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Calculation of hyperfine splitting in mesons using configuration interaction approach

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Abstract. The spin–spin mass splitting of light, heavy and mixed mesons are described within a good accuracy in the potential model with screened potential. We conclude that the long-range part of the potential cannot be pure scalar and that a vector–scalar mixture is favored. Excellent spin–spin splittings of heavy quarkonia are obtained with the same parameters as the ones which give the correct average mass spectrum. The results are obtained by going beyond the usually used perturbation method, namely using a configuration interaction approach.

1 Introduction

The problem of hyperfine splitting in mesons still attracts wide interest. It is widely accepted that the quark potential model gives a rather good description of the spin average mass spectrum of hadrons, considered as a composite system of quarks [1]. However, the question of explaining the influence of spin, namely the spin–orbit ("fine") and the spin–spin ("hyperfine") interaction, is not solved yet. The problem of mass splitting is due to the spin structure and it is closely connected with the Lorentz structure of the quark potential. These effects are far from being solved yet. One of the first works in these fields were [2,3] and references cited therein. Quite recently a QCD motivated potential was applied to heavy quarkonia. Unfortunately, the authors of [4] restricted themselves to comparing the results to a single experimental value, the J/Ψ–η_c mass difference. No attempt was made to calculate other quark–antiquark pairs. The spectroscopy of heavy mesons is studied also in [5]. The authors calculated only the spectra mass of heavy mesons B, B_S, D, D_S including the spin–spin interaction. In [5] the first- and second-order calculations of the masses are in good agreement with the experimental data, except for the higher spin states. Besides, there are several works in which the authors take a limited number of pairs (like $u\overline{d}$) of particles for which the splitting is calculated. Their results are quite good, but in [6] the authors limited themselves to considering only the π, ρ, K, K^* mesons. Analogously Buchmuller and Tye [7] also consider only two systems, namely $c\bar{c}$ and $b\bar{b}$, though in the states $1S$ and $2S$. They obtained good results for these particular pairs. There is another paper [8] in which both splitting and decay properties are studied. In this paper, also good results for hyperfine splitting are obtained. But the authors calculate only a few special mesons (J/Ψ–η_c). The authors in this paper come to the conclusion that only the one-gluon term gives a contribution in the hyperfine splitting. From our paper, it is clear that confinement also plays an essential role in the calculations of the hyperfine effects in mesons. To illustrate this, we show the results of the mass differences of the (J/Ψ–η_c) mesons, where the one-gluon term gives in hyperfine splitting only 62 MeV, while the confinement part gives 38.8 MeV (which is very essential).

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2 Interaction potential

We turned to the use of a realistic potential, namely, a screened potential, for obtaining acceptable results. In our case the choice of the potential itself is motivated by the consideration of the most accurate description of the averaged mass spectrum.

Our approach is based on the model of the nonperturbative gluon propagator, which was saggested by Chikovani, Jenkovszky and Paccanoni (CJP) [9]. In this model the propagator has the form

$$D\left(q^2\right) = \frac{c}{\left(q^2-\mu^2\right)} - \frac{1}{q^2-M^2}, \tag{1}$$

and the corresponding potential assumes the form [9]

$$V\left(r\right) = \frac{g^2}{6\pi\mu}\left(1-\exp\left[-\mu r\right]\right) - \frac{16\pi}{25}\frac{\exp\left[-Mr\right]}{r\cdot\ln\left(b+1/\left(\Lambda r\right)^2\right)}. \tag{2}$$

In fact, we used the simple form for the one-gluon-exchange type of the α_s/r term with QCD-like asymptotic freedom, in a numerical calculation, where

$$\alpha_s(r) = \frac{12\pi}{33-2N} \cdot \frac{1}{\ln\left(1/\left(\widetilde{\Lambda} r\right)^2\right)}, \tag{3}$$

where $\widetilde{\Lambda}$ is taken to be equal to $\widetilde{\Lambda} = 0.14\,\mathrm{GeV}$. The initial value of α_s was defined via calculating the $q\overline{q}$ masses.

As Gerasimov pointed out [10], the QCD lattice calculations indicate that the spin–spin forces are rather short-range. Also Laerman et al. [11] showed that the quark confinement potential is lower than the linear confinement potential, when $\boldsymbol{r}$, the distance between the quarks, becomes larger. The screened potential just satisfies this condition. In this case, even the basic solutions for the unperturbed Hamiltonian cannot be found in analytical form. We found these solutions numerically and evaluated the matrix elements numerically too. The final results for hyperfine splitting for the screened potential are given in Tables 1 and 2. The following parameters were used in the screened potential: $g^2/(6\pi) = 0.224\,\mathrm{GeV}^2$, $\mu = 0.054\,\mathrm{GeV}$. All parameters were taken from [12]. Experimental values were taken from [13].

However, there are more "exact" QCD-motivated potentials, which are based on two loop back diagrams [14]. But by a more detailed consideration it appears that these accounts were carried out only for the one-gluon-exchange part.

Following many authors we assume an admixture of the vector–scalar potential (soft model). We consider the vector and scalar parts of a static potential [1]:

$$V(r) = V_S(r) + V_V(r), \tag{4}$$

where

$$V_V = -\frac{a_S}{r} + \varepsilon\frac{g^2}{6\pi\mu}\left(1-\mathrm{e}^{-\mu r}\right),$$
$$V_S = (1-\varepsilon)\frac{g^2}{6\pi\mu}\left(1-\mathrm{e}^{-\mu r}\right), \tag{5}$$

and ε is the mixing constant. Here the Lorentz nature of the one-gluon and the confining potential is different, the one-gluon potential being totally of vector type, while the confining part has a vector–scalar mixture character. This choice seems to be reasonable, since non-perturbative vertex corrections are important at small q^2, i.e. large distances and naturally are coupled only with the long-range term in the potential. A very interesting review was given in the work of Brambilla concerning the choice of the interaction potential in [15].

3 Configuration interaction approach

The main problem of our work is to clarify some aspects of the hyperfine interaction in the framework of the configuration interaction approach (CI approximation, or CIA [16]). As is well known, the CIA approach was successfully used in atomic physics [16], where the authors show that the configuration interaction approach is more reasonable than the perturbation method. The main advantage of the CIA lies in the fact that in this approach one does not use the interaction parameter α, which in this region of QCD is rather large; because of that the perturbation theory is out of use in CIA. The differences beetwen CIA and perturbation theory are shown in [17], but the most important difference is that in CIA we take into account the fifth order of the expansion, while in perturbation theory we can obtain only the first and second orders; because of this the CIA is more precise.

The Hamiltonian can be written as

$$H = H_0 + H_{SS}, \tag{6}$$

where for the screened potential

$$H_0 = \frac{1}{2m}\nabla^2 - \frac{a_S}{r} + \varepsilon\frac{g^2}{6\pi\mu}\left(1-\mathrm{e}^{-\mu r}\right), \tag{7}$$

where m is the reduced mass of the $q\overline{q}$ system and $h = c = 1$ units are used. We take into account H_{SS} in (6), because we calculate hyperfine splitting in S-waves. Then all terms in which the orbital quantum number l are contained are absent.

In the framework of the Breit–Fermi approach the spin–spin interaction term is

$$H_{SS} = \frac{2}{3m_{q_1}m_{q_2}}\boldsymbol{S_1 S_2}\Delta V_V, \tag{8}$$

where $\boldsymbol{S_{1,2}}$ are the spins of the particles; $\boldsymbol{S_1 S_2} = -3/4$ for pseudo-scalar mesons and $\boldsymbol{S_1 S_2} = 1/4$ for vector mesons.

Now, we consider the Schroedinger equation

$$(H_0 + H_{SS})\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r}). \tag{9}$$

Here we suggest to use the CI approach. The essence of this approximation is that the wave function $\Psi(\boldsymbol{r})$, which is expanded in a set of eigenfunctions $\varphi_n(\boldsymbol{r})$ of the Hamiltonian H_0, is

$$\Psi(\boldsymbol{r}) = \sum_n a_n\varphi_n(\boldsymbol{r}). \tag{10}$$

On substituting (10) into (9) and using the eigenvalue E_n^0, we obtain a homogeneous system of linear equations for a_n:

$$a_n\left(E_m^0 - E\right) = -\sum_n a_n\langle\varphi_m|H_{SS}|\varphi_n\rangle \tag{11}$$

or

$$a_1\left(E - E_1^0 - W_{11}\right) - a_2W_{12} - a_3W_{13} - \ldots - a_nW_{1n} = 0,$$
$$-a_1W_{21} + a_1\left(E - E_2^0 - W_{22}\right) - a_3W_{23} - \ldots - a_nW_{2n} = 0,$$
$$\ldots$$
$$-a_1W_{n1} - a_2W_{n2} - a_3W_{n3} - \ldots + a_n\left(E - E_n^0 - W_{nn}\right) = 0, \tag{12}$$

Table 1. Hyperfine splitting for light and mixed quark systems

	[18] $\Delta M_{\mathrm{THEOR}}$ (MeV)	[19] $\Delta M_{\mathrm{THEOR}}$ (MeV)	[20] $\Delta M_{\mathrm{THEOR}}$ (MeV)	Our results $\Delta M_{\mathrm{THEOR}}$ (MeV)	ΔM_{EXP} (MeV)
$\Delta M_{\rho-\pi}$	634	550	651	923	635
$\Delta M_{\rho'-\pi'}$	329	-	-	411	150
$\Delta M_{\varphi-\eta}$	217	-	270	580	320
$\Delta M_{\varphi'-\eta'}$	135	-	-	285	-
ΔM_{K^*-K}	405	461	393	707	398
$\Delta M_{K^{*\prime}-K'}$	195	-	-	336	200
ΔM_{D^*-D}	92	147	150	186	143
$\Delta M_{D^{*\prime}-D'}$	-	-	-	112	-
ΔM_{B^*-B}	32	52	58	57	45.9
$\Delta M_{B^{*\prime}-B'}$	-	-	-	36	-

which have to be truncated for a reasonable large n. In (11) E_m^0 is the eigenvalue of the non-perturbative Hamiltonian,

$$H_0\varphi_m = E_m^0\varphi_m, \tag{13}$$

and $W_{ij} = \langle\varphi_i|\widehat{W}|\varphi_j\rangle$.

It is evident that the solution of (12) exists only if the determinant which contains the coefficients a_n is equal to zero. The diagonalization of this determinant gives the values of energies we are looking for. This is a good method for the eigenvalues E_n. This procedure goes far outside the perturbative method. Our calculations showed that the next terms of the CIA method give a contribution of order of 10% for heavy mesons (See Appendix A) and 35% for light mesons. Let us stress that the first term of the CIA method is in fact just a perturbative method result. The CIA expansion better takes into account the interaction between particles. A similar approach was suggested in [18], where the expansion was carried out in the basis functions of the oscillator potential. The proposed method is of considerable interest, since the perturbation method is still used as the practical method [5,6,19].

4 Hyperfine splitting

We compare our results with results obtained in the works [18–20]. In [18] good results are obtained, but the authors introduced additional parameters r_0. In [19], the hyperfine splitting is calculated in a first account of perturbation theory. There it is shown that the first-order perturbation theory gives a good description of the experimental data. The best results are obtained in [20], but only the one-gluon term is taken in the spin–spin forces into account. In all these works there is one common flaw: they are restricted to viewing a limited number of mesons.

In Table 1 we show the final data for light-quark systems, which have mainly a relativistic character, and we compare them with other data. The results obtained for light mesons indicates that a non-relativistic model is not applicable. In the case of the average spin spectrum, this model for light mesons gives similar results as in the relativistic approach [12,21]. The calculations are carried out by using (1)–(13). In our quasi-relativistic approach we should take into account in the Hamiltonian also the term of order p^4. In [20] it was shown that the spectrum of the Hamiltonian in non-relativistic potential models and the spectrum of the relativistic Hamiltonian for the bound state and for the first radial excited state are equivalent. On the other hand, Lucha and Shoeberl [1] have shown that in some cases this relativistic kinematical term changes the mass spectrum drastically. In other papers, the authors introduce new additional parameters and obtain a good description of hyperfine splitting. In [22] Faustov et al. also have made similar calculations using a quasi-potential, but evidently this potential was chosen unsuccessfully, as ε turned out to be -0.9. Evidently this negative value lacks a clear physical meaning, as a mixing parameter. Therefore this approach lost very much of its heuristic value in understanding the results obtained.

In Table 2 we present the results of a hyperfine splitting calculation in heavy-quark systems. Namely, exactly for these systems our Breit–Fermi approach must be true to a maximal extent. The obtained results for the hyperfine splittings of the S-wave states agree with the measured splittings. As is seen, most of our results have mainly a predictive character. The results in Table 2 showed that for $2S$-states we obtain somewhat worse results for the hyperfine splitting than for the $1S$-states. This may mean that in this state the mixing of S- and D-waves is very essential. In the $2S$-state, as is shown in [23], the mixing can give a contribution of 10%, while in the case of $1S$-states the mixing correction is only of the order of 1%. In other words, we believe that taking into account the mixture of S- and D-waves would considerable improve our results.

We suggest that the potential consists of a sum of vector and scalar parts. This idea of scalar–vector mixing was discussed in [24–27]. The authors of these papers also came to the conclusion that its mixing parameter must be different from zero. Franzinis et al. [24] showed that V_{CONF}

Table 2. Hyperfine splitting for mixed and heavy-quark systems

	[18] $\Delta M_{\mathrm{THEOR}}$ (MeV)	[19] $\Delta M_{\mathrm{THEOR}}$ (MeV)	[20] $\Delta M_{\mathrm{THEOR}}$ (MeV)	Our results $\Delta M_{\mathrm{THEOR}}$ (MeV)	ΔM_{EXP} (MeV)
$\Delta M_{D_S^*-D_S}$	87	190	128	163	144
$\Delta M_{D_S^{*\prime}-D_S'}$	-	-	-	100	-
$\Delta M_{B_S^*-B_S}$	-	-	-	50	47
$\Delta M_{B_S^{*\prime}-B_S'}$	-	-	-	33	-
$\Delta M_{B_c^*-B_c}$	-	-	-	49	-
$\Delta M_{B_c^{*\prime}-B_c'}$	-	-	-	31	-
$\Delta M_{\gamma-\eta_b}$	31	39	82	46	-
$\Delta M_{\gamma'-\eta_b'}$	9	-	-	26	-
$\Delta M_{J/\Psi-\eta_c}$	65	100	112	110	117
$\Delta M_{\Psi-\eta_c'}$	32	54	-	67	95

must be totally scalar, while V_{OGE} must be totally vector, but nevertheless they cannot give an adequate description of the data concerning the fine splitting. In [25–27] Deoghuria and Chakrabarty chose a confining potential of the form

$$V = \varepsilon V_{\mathrm{OGE}} + (1-\varepsilon)\, V_{\mathrm{CONF}} \tag{14}$$

and found $\varepsilon = 0.2$, to be treated as an adjustable parameter. In this paper we have used the same approach for a CJP-type potential and consider $\varepsilon = 0.5$, because with this value of ε we obtained a good description of spin–spin splitting in heavy mesons. With this value of ε we described the hyperfine splitting of all mesons, from heavy to light ones.

5 Conclusion

The results for heavy quarkonium are quite good for the value of $\varepsilon = 0.5$, which coincides with results obtained by Lai-Him Chan [28]. For light quarkonium the results are worse, which means that relativistic effects have to be taken into account more carefully. We believe that the difference between these cases exactly reflects this fact.

We conclude, that it is very important to take confinement into account in the spin–spin effect calculations. For example the one-gluon term gives only 56% in the case of the $u\overline{u}$ system and 38% in the $c\overline{c}$ system in the splitting.

Our calculations show that CIA gives better results than the perturbation method. (See for example [16].)

Appendix

A Calculation of hyperfine splitting using oscillator potential

In this part we obtain the hyperfine splitting for the J/Ψ and η_c mesons using the configuration interaction approach with an oscillator potential.

Let us write down for this case the Fermi–Breit equation for the two-quark system in the form

$$\left(-\frac{1}{2m}\Delta + Ar^2 - \frac{\alpha_S}{r} + H_{SS}\right)\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r}), \tag{A.1}$$

where

$$H_0 = -\frac{1}{2m}\Delta + Ar^2, \tag{A.2}$$

$$H_{SS} = \frac{2}{3m_{q_1}m_{q_2}}\boldsymbol{S_1}\boldsymbol{S_2}\Delta V_V, \tag{A.3}$$

and (A.3) is an addition to the non-perturbed Hamiltonian.

In this case the vector and scalar parts of the potential are equal accordingly:

$$V_V = -\frac{\alpha_S}{r} + \varepsilon Ar^2 \quad \text{and}$$
$$V_S = (1-\varepsilon)Ar^2. \tag{A.4}$$

On the one hand the oscillator potential is not so bad in describing the quarkonia data, on the other hand it allows us to obtain analytic basis solutions. H_{SS} is the additional term which has to be taken into account in CIA. Also we put the one-gluon-exchange term into the CIA. Substituting the expanded wave function into (A.1) and using the condition of orthonormality of the basis functions, we obtain an algebraic system of equations for the coefficients a_n.

Two remarks are to be made in connection with (A.3). First, we shall consider the contribution to the spin–spin term of the potential consisting of both one-gluon and many-gluon exchanges (A.4). Then we have

$$H_{SS} = \frac{2}{3m_{q_1}m_{q_2}}\boldsymbol{S_1}\boldsymbol{S_2}\left(4\varepsilon\pi\alpha_S\delta(\boldsymbol{r}) + 6A\right). \tag{A.5}$$

The second remark concerns the meaning of the parameter ε. According to the general point of view [1] only a vector contribution is present in (A.3). In order to avoid

Table 3. Hyperfine splitting between J/Ψ and η_c mesons

	1	2	3	4	5	6	7	8	9	$\Delta M_{\rm EXP}$
$1S$	37	41	44	46	47	48	49	50	51	117
$2S$	–	39	43	46	48	49	51	52	53	95

introducing additional parameters, we shall consider the mixing parameter ε to be same for many-gluon terms approximately equal to $\varepsilon = 1$ in the case of an oscillator potential.

If we restrict ourselves to one term in the expansion (7) then (8) is reduced to the perturbation method. We represent the wave function in (A.1) as the two basis functions of (7),

$$\Psi(\boldsymbol{r}) = a_1\varphi_1(\boldsymbol{r}) + a_2\varphi_2(\boldsymbol{r}), \tag{A.6}$$

which immediately leads to a much better approximation. Namely, one obtains for the energies

$$E_{1,2} = \frac{E_1^0 + E_2^0 + H_{11} + H_{22}}{2} \pm \frac{1}{2}\sqrt{\left(E_1^0 - E_2^0 + H_{11} + H_{22}\right)^2 + 4H_{12}H_{21}}, \tag{A.7}$$

where $H_{nm} = \langle\varphi_m|H_{\rm SS}|\varphi_n\rangle$.

Let us write for completeness the first two terms of the oscillator wave functions:

$$\varphi_{1S}(r) = \left[\frac{2\beta^{3/4}}{\pi^{1/4}}\right]\mathrm{e}^{-\beta r^2/2}Y_m^l,$$
$$\varphi_{1S}(r) = \left[\frac{\sqrt{6}\beta^{3/4}}{\pi^{1/4}}\right]\left(1 - \frac{2}{3}\beta r^2\right)\mathrm{e}^{-\beta r^2/2}Y_m^l,$$

where $\beta = (Am_q)^{1/2}$. In this case the corresponding matrix elements should have the form

$$H_{11} = -\frac{2\alpha_{\rm S}\beta^{1/2}}{\pi^{1/4}} + \frac{\boldsymbol{S_1}\boldsymbol{S_2}}{m_q^2}\left\{4A + \frac{8\alpha_{\rm S}\beta^{3/2}}{3\pi^{1/2}}\right\},$$
$$H_{22} = -\frac{5\alpha_{\rm S}\beta^{1/2}}{3\pi^{1/4}} + \frac{\boldsymbol{S_1}\boldsymbol{S_2}}{m_q^2}\left\{4A + \frac{4\alpha_{\rm S}\beta^{3/2}}{\pi^{1/2}}\right\},$$
$$H_{12} = H_{21} = -\frac{2\alpha_{\rm S}\beta^{1/2}}{\sqrt{6}\pi^{1/2}} + \frac{\boldsymbol{S_1}\boldsymbol{S_2}}{m_q^2}\left\{\frac{4\sqrt{6}\alpha_{\rm S}\beta^{3/2}}{3\pi^{1/2}}\right\}. \tag{A.8}$$

Substituting (A.8) into (A.7) and using the standard values of E_m^0 for the oscillator potential one can obtain the values for the hyperfine splitting which are given in Table 3. The parameters are chosen to be $A = 0.014\,\mathrm{GeV}^3$, $\varepsilon = 1$, $m_q = 1.5\,\mathrm{GeV}$, $\alpha_{\rm S} = 0.32$, since exactly these parameters give the best results for the oscillator potential in the meson masses.

Certainly, using the oscillator potential we obtain only didactically relevant values.

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