model and the experiment is within the limits of 3-5 %, except of masses of states $P_{3/2}$ and $P_{1/2}$ of the system $c\bar{s}$, where the deviation depends on the interpretation of D_{s1} (2536)$^{\pm}$ meson and equals about 10 %, when considered as a vector state $J^P = 1^+$, belonging to the doublet $j = 3/2^+$, or 4%, when attributed to the state $J^P = 1^+$ of the doublet $j = 1/2^+$.

For systems $b\bar{u}$ and $b\bar{s}$ (the tab. 3) a good coincidence of results represented by us with experimental

Table 2: The mass spectrum of D-, D_s-mesons (in MeV).

		$c\bar{u}$, $c\bar{d}$			$c\bar{s}$		
$^{2s+1}N_j$	(n_r, k)	E_{theor}	M_{theor}	M_{exp}	E_{theor}	M_{theor}	M_{exp}
$^2S_{1/2}$	(0,-1)	427.81	2001.54	1971.05	514.35	2069.04	2072
	(1,-1)	880.47	2632.28	< 2637	957.76	2737.42	–
$^2P_{3/2}$	(0,-2)	752.2	2443.17	2447.3	854.32	2552.06	2530.68
	(1,-2)	1106.36	2981.9	–	1207.2	3107.16	–
$^2P_{1/2}$	(0,1)	724.73	2403.67	2407.75	825.19	2508.48	2480.86
	(1,1)	1075.78	2933.44	–	1177.35	3058.48	–

Table 3: The mass spectrum of B-, B_s-mesons (in MeV).

		$b\bar{u}$, $b\bar{d}$			$b\bar{s}$		
$^{2s+1}N_j$	(n_r, k)	E_{theor}	M_{theor}	M_{exp}	E_{theor}	M_{theor}	M_{exp}
$^2S_{1/2}$	(0,-1)	440.76	5329.5	5313.55	532.22	5415.63	5404.85
	(1,-1)	883.82	5832.22	–	979.12	5931.2	–
$^2P_{3/2}$	(0,-2)	737.25	5661.58	< 5698	839.24	5765.57	< 5853
	(1,-2)	1089.21	6078.39	–	1188.99	6186.8	–
$^2P_{1/2}$	(0,1)	729.28	5652.42	–	827.84	5752.24	–
	(1,1)	1073.25	6058.97	–	1172.77	6166.75	–

data obtained for a ground state with $j = 1/2^-$ and p-state with $j = 3/2^+$. For states of the doublet $j = 1/2^+$ there are only theoretical predictions of other authors. In the case of the system $b\bar{u}$ our results agree with the data obtained in [17], and in the case of $b\bar{s}$ systems we see the amazing coincidence with results of papers [18].

For the first radial excitations in $Q\bar{q}$ systems the precision of determination of masses has appeared to be better ($\lesssim$50 MeV) than for ground states. It is not surprising, as it is known, that the applicability of WKB approximation is justified only for highly excited states corresponding to rapidly oscillating wave functions. Hence, we obtain the additional confirmation of validity of usage of the quasiclassical approximation for the Dirac equation with a scalar-vector coupling to heavy-light mesons as well as a clearer understanding of Lorentz structure of a long-range part $v(r)$ of interquark interaction potential.

Appendix

$$\nu_\pm = \frac{\lambda_\pm - c}{\lambda_\pm - b}\nu, \quad \Re = (1-\nu)\left(\chi^2 - \nu\right), \quad \aleph = \chi^2(3-2\nu) + \nu(\nu-2),$$

$$N_1 = \frac{\chi^2(b-c)}{4} - \frac{3\aleph(b-c)}{8(1-\nu)} - \frac{(\chi^2-\nu)}{2}(f+3c) + \frac{\Re}{(b-c)^2}\left(c^3 + c^2 f + cg + h + l/c\right),$$

$$N_2 = -\frac{\nu}{2}\left[f + 3c + \frac{3}{4}\frac{(b-c)\aleph}{\Re}\right],$$

$$N_3 = \frac{1}{2}\left[\frac{3}{4}\frac{(b-c)\aleph^2}{\Re} + \frac{2\Re}{(b-c)}\left(3c^2 + 2cf + g\right) + (b-c)\left(\left(1+\chi^2\right)\nu - 3\chi^2\right) + \aleph(f+3c)\right],$$

$$N_4 = -\frac{\Re}{(b-c)}\frac{l}{bc}, \quad N_5 = [(b-\lambda_+)(\lambda_+ - c)]^{-1}, \quad N_6 = [(b-\lambda_-)(\lambda_- - c)]^{-1},$$

$$N_7 = \frac{2}{(\lambda_+ - c)(\lambda_- - c)} \left(c + \frac{\widetilde{E} + \widetilde{m}}{2(1-2\lambda)\sigma} \right).$$

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