DUST LATTICE WAVES IN A HEXAGONAL LATTICE OF PARAMAGNETIC GRAINS

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Dust lattice (DL) modes are studied in a hexagonal two-dimensional lattice in plasma crystal, include paramagnetic dust particles. The gradients of magnetic field, electric fields and dust charge taken into account. A novel type of coupling between transverse and longitudinal modes, due to these gradients is found. Also these gradients modifies the levitation condition and affect the frequencies of dust lattice waves.

Introduction

Dust lattice waves are produced by the oscillations of regularly spaced charged micro particles suspended in a plasma crystal, which form as a result of strong mutual coulomb interaction. Crystalline complex plasma structures has been observed in recent rf discharge experiments [1], in which the plasma sheath was embedded in an external magnetic field. Theoretical studies then followed for the investigation of conditions for magnetic-field-assisted crystal equilibria involving paramagnetic charged dust grains. The role of various forces acting on paramagnetic grains has been discussed in Ref. [2], where magnetic forces have been shown to prevail over the (weaker) electric polarization forces. Also effect of magnetic field in dusty plasma lattice has studied in Ref. [3,4]

Recently, Yaroshenko et al, studied the vertical vibrations of a one-dimensional string of magnetized particles, taking into account the magnetic force associated with gradients of an external magnetic field, and they founded a new low-frequency oscillatory mode [5]. The influence of an inhomogeneous magnetic field, ion focusing effect and equilibrium charge gradient on the propagation of DL modes in a 1D string by paramagnetic particles is considered in Ref [6], and they founded the modification of DL waves. Dust lattice waves in hexagonal dusty plasma crystal studied before [7-8]. Bending mode in hexagonal dusty plasma crystal has studied by S.V.Vladimirov [8].

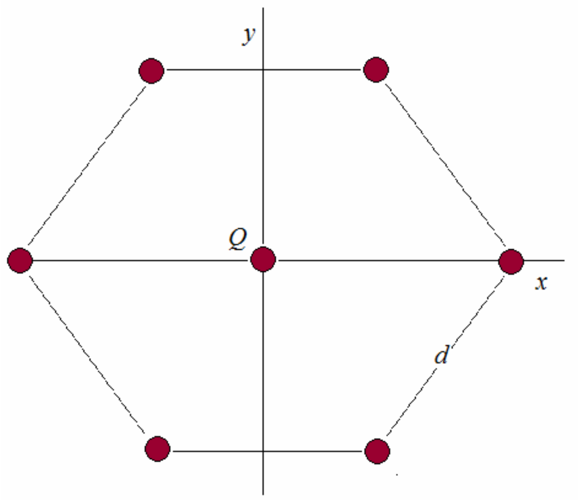

Figure 1. Hexagonal structure of dusty plasma crystal.

Vibrational modes in a hexagonal lattice of paramagnetic grains

The hexagonal structure of dusty plasma crystal show in Figure 1.

In this section to describe the modes in a dusty plasma with magnetized grains, we consider a hexagonal crystal, where the spherical dust grains have magnetic moment $\vec{m}$, parallel to the external magnetic field. The magnetic moment of a particle with radius a and magnetic permeability μ, in an external magnetic field B is

$$m = \frac{\mu - 1}{\mu + 2} a^3 B = \alpha B . \qquad (1)$$

The influence of a inhomogeneous magnetic field on a magnetized grains is

$$F_m = -\frac{\partial(-\vec{m}\cdot\vec{B})}{\partial z} = 2\alpha B(\partial B / \partial z) \approx 2\alpha B_0 B_0' + 2\alpha (B_0 B_0'' + B_0'^2) z + \cdots , \qquad (2)$$

where a series expansion used for B. The electric force, by using from the same series expansion for Q and E is

$$F_E = QE = Q_0 E_0 + (Q_0 E_0' + Q_0' E_0) z + \cdots \qquad (3)$$

The electrostatic and magnetic energy due to interaction between the origin (o'th) grain and its neighbors (i'th grains) of crystal can be written as

$$U_{o,i} = \frac{Q_o Q_i}{4\pi\varepsilon_o |r_{o,i}|} \exp\left(-\frac{|r_{o,i}|}{\lambda_D}\right) - \frac{\mu_o}{4\pi}\left[\frac{\vec{m}_o \cdot \vec{m}_i}{|r_{o,i}|^3} - \frac{3(\vec{m}_o \cdot \vec{r}_{o,i})(\vec{m}_i \cdot \vec{r}_{o,i})}{|r_{o,i}|^5}\right] . \qquad (4)$$

The particle interaction force acting on the o'th particle can be presented as

$$\vec{F}_{o,i} = -\partial U_{o,i} / \partial \vec{r}_o . \qquad (5)$$

The dipole interactions are also short ranged, so that we need only to consider the nearest neighbor particle interactions. The equation of motion for the origin particle in crystal is

$$\vec{F} = -\sum_i \nabla U_{o,i} + \vec{F}_E + \vec{F}_m - M\vec{g} . \qquad (6)$$

Using from Eqs. (2)-(6), and considering only small oscillations (u, v and $z << d$) around the equilibrium position, it gives he component of linear equation of motion for origin particle

$$\begin{aligned}
M\ddot{u} + 2\gamma M\dot{u} = {} & \left(U_o'' - \frac{3\mu_o m_o^2}{\pi d^5}\right)(u_{m+1,n} + u_{m-1,n} - u_{m,n}) \\
& + \left(\frac{3U_o'}{4d} + \frac{U_o''}{4} - \frac{9\mu_o m_o^2}{8\pi d^5}\right)[u_{m+\sqrt{3}/2,n+1/2} + u_{m-\sqrt{3}/2,n+1/2} + u_{m-\sqrt{3}/2,n-1/2} \\
& + u_{m+\sqrt{3}/2,n-1/2} - 4u_{m,n}] + \left(-\frac{\sqrt{3}U_o'}{4d} + \frac{\sqrt{3}U_o''}{4} - \frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5}\right)[v_{m+\sqrt{3}/2,n+1/2} \\
& - v_{m-\sqrt{3}/2,n+1/2} + v_{m-\sqrt{3}/2,n-1/2} - v_{m+\sqrt{3}/2,n-1/2}] \\
& + \left(\frac{3\mu_o m_o m_o'}{4\pi d^4}\right)[z_{m+1,n} - z_{m-1,n}] \\
& + \left(\frac{3\mu_o m_o m_o'}{8\pi d^4}\right)[z_{m+\sqrt{3}/2,n+1/2} - z_{m-\sqrt{3}/2,n+1/2} - z_{m-\sqrt{3}/2,n-1/2} + z_{m+\sqrt{3}/2,n-1/2}]
\end{aligned} \qquad (7)$$

$$
\begin{aligned}
M\ddot{v}+2\gamma M\dot{v} &= \left(\frac{U_o'}{d}-\frac{3\mu_o m_o^2}{4\pi d^5}\right)(v_{m+1,n}+v_{m-1,n}-v_{m,n}) \\
&+\left(\frac{U_o'}{d}+\frac{3U_o''}{4}-\frac{39\mu_o m_o^2}{8\pi d^5}\right)[v_{m+\sqrt{3}/2,n+1/2}+v_{m-\sqrt{3}/2,n+1/2} \\
&+v_{m-\sqrt{3}/2,n-1/2}+v_{m+\sqrt{3}/2,n-1/2}-4v_{m,n}] \\
&+\left(\frac{\sqrt{3}U_o''}{4}-\frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5}\right)[u_{m+\sqrt{3}/2,n+1/2}-u_{m-\sqrt{3}/2,n+1/2} \\
&+u_{m-\sqrt{3}/2,n-1/2}-u_{m+\sqrt{3}/2,n-1/2}] \\
&+\left(\frac{3\sqrt{3}\mu_o m_o m_o'}{8\pi d^4}\right)[z_{m+\sqrt{3}/2,n+1/2}+z_{m-\sqrt{3}/2,n+1/2}-z_{m-\sqrt{3}/2,n-1/2}-z_{m+\sqrt{3}/2,n-1/2}]
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
M\ddot{z}+2\gamma M\dot{z} &= \left(\frac{U_o'}{d}+\frac{9\mu_o m_o^2}{4\pi d^5}\right)(z_{m+1,n}+z_{m-1,n}+z_{m+\sqrt{3}/2,n+1/2}+z_{m-\sqrt{3}/2,n+1/2} \\
&+z_{m-\sqrt{3}/2,n-1/2}+z_{m+\sqrt{3}/2,n-1/2}-6z_{m,n})+\left(-\frac{3\mu_o m_o m_o'}{4\pi d^4}\right)[u_{m+1,n}-u_{m-1,n}] \\
&+\left(-\frac{3\mu_o m_o m_o'}{8\pi d^4}\right)[u_{m+\sqrt{3}/2,n+1/2}-u_{m-\sqrt{3}/2,n+1/2}-u_{m-\sqrt{3}/2,n-1/2}+u_{m+\sqrt{3}/2,n-1/2}] \\
&+\left(-\frac{3\sqrt{3}\mu_o m_o m_o'}{8\pi d^4}\right)[v_{m+\sqrt{3}/2,n+1/2}+v_{m-\sqrt{3}/2,n+1/2}-v_{m-\sqrt{3}/2,n-1/2}-v_{m+\sqrt{3}/2,n-1/2}] \\
&+(QE)_o' z-2\alpha(BB')_o' z
\end{aligned}
\tag{9}
$$

where *u*, *v* and *z*, are displacement components of the origin particle; m_o stands for the equilibrium magnetic moment of the grains, and $U_o' = -\frac{Q_o^2}{4\pi\varepsilon_o d^2}e^{-\kappa}(1+\kappa)$, $U_o'' = \frac{Q_o^2}{4\pi\varepsilon_o d^3}e^{-\kappa}(2+2\kappa+\kappa^2)$. Subscript ‘‘0’’ denotes the equilibrium position *z=0*. There is always a position where gravitation can be compensated by the electric and magnetic field

$$
Mg = Q_o E_o + 2\alpha B_o B_o' - 6\frac{Q_o Q_o'}{4\pi\varepsilon_o d} + 6\frac{\mu_o m_o m_o'}{4\pi d^3}. \tag{10}
$$

Assuming now that $u_{m,n}, v_{m,n}, z_{m,n}$ varies as $\propto \exp[i(kmd+knd+kld-\omega t)]$, yields from the equation of motion

$$
[\omega^2+2i\gamma\omega+D_1]u+D_{12}v+iD_{13}z=0 \tag{11}
$$

$$
D_{21}u+[\omega^2+2i\gamma\omega+D_{22}]v+iD_{23}z=0 \tag{12}
$$

$$
iD_{31}u+iD_{32}v+[\omega^2+2i\gamma\omega+D_{33}]z=0 \tag{13}
$$

where

$$
\begin{aligned}
D_{11} &= 4\left(U_o''-\frac{3\mu_o m_o^2}{\pi d^5}\right)\sin^2\left(\frac{kd\cos\theta}{2}\right) \\
&+4\left(\frac{3U_o'}{4d}+\frac{U_o''}{4}-\frac{9\mu_o m_o^2}{8\pi d^5}\right)\left[\sin^2\left(\frac{kd\cos(\theta+\pi/3)}{2}\right)+\sin^2\left(\frac{kd\cos(\theta-\pi/3)}{2}\right)\right]
\end{aligned}
$$

$$
D_{12} = -2\left(-\frac{\sqrt{3}U_o'}{4d}+\frac{\sqrt{3}U_o''}{4}-\frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5}\right)[\cos(kd\cos(\theta+\pi/3))-\cos(kd\cos(\theta-\pi/3))]
$$

$$D_{13} = -\frac{3\mu_o m_o m_o'}{2\pi d^4}\sin(kd\cos(\theta)) - \frac{3\mu_o m_o m_o'}{4\pi d^4}[\sin(kd\cos(\theta+\pi/3)) - \sin(kd\cos(\theta-\pi/3))]$$

$$D_{21} = -(\frac{\sqrt{3}U_o''}{2} - \frac{15\sqrt{3}\mu_o m_o^2}{4\pi d^5})[\cos(kd\cos(\theta+\pi/3)) - \cos(kd\cos(\theta-\pi/3))]$$

$$D_{22} = 4(\frac{U_o'}{d} - \frac{3\mu_o m_o^2}{4\pi d^5})\sin^2(\frac{kd\cos\theta}{2}) + 4(\frac{U_o'}{d} + \frac{3U_o''}{4} - \frac{39\mu_o m_o^2}{8\pi d^5})[\sin^2(\frac{kd\cos(\theta+\pi/3)}{2}) + \sin^2(\frac{kd\cos(\theta-\pi/3)}{2})]$$

$$D_{23} = -\frac{3\sqrt{3}\mu_o m_o m_o'}{4\pi d^4}[\sin(kd\cos(\theta+\pi/3)) + \sin(kd\cos(\theta-\pi/3))]$$

$$D_{31} = -(-\frac{3\mu_o m_o m_o'}{2\pi d^4} + \frac{Q_o Q_o' e^{-\kappa}}{2\pi\varepsilon_0 d^2}(1+\kappa))[\sin(kd\cos(\theta)) + \frac{1}{2}(\sin(kd\cos(\theta+\pi/3)) - \sin(kd\cos(\theta-\pi/3)))]$$

$$D_{32} = -\sqrt{3}(-\frac{3\mu_o m_o m_o'}{4\pi d^4} + \frac{Q_o Q_o' e^{-\kappa}}{4\pi\varepsilon_0 d^2}(1+\kappa))[\sin(kd\cos(\theta+\pi/3)) + \sin(kd\cos(\theta-\pi/3))]$$

$$D_{33} = 4(-\frac{Q_o^2 e^{-\kappa}}{4\pi\varepsilon_0 d^2}(1+\kappa) + \frac{9\mu_o m_o^2}{4\pi d^5})[\sin^2(\frac{kd\cos(\theta)}{2}) + \sin^2(\frac{kd\cos(\theta+\pi/3)}{2}) + \sin^2(\frac{kd\cos(\theta-\pi/3)}{2})] - \frac{Q_o'^2 e^{-\kappa}}{2\pi\varepsilon_0 d}\cos(kd\cos(\theta)) + \frac{3Q_o Q_o'' e^{-\kappa}}{2\pi\varepsilon_0 d} + \frac{Q_o'^2 e^{-\kappa}}{2\pi\varepsilon_0 d}[\cos(kd\cos(\theta+\pi/3)) + \cos(kd\cos(\theta-\pi/3))] + (QE)_o' + 2\alpha(BB')_o'$$

If we set $Q' = 0, B' = 0, E' = 0$ in Eqs. (11)-(13), the vertical oscillatory mode is a independent mode, while two other modes are coupled yet. In this case the vertical component of equation of motion is

$$M\ddot{z} + 2\gamma M\dot{z} = (\frac{U_o'}{d} + \frac{9\mu_o m_o^2}{4\pi d^5})(z_{m+1,n} + z_{m-1,n} + z_{m+\sqrt{3}/2,n+1/2} + z_{m-\sqrt{3}/2,n+1/2} + z_{m-\sqrt{3}/2,n-1/2} + z_{m+\sqrt{3}/2,n-1/2} - 6z_{m,n}) \qquad (14)$$

which in accordance to Eq.(1) of Ref.[8], else in second term in vertical frequency due to dipole interactions.

Two another components of equation of motion is same as Eqs. (14), (15) of Ref. [7], approximately. These equations are include the effect of dipole-dipole interactions, and it leads to modified coefficients

$$M\ddot{u} + 2\gamma M\dot{u} = (U_o'' - \frac{3\mu_o m_o^2}{\pi d^5})(u_{m+1,n} + u_{m-1,n} - u_{m,n}) + (\frac{3U_o'}{4d} + \frac{U_o''}{4} - \frac{9\mu_o m_o^2}{8\pi d^5})[u_{m+\sqrt{3}/2,n+1/2} + u_{m-\sqrt{3}/2,n+1/2} + u_{m-\sqrt{3}/2,n-1/2} + u_{m+\sqrt{3}/2,n-1/2} - 4u_{m,n}] + (-\frac{\sqrt{3}U_o'}{4d} + \frac{\sqrt{3}U_o''}{4} - \frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5})[v_{m+\sqrt{3}/2,n+1/2} - v_{m-\sqrt{3}/2,n+1/2} + v_{m-\sqrt{3}/2,n-1/2} - v_{m+\sqrt{3}/2,n-1/2}] \qquad (15)$$

$$
\begin{aligned}
M\ddot{v}+2\gamma M\dot{v} &= (\frac{U_o'}{d}-\frac{3\mu_o m_o^2}{4\pi d^5})(v_{m+1,n}+v_{m-1,n}-v_{m,n}) \\
&+(\frac{U_o'}{d}+\frac{3U_o''}{4}-\frac{39\mu_o m_o^2}{8\pi d^5})[v_{m+\sqrt{3}/2,n+1/2}+v_{m-\sqrt{3}/2,n+1/2} \\
&+v_{m-\sqrt{3}/2,n-1/2}+v_{m+\sqrt{3}/2,n-1/2}-4v_{m,n}] \\
&+(\frac{\sqrt{3}U_o''}{4}-\frac{15\sqrt{3}\mu_o m_o^2}{8\pi d^5})[u_{m+\sqrt{3}/2,n+1/2}-u_{m-\sqrt{3}/2,n+1/2} \\
&+u_{m-\sqrt{3}/2,n-1/2}-u_{m+\sqrt{3}/2,n-1/2}]
\end{aligned}
\qquad (16)
$$

But if gradients of fields and charge to be account, Eqs.(11)-(13) shows a coupling between three modes. Dispersion relation can obtain from simultaneous solution of these equations, so one can obtain dispersion relation

$$
\det\begin{pmatrix} \omega^2+2i\gamma\omega+D_{11} & D_{12} & D_{13} \\ D_{21} & \omega^2+2i\gamma\omega+D_{22} & iD_{23} \\ iD_{31} & iD_{32} & \omega^2+2i\gamma\omega+D_{33} \end{pmatrix}=0 \qquad (17)
$$

Three dust lattice modes are mixed, via Eq. (17), which for study of coupling of modes it should be plotted and be compared with modes in absent of coupling.

If a complex form as $\omega \to \omega_r + i\omega_i$ considered for frequency, and using from $\omega_i << \omega_r$ condition, then substituting this form of frequency in Eq. (17), one can obtain real part and imaginary part of this equation, respectively

$$
\begin{aligned}
&\omega_r^6+\omega_r^4[D_{11}+D_{22}+D_{33}]+\omega_r^2[D_{22}D_{33}+D_{23}D_{32}-D_{12}D_{21} \\
&\quad +D_{11}(D_{33}+D_{22})]+\omega_r[2(\omega_i+\gamma)D_{13}D_{31}] \\
&\quad +D_{11}(D_{22}D_{33}+D_{23}D_{32})-D_{12}D_{21}D_{33}+D_{12}D_{23}D_{31}=0
\end{aligned}
\qquad (18)
$$

$$
\begin{aligned}
&\omega_r^5[6\omega_i+6\gamma]+\omega_r^3[4\omega_i(D_{33}+D_{22}+D_{11}) \\
&\quad +4\gamma(D_{33}+D_{22}+D_{11})]+\omega_r^2[-D_{13}D_{31}] \\
&\quad +\omega_r[2(\omega_i+\gamma)(D_{33}D_{22}+D_{32}D_{23}) \\
&\quad +2D_{11}(\omega_i+\gamma)(D_{33}+D_{22})-2(\omega_i+\gamma)D_{12}D_{21}] \\
&\quad +D_{13}D_{21}D_{32}-D_{13}D_{31}D_{22}=0
\end{aligned}
\qquad (19)
$$

Eqs. (18) and (19) should solve simultaneously, to obtain a numerical expression for real and imaginary part of frequency. Then by using of these numerical expressions, and plotting of these two part of frequency, one can study coupling of dust lattice modes. The behavior of imaginary part of frequency, shows probably instability of wave propagation.

In summary the propagation of dust lattice modes in a hexagonal paramagnetic dust crystal has studied, including gradients of magnetic and electric fields and dust charge. Paramagnetic property of dusts, leads to modification of frequencies of dust lattice waves. When these gradients taken into account, the main conclusion is coupling of three modes. Also these gradients modifies the levitation condition and affect the frequencies of dust lattice waves. This implies that the characteristics of dust lattice mode coupling can be effectively controlled externally, due to gradients by experimental conditions.

Conclusion

The propagation of dust lattice mode in a hexagonal paramagnetic dust crystal has studied, including gradients of magnetic and electric fields and dust charge. Paramagnetic property of dusts, leads to modification of frequencies of dust lattice waves. When these gradients taken into account, the main conclusion is coupling of three modes. Also these gradients modifies the levitation condition and affect the frequencies of dust lattice waves. This implies that the characteristics of dust lattice mode coupling can be effectively controlled externally, due to gradients by experimental conditions.

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ХВИЛІ ПИЛОВОЇ ҐРАТКИ У ГЕКСАГОНАЛЬНОМУ КРИСТАЛІ ПАРАМАГНІТНИХ ЗЕРЕН

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Вивчаються моди пилової гратки у гексагональній двовимірній гратці у кристалі плазми, що включає парамагнітні частинки пилу. Враховано градієнти магнітного поля, електричного поля і заряду пилинок. Виявлено новий тип взаємодії між поперечними і поздовжніми модами внаслідок наявності цих градієнтів. Вказані градієнти також змінюють умови левітації і впливають на частоти хвиль пилової ґратки.