RANKING METHOD OF ALTERNATIVE OPTIONS OF INHOMOGENEOUS NATURE

Nikola MALYAR[1] *- Volodimir POLISHCHUK*[2]

Abstract: The article shows a mathematic model, which allows to construct a ranking range of inhomogeneous alternatives and which is approved on the example of evaluation of promising economics fields functioning.
Key words: inhomogeneous alternatives, multicriterial selection, aggregated evaluations, fields of economics, investments.

Introduction

Let us cite a mathematic model, which will allow to estimate and build a ranking range of inhomogeneous alternatives. Inhomogeneous alternatives – are alternatives which are different by their nature and cannot be estimated by the common set of criteria. We can divide on groups the set of inhomogeneous alternatives by some common features, which can be rated by appropriate set of criteria. Each group of alternatives with their criteria will be called corresponding "category alternatives".

Mathematic model

In problems of multicriterial choice of alternative set in respect of evaluating criteria, they can be classified as follows:
1. comparative by the common set of criteria;
2. not comparative by the common set of criteria;
3. partially comparative by the common set of criteria.

Problem solution of first class comes down to the common multicriterion problem of decision taking, which is specifically defined by the scope of its application.
In the second class alternatives need to be estimated separately by own criteria set of rating one alternative or a group and to take decisions on their basis.

The third class includes inhomogeneous alternatives, which have a common set of criteria, but rating by their assistance does not give detailed information. For each alternative, there are additional criteria of themselves, with the application of which we will get improved and adequate mark. Such set of alternatives arises in problems where they are combined in one area, but each of them has their own concrete functional direction. For example, they may include the following tasks:
- Evaluating branches of economics;
- Evaluating investment projects in different areas of activity;
- Evaluating and choosing the venue of music festivals and etc.

Let us consider the setting and approaches to problem solution, which belong to the class of partially comparative alternatives by the common set of criteria.

Depending on task the set of inhomogeneous alternatives $X = \{x_1, x_2, ..., x_n\}$ is divided into $A = \{A_1, A_2, ..., A_\alpha\}$ categories by common features, $A_i = \{x^i_1, x^i_2, ...\}$, $i = \overline{1, \alpha}$, where A_i is the i category of alternatives. All alternatives will be estimated by common criteria set of effectiveness $\{K_1, K_2, ..., K_{p-1}\}$ and each category of alternative, in turn, we will rate by own set of criteria $K_p = \{K_1, K_2, ..., K_{m_i}\}$.

The problem can be formulated as follows: build a ranking range and choose the best alternative from X set, when they are known on this set of criteria estimation. The model of the problem can be represented as table 1.

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[1] *Nikola Malyar Ph.D., Associate Professor, doctoral student modeling complex systems, the Department of Cybernetics, Taras Shevchenko National university of Kyiv. Scope of scientific interests: Decision making, modeling social and economic processes and systems, methods of fuzzy logic. Home. phone. - (0312) 64-47-40, E-mail: malyarmm@gmail.com*

[2] *Volodimir Polishchuk PhD in technical sciences, Lecturer the Department of Software Systems Uzhgorod National University Scope of scientific interests: Decision making in investment activity, methods of fuzzy logic. Contact tel.: +380664207484, E-mail: v.polishchuk87@gmail.com*

Table 1. Table of estimating by criteria

	x_1	x_2	$\dots$	x_n
K_1	O_{11}	O_{12}	$\dots$	O_{1n}
K_2	O_{21}	O_{22}	$\dots$	O_{2n}
$\vdots$				
K_{p-1}	O_{p-11}	O_{p-12}	$\dots$	O_{p-1n}
K_p	O_{p1}	O_{p2}	$\dots$	O_{pn}

Or as a matrix of solutions:

$$O = (O_{gj}),\ g = 1, \dots, p;\ j = 1, \dots, n; \tag{1}$$

where O_{gj} is a mark of j- alternative by the g- criterion. Each column of the matrix is a vector of marks, which characterizes the alternative, and each line of the matrix is a criterion. $O_{p1}, O_{p2}, \dots, O_{pn}$ are aggregated assessments of alternatives, which are received by the criteria set of concrete category. The task of selection is divided into two stages:

on the first stage of problem solution it is necessary to find aggregated marks of $O_{p1}, O_{p2}, \dots, O_{pn}$ alternatives considering their category;

on the second stage, with all marks of alternatives by criteria we can build a ranking range of matrix solutions (1).

Let us examine the cases when there are alternative marks by criteria in different time, i.e. for static and dynamic criteria. Criteria by which we can monitor the dynamics of criterial marks of l periods we will call dynamic.

In the solution of a concrete applied problem, firstly, we group alternatives into different categories by some common features. The scheme of problem solution, for obtaining aggregate marks, is selected depending on the number of alternatives in each category and criterial set.

Let us consider two cases of problem solution depending on the set of criteria: problem C1 – criteria of evaluating are static, problem C2 – dynamic.

Problem C1

Let us have in this problem some alternatives in one $A_i = \{x^i_1, x^i_2, \dots, x^i_k\}$ category, $k < n$ which are evaluated by static criteria of $\{K^i_1, K^i_2, \dots, K^i_{m_i}\}$, where i is a category of alternatives, $i = \overline{1, \alpha}$. The model of the problem can be represented as table 2.

Table 2. Table of alternative marks by criteria for the C1 problem

	x^i_1	x^i_2	$\dots$	x^i_k
K^i_1	O^i_{11}	O^i_{12}	$\dots$	O^i_{1k}
K^i_2	O^i_{21}	O^i_{22}	$\dots$	O^i_{2k}
$\vdots$				
$K^i_{m_i}$	$O^i_{m_i1}$	$O^i_{m_i2}$	$\dots$	$O^i_{m_ik}$

Or as a matrix of solutions:

$$Z^i_1 = \left(O^i_{df}\right), d = \overline{1, m_i}; f = \overline{1, k}; i = \overline{1, \alpha}, \tag{2}$$

where O^i_{df} is the mark of f-alternative by d-criterion for i-category of alternatives.

The number of matrix solutions with be determined by the number of alternative categories. On the basis of matrix solutions it is necessary to obtain vector of $V_1, V_2, \dots, V_\alpha$ alternative ratings, which will include all desired $O_{p1}, O_{p2}, \dots, O_{pn}$ marks for K_p criterion. Such task is a problem of multicriterial selection, therefore vectors of alternative marks $V_1, V_2, \dots, V_\alpha$ can be found by one of approaches [1-3].

Problem C2

The following type of problems arises, when a criteria set of marks is dynamic $\{K^i_1, K^i_2, ..., K^i_{mi}\}$ and the category of alternatives has one or more alternatives: $A_i = \{x^i_1, x^i_2, ..., x^i_k\},\ k < n$. Without reducing the generality let us consider the problem solution when some category has one alternative. In other case the problem solution will be gradual as many times , as there are alternatives in each category.

The value of criteria for all periods we represent as table 3, separately for each category of alternatives $i = 1,2,...,\alpha$, where ε_{l_i} are periods.

Table 3. Table of alternative marks by criteria for the C2 problem

A_i	ε_1	ε_2	$\dots$	ε_{l_i}
K^i_1	Q^i_{11}	Q^i_{12}	$\dots$	$Q^i_{1l_i}$
K^i_2	Q^i_{21}	Q^i_{22}	$\dots$	$Q^i_{2l_i}$
$\vdots$				
$K^i_{m_i}$	$Q^i_{m_i1}$	$Q^i_{m_i2}$	$\dots$	$Q^i_{m_il_i}$

Each category of alternatives can have its own set of criteria and number of periods.

For each category of alternatives let us build aggregated evaluations $O_{pi}, i = \overline{1,\alpha}$ using the following steps.

Step 1.

On the first step we normalize the evaluations $Q^i_{m_il_i}$. For this review, we introduce a "point of satisfaction" [4] $T^i = \left(t^i_1, t^i_2, ..., t^i_{m_i}\right)$, i.e. imaginary alternatives for each category in which evaluations for all criteria could satisfy a person, who is making decisions (DM).

Let us determine the sets of values as follows:

$$z^i_{de} = 1 - \frac{\left|t^i_d - Q^i_{de}\right|}{\max\{t^i_d - \min\limits_e Q^i_{de}; \max\limits_e Q^i_{de} - t^i_d\}}, d = \overline{1,m_i};\quad e = \overline{1,l_i};\quad i = \overline{1,\alpha}. \tag{3}$$

Matrices $Z_i = \{z^i_{de}\}$ defined in this way characterize by columns relative evaluations of alternative intimacy to the “points of satisfaction” for each concrete criterion and remove the issue of different evaluation scales. The *Z* matrix is being built for each alternative and one “point of satisfaction” is determined for each category.

Step 2.

For each $\{K^i_1, K^i_2, ..., K^i_{mi}\}$ criterion the decision maker knows or can set weight coefficients $\{p^i_1, p^i_2, ..., p^i_{mi}\}$ from the [1; a] interval. Then you can determine the normalized weight coefficients for each criterion by different categories of alternatives:

$$\alpha^i_d = \frac{p^i_d}{\sum\limits_{d=1}^{m_i} p^i_d},\ d = \overline{1,m_i}\ ;\ \alpha^i_d \in [0;1]; \tag{4}$$

which meet the $\sum\limits_{d=1}^{m_i} \alpha^i_d = 1$ requirement .

Let us apply one of the convolutions to build a vector of evaluations [5]. For example, take the average weighted convolution:

$$w_e^i = \sum_{d=1}^{m_i} \alpha_d^i z_{de}^i, i = \overline{1,\alpha}; d = \overline{1,m_i}; e = \overline{1,l_i}. \quad (5)$$

Vectors of evaluations will look like:

$$W^i = \left(w_1^i; \quad w_2^i; \quad ... \quad w_{l_i}^i\right), i = \overline{1,\alpha}. \quad (6)$$

Each vector will include aggregated and normalized marks on the set of criteria, separately for each period and alternative category. Thus, vector will have *l* dimension – i.e. a number of periods in concrete alternative category. In case when then number of alternatives is greater than one, then vector will be presented as a matrix.

Step 3.

As well as each $w_{l_i}^i$ mark includes aggregated information of alternative mark for a concrete period, then we have to somehow obtain one mark, which could include activity tendencies of all considered periods. For this, let us forecast $w_{l_i}^i$ marks for the next period $L = l_i + 1$, for example, based on the pair linear regression [6-7]:

$$Y^i(L) = a^i + b^i \cdot L, \qquad i = \overline{1,\alpha}. \quad (7)$$

The value of *a, b* coefficients we will calculate by method of least squares according to the formulas [6]:

$$b^i = \frac{l_i \cdot \sum_{e=1}^{l_i} \varepsilon_e \cdot w_e^i - \sum_{e=1}^{l_i} \varepsilon_e \cdot \sum_{e=1}^{l_i} w_e^i}{l_i \cdot \sum_{e=1}^{l_i} {\varepsilon_e}^2 - \left(\sum_{e=1}^{l_i} \varepsilon_e\right)^2}, \quad (8)$$

$$a^i = \overline{d^i} - b^i \cdot \overline{q}, \quad (9)$$

where $\overline{d^i} = \frac{1}{l_i} \sum_{e=1}^{l_i} w_e^i, \quad \overline{q} = \frac{1}{l_i} \sum_{e=1}^{l_i} \varepsilon_e, \quad i = \overline{1,\alpha}.$

Step 4.

Substituting the values of *a, b* coefficients into the regressive equation (7) we will get the desired evaluation:

$$O_{pi} = Y^i(l_i + 1), \qquad i = \overline{1,\alpha}. \quad (10)$$

These evaluations will contain predictive information, estimates of alternative by dynamic criteria, for the next period received considering the trend of previous periods.

Thus we got all alternative evaluations for the matrix of decisions (1) and completed the first stage of the given task. On the second stage with having all the alternative evaluations by criteria, we can construct a ranking range of alternatives on the basis of decision matrix (1). Without reducing the generality we assume the $O = \left(O_{gj}\right), g = \overline{1, p-1}; j = \overline{1,n}$ elements of matrix are normalized. In other case depending on a concrete applied task, the normalization of evaluations can be managed with the usage of the "point of satisfaction" or other approaches are described in [8].

Let the DM know or could set the weight coefficients to each criterion of effectiveness $\{p_1, p_2, ..., p_p\}$ from the [1; a] interval. Then we determine the normalized weight coefficients for each criterion by the usage of formula (4).

Further, let us take one of the convolutions for the construction of an aggregated evaluation of alternatives from the matrix of decisions (1) [3]. For example the average weighted convolution, in this case will look like:

$$A(x_j) = \sum_{g=1}^{p} \alpha_g O_{gj}, j = \overline{1,n}. \quad (11)$$

On the basis of $A(x_j)$ values we build a ranking range of inhomogeneous alternatives:

$$A = (A_1, A_2, ..., A_n). \quad (12)$$

Therefore, the mathematic model with the usage of which we can construct a ranking range of inhomogeneous alternatives and having both static and dynamic criteria of evaluations is given.

Example of mathematic model usage

We will use the mathematic model for evaluating the promise of economics fields. For example it is needed to evaluate fields of economics and determine between then the most perspective for investing. The difficulty of evaluation is that each field works in its own conditions and has both common and own set of criteria for evaluation. Therefore, there are inhomogeneous alternatives:

The set of alternatives we select as follows:

- Wood, pulp and paper industry - x_1 ;
- Light industry goods - x_2 ;
- Agriculture - x_3 ;
- Tourism - x_4 .

Grouping alternatives in categories we carry separately on braches. I.e. each separate field will be a category $A_1 = (x_1), A_2 = (x_2), A_3 = (x_3), A_4 = (x_4)$.

The set of common criteria for evaluating we determine as follows:

1. K_1 - level of profitability (determined in percentage from the previous period);
2. K_2 - the speed of technological innovation (low – [0; 0,3]; medium – [0,3; 0,7]; high – [0,7; 1]).
3. K_3 - competence level (low – [0,7; 1]; medium – [0,3; 0,7]; high – [0; 0,3]);
4. K_4 - attitude of financial institutions and intermediaries to industry (low trust level – [0; 0,3]; medium – [0,3; 0,7]; high – [0,7; 1]).

Let the investor known the evaluations by common criteria for each field and necessity of criteria. Let us represent this information in table 4, where the last line shows the aggregated evaluations of alternatives by own set of criteria in each field [9].

Table 4. Evaluations of alternatives by common criteria

	x_1	x_2	x_3	x_4	Weight
K_1	7,65	8,6	11,2	15,11	7
K_2	0,1	0,35	0,15	0,65	4
K_3	0,33	0,67	0,77	0,35	9
K_4	0,7	0,4	0,3	0,8	3
K_5	O_{51}	O_{52}	O_{53}	O_{54}	10

Evaluations of each category by own criteria set exists in the different period of time.

For all listed above fields we represent estimates data by years 2011,2012,2013, their necessity (from interval [1;10]) and the "point of satisfaction"(PS) in tables 5-8 [9].

Table 5. Wood, pulp and paper industry - x_1

Evaluation criteria	Criteria name	2011	2012	2013	Weight	PS
K_1^1	The wood is sawn lengthwise or split, thousands of m3	1888	1823	1804	9	1900
K_2^1	Bars, strips and friezes for parquet or wooden flooring, thousands of m2	3765	3619	3563	9	3700
K_3^1	Wooden windows and doors, thousands of m2	2921	2642	3309	8	3300
K_4^1	Paper and cardboard of graphic destination, thousands of tons	33,4	17,8	13,7	5	30
K_5^1	Paper for manufacturing hygienic or cosmetic wipes, towels, diapers, cloths, cellulose wadding and webs of cellulose fibers	131	138	146	8	140

Table 6. Field – light industry goods - x_2

Evaluation criteria	Criteria name	2011	2012	2013	Weight	PS
K_1^2	cloths, millions of m2	89,0	105,7	93,6	9	90
K_2^2	Woolen yard, thousands of tons	1,7	1,8	1,9	4	2
K_3^2	Rugs and carpet products, millions of m2	7,6	8,2	8,5	5	8
K_4^2	footgear, millions of pairs	28,1	28,3	30,5	6	30

Table 7. Agricultural field x_3

Evaluation criteria	Criteria name	2011	2012	2013	Weight	PS
K_1^3	Fresh or chilled bovine animals meat, thousands of tons	64,0	61,8	62,8	7	62
K_2^3	Fresh or chilled domestic poultry meat, thousands of tons	689	691	778	10	700
K_3^3	Flour, thousands of tons	2596	2605	2542	9	2600
K_4^3	Groats, thousands of tons	356	365	367	8	400
K_5^3	White sugar, thousands of tons	2586	2143	1263	8	2000

Table 8. Field of tourism x_4

Evaluation criteria	Criteria name	2011	2012	2013	Weight	PS
K_1^4	The number of Ukrainian citizens who travelled abroad – total number, millions.	19,77	21,43	23,76	7	23
K_2^4	The number of foreign citizens who visited Ukraine – total number, millions	21,41	23,01	24,67	8	30
K_3^4	The number of tourists who were serviced by the subjects of Ukrainian tourist activity – total number, millions	21,99	30,00	34,54	9	30

On the first stage we get aggregated evaluations for each field. Such a problem we assign to the C2 type. To do this we follow these steps

Step 1.

Let us construct matrices $Z_1, \ldots, Z_4$, elements of which are calculated by the formula (3).

$$Z_1 = \begin{pmatrix} 0,88 & 0,20 & 0,00 \\ 0,53 & 0,41 & 0,00 \\ 0,42 & 0,00 & 0,99 \\ 0,79 & 0,25 & 0,00 \\ 0,00 & 0,78 & 0,33 \end{pmatrix}; \quad Z_2 = \begin{pmatrix} 0,94 & 0,00 & 0,77 \\ 0,00 & 0,33 & 0,67 \\ 0,20 & 0,60 & 0,00 \\ 0,00 & 0,11 & 0,74 \end{pmatrix}; \quad Z_3 = \begin{pmatrix} 0,00 & 0,90 & 0,60 \\ 0,86 & 0,88 & 0,00 \\ 0,93 & 0,91 & 0,00 \\ 0,00 & 0,20 & 0,25 \\ 0,20 & 0,81 & 0,00 \end{pmatrix};$$

$$Z_4 = \begin{pmatrix} 0,00 & 0,51 & 0,76 \\ 0,00 & 0,19 & 0,38 \\ 0,00 & 1,00 & 0,43 \end{pmatrix}.$$

Step 2.

Let us determine the normalized weight coefficients for each criterion and category of alternatives by formula (4).

$\alpha^1 = (0{,}23;\ \ 0{,}23;\ \ 0{,}21;\ \ 0{,}13;\ \ 0{,}21)$; $\alpha^2 = (0{,}38;\ \ 0{,}17;\ \ 0{,}21;\ \ 0{,}25)$;

$\alpha^3 = (0{,}17;\ \ 0{,}24;\ \ 0{,}21;\ \ 0{,}19;\ \ 0{,}19)$; $\alpha^4 = (0{,}29;\ \ 0{,}33;\ \ 0{,}38)$.

On the basis of the average weighted convolution, we calculate evaluation vectors by the formula (5).

$$W^1 = \begin{pmatrix} 0{,}512 \\ 0{,}332 \\ 0{,}271 \end{pmatrix}; \quad W^2 = \begin{pmatrix} 0{,}393 \\ 0{,}207 \\ 0{,}584 \end{pmatrix}; \quad W^3 = \begin{pmatrix} 0{,}443 \\ 0{,}749 \\ 0{,}148 \end{pmatrix}; \quad W^4 = \begin{pmatrix} 0{,}000 \\ 0{,}587 \\ 0{,}512 \end{pmatrix}.$$

Each vector includes aggregated and normalized evaluations for criteria set, separately by periods 2011, 2012, 2013. I.e., it characterized how effective was the field functioning in some particular year.

Step 3.

Let us forecast the aggregated evaluation of vectors for 2014. Firstly, we calculate the values of *a,b* coefficients for each field using the formulas (8)-(9).

$b^1 = -0{,}12$, $a^1 = 242{,}82$, $b^2 = 0{,}10$, $a^2 = -191{,}75$, $b^3 = -0{,}15$, $a^3 = 297{,}22$, $b^4 = 0{,}26$, $a^4 = -514{,}71$.

Substituting the values of *a ,b* coefficients into a regressive equation (7) we will get the desired evaluation (10), which includes the forecasting information considering the tendency of previous periods: $O_{51} = Y^1(2014) = 242{,}82 - 0{,}12 \cdot 2014 = 0{,}131$; $O_{52} = Y^2(2014) = 0{,}586$; $O_{53} = Y^3(2014) = 0{,}152$; $O_{54} = Y^4(2014) = 0{,}878$.

The first stage of the task is complete, let us proceed to the evaluation and construction of the economics fields ranking range. All evaluation for economics fields are normalized, except the profitability criteria. The evaluation normalizing of this criterion we propose to manage by dividing each evaluation on the maximal criteria evaluation.

All evaluations for economics fields and normalized weight coefficients (calculate by the formula (4)) we represent as table 9.

Table 9. Economics field evaluations by criteria

	x_1	x_2	x_3	x_4	Normalized scales
K_1	0,51	0,57	0,74	1,00	0,21
K_2	0,1	0,35	0,15	0,65	0,12
K_3	0,33	0,67	0,77	0,35	0,27
K_4	0,7	0,4	0,3	0,8	0,09
K_5	0,131	0,586	0,152	0,878	0,31

To construct an aggregated evaluation we use formula (11): $A = (0{,}313;\ 0{,}560;\ 0{,}459;\ 0{,}725)$. Organize alternatives by decending: x_4 - tourism; x_2 - light industry goods; x_3 - agriculture; x_1 - wood, paper and pulp industry.

According to the constructed ranking range we can define the most promising fields of economics, which are examined for the last three years, such as: tourism; light industry goods. On the other hand fields such as agriculture, wood, paper and pulp industry show a lower tendency and require measures to change the situation for the better.

Conclusion

Thus, a mathematic model is shown, which allows to construct a ranking range of inhomogeneous alternatives. An algorithm is offered, which allows to solve adequately such a difficult problem, as evaluation of the prospects of economics field functioning for possible investments. The given example illustrates only the fragment by criterial set. For real evaluation on the basis of which a decision of investing will be taken it is necessary to include more evaluation criteria from different aspect and perspectives of fields activity.

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Recenzenti:
Dr.h.c. prof. Ing. Vladimír Klimo, CSc. - VŠBM v Košiciach, Ústav humanitných a technologických vied
Ing. Alena Olehlová, Ph.D. - Fakulta vojenského leadershipu, Univerzita obrany, Brno