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CONTENTS

PLENARY LECTURES	7
Danielewski M., Sapa L., <i>Quaternionic quantum wave and gravity in the ideal elastic crystal</i>	7
Gerasimenko V.I., <i>On evolution equations of processes of creation and propagation of correlations in quantum systems</i>	7
Gusak A., Zaporozhets T., Tu K.N., <i>Solid-state reactions as phase transformations in open sharply inhomogeneous systems - physical and mathematical challenges</i> . . .	8
Minaev B.F., Baryshnikov G.V., Ågren H., <i>Quantum chemistry and molecular electronics</i>	9
Mishura Yu., <i>Statistics of fractional diffusion processes</i>	10
Samoilenko V.Hr., Samoilenko Yu.I., <i>Asymptotic soliton like solutions to the singularly perturbed Benjamin-Bona-Mahony equation with variable coefficients</i>	11
Vakhnenko V.O., <i>From simple poles to multiple poles in the inverse scattering method</i>	12
Zolotaryuk A.V., <i>Resonant tunneling of quantum particles through single-point potentials: A countable splitting phenomenon</i>	14
Zuyev A.L., <i>Stabilization of nonlinear controllable systems with oscillating input functions</i>	16
DYNAMICAL SYSTEMS, OPTIMAL CONTROL AND PROBLEMS OF MECHANICS	18
Ak T., <i>Computational investigation of the dynamics of shallow water waves</i>	18
Bihun Ya.I., <i>Multifrequency systems with linearly transformed argument</i>	18
Chepok O.O., <i>The asymptotic properties of regularly varying solutions of second order differential equations with regularly and rapidly varying nonlinearities</i>	19
Dronyuk I.M., <i>Q-analogs for Ateb-functions: properties and applications</i>	21
Filipkovska M.S., <i>A numerical method for the global analysis of implicit semilinear differential equations</i>	23
Gerzhanovskaya G.A., <i>Conditions of existence of solutions with slowly varying derivatives of second order essential nonlinear differential equations</i>	25
Gotsulenko V.V., <i>Discontinuous and chaotic self-oscillations in singularly perturbed dynamical systems</i>	27
Grushkovskaya V.V., Zuyev A.L., <i>Approximate path following problem for step-3 bracket generating systems</i>	28
Kalosha Ju.I., Zuyev A.L., <i>Finite-dimensional model of the beam oscillations with distributed and lumped controls</i>	30
Kononov Yu.M., Lymar A., <i>Oscillations of rectangular plate which separates ideal liquids of different density in a rectangular channel with the elastic basis</i>	31
Kononov Yu.N., Vasylenko V.Yu., <i>On the stability rotation of non symmetrical rigid body under stimulation in the resisting environment</i>	33
Kononov Yu.N., Shevchenko V.P., Dzhukha Yu.O., <i>Coupled vibrations of circular bases and ideal liquid in a rigid circular cylindrical container</i>	35
Kyrychenko V.V., Lesina Ye.V., <i>Visual simulation of chaotic dynamics of gyroscope</i> . .	37
Limanska D.E., Samkova G.E., <i>Existence of some solutions to the systems of ordinary differential equations, which are partially resolved relatively to the derivatives in the case of removable singularity</i>	38
Marynets K., <i>On one antiperiodic boundary-value problem for fractional differential systems</i>	39
Mykulyak S., Skurativskyi S., <i>Nonlinear dynamics of the system of hierarchically coupled oscillators with power law interactions</i>	42
Myslo Yu.M., <i>Asymptotically almost periodic solutions of equations with delay and state-dependent impulses</i>	44

Ocheretniuk Ye.V., <i>Rigid body on string suspension</i>	45
Shcherbak V.F., <i>Nonlinear observer design for dynamical systems with uncertainty</i>	45
Skurativska I., Skurativskiy S., <i>Periodic and solitary waves in the cubically nonlinear model for mutually penetrating continua</i>	46
Slyn'ko V.I., Denysenko V.S., <i>Interval stability of switched linear differential systems</i>	48
Solodun A.V., <i>Modeling of nonlinear fluid sloshing in coaxial conical tanks</i>	50
Tarasenko O.V., <i>The optimal control problem for a linear system of differential equations</i>	50
Vasylieva I., Zuyev A., <i>Stability analysis for a nonholonomic mechanical system using Sussmann-type processes</i>	51
PARTIAL DIFFERENTIAL EQUATIONS AND MATHEMATICAL PHYSICS	53
Anikushyn A.V., Kostejeva L.O., <i>A priori estimations and generalized solutions for a class of partial integro-differential equations</i>	53
Bandura A.I., Skaskiv O.B., <i>Properties of entire solutions of some linear PDE's</i>	55
Baranetskij Ya.O., <i>Boundary-value problem for functional-differential equations of even order with involution</i>	56
Betsko I.V., <i>On continuous solutions of systems of nonlinear functional-difference equations</i>	57
Bonafede S., Voitovych M., <i>On regularity up to the boundary of solutions to nonlinear fourth-order elliptic equations with natural growth terms</i>	57
Buryachenko K., <i>A priori estimate of Keller-Osserman type for quasilinear elliptic equations with absorption lower order term</i>	59
Evgenieva E.A., <i>Boundary LS-Regimes for quasilinear parabolic equations</i>	60
Huzyk N.N., <i>Inverse free boundary problem for a weakly degenerate parabolic equation</i>	62
Ilkiv V.S., <i>The moments type problem for strictly hyperbolic equations</i>	63
Khilkova L.O., <i>Robin's nonlinear problem in domains with a fine-grained random boundary</i>	64
Kudrych Yu., Buryachenko K., <i>Pointwise estimates of solutions to the divergence type quasilinear elliptic equation with lower terms</i>	65
Kuduk G., <i>Nonlocal problem with integral conditions for the system of partial differential equations</i>	66
Lopushanskyy A.O., Lopushanska H.P., Myaus O.M., <i>Inverse source fractional problems with distributions</i>	67
Nytrebych Z.M., Malanchuk O.M., <i>Two-point problem for homogeneous PDE of second order in time with nonhomogeneous conditions</i>	68
Panasenko Ye.V., Pokutnyi O.O., <i>Bifurcation conditions of solutions of boundary-value problem for the Lyapunov equation in the Hilbert space</i>	69
Pokrovskii A.V., <i>Removable singularities of plurisubharmonic functions</i>	70
Protsakh N.P., <i>On the determination of the minor coefficient in semilinear ultraparabolic equation</i>	72
Sandrakov G.V., <i>Homogenization of Stokes and Navier-Stokes equations with oscillating data and a vanishing viscosity</i>	73
Savka I.Ya., <i>Nonlocal multipoint problem with jump conditions for hyperbolic-parabolic system of differential equations</i>	74
Shan M.A., <i>Removability result for the anisotropic porous medium equation with gradient absorption term</i>	76
Shepelsky D., <i>The Riemann-Hilbert approach to the short pulse equation</i>	77
Skrypnik I.I., <i>Harnack's inequality for porous medium equation with singular absorption term</i>	79
Stiepanova K.V., <i>Local vanishing property of solutions for parabolic PDEs</i>	81
Stolyarchuk R., Auzinger W., <i>Error estimation in exponential integrators for evolution equations</i>	83

Suvorova I.G., <i>Investigation the hydrodynamics of flows in channels of complex geometric forms</i>	84
Trofymenko O.D., <i>Convolution equations and two-radii theorems for solutions of linear elliptic equations with constant coefficients in the complex plane</i>	85
Vorobyova A., <i>Group analysis of wave equation with special right-hand side</i>	87
Yehorchenko I., <i>Relative differential invariants and invariant equations equations</i>	88
STOCHASTIC DIFFERENTIAL EQUATIONS	90
Borysenko O.D., Borysenko D.O., <i>Global solution to stochastic logistic equation</i>	90
Buchak Kh.V., Sakhno L.M., <i>Investigation of properties of time-changed Poisson processes</i>	91
Doobko V.A., <i>Derivation of the diffusion equation in random fields from the special Langevin equation</i>	92
Ivanov I.L., <i>Correlation results for some foreign exchange markets</i>	93
Kozachenko Yu.V., Petranova M.Yu., <i>Simulation of the Ornstein-Uhlenbeck process</i>	94
Lohvinenko S.S., <i>Parameters estimation in fractional Vasicek model</i>	95
Osyphuk M.M., Portenko M.I., <i>Jump theorem and its applications</i>	97
Pashko A., Vasylyk O., <i>Accuracy of simulation of fractional Brownian motion in $L_p([0, T])$ and $C([0, T])$</i>	98
Pryhara L.I., Shevchenko G.M., <i>Regularity of solution to the wave equation with a coloured stable noise</i>	100
Radchenko V.M., <i>Averaging principle for heat equation driven by general stochastic measure</i>	101
Ralchenko K.V., <i>Statistical inference for fractional Ornstein-Uhlenbeck process</i>	102
Ralchenko S.A., <i>Existence and uniqueness of solutions to stochastic differential equations driven by fractional Brownian field</i>	103
Rozora I.V., <i>Some properties of impulse response function</i>	105
Sakhno L.M., <i>CLT for quadratic forms for random fields with tapered data and asymptotic properties of minimum contrast estimators</i>	106
Shklyar S., Ralchenko K.V., Mishura Yu., <i>Maximum likelihood estimation of the drift slope of a Gaussian process</i>	107
Tsukanova A.O., <i>On comparison theorem for the Cauchy problems to neutral stochastic integro-differential equations</i>	107
Yamnenko R.E., Yancyvych T.O., <i>Some properties of random processes from Orlicz spaces of exponential type</i>	110
MATHEMATICAL PROBLEMS OF PHYSICS AND MATERIALS SCIENCE	111
Auzinger W., <i>Adaptive integration of large linear systems of Schrödinger type with time-dependent coefficients using Magnus-type methods</i>	111
Bezpalchuk V., Kozubski R., Gusak A., <i>Simulation of the tracer diffusion and ordering kinetics in FCC structures - Stochastic Kinetic Mean-Field Method</i>	112
Bhat A.P., Dhoble S.J., Rewatkar K.G., <i>Effect of rare earth doped metamaterial towards the antenna miniaturisation and substrate primitive</i>	113
Bobrov O., Pasichnyy M., Gusak A., <i>Size effect on distributions of times to failure and times to transformations</i>	114
Bondarchuk S.V., Minaev B.F., <i>First principles DFT study of structural, electronic and mechanical properties of two-dimensional nitrogen monolayers</i>	115
Borisenko A., <i>Nominal vs. active supersaturation of solutions</i>	116
Gavriliuk A.M., Kachurik I.I., <i>Hermitian Hamiltonian built from Non-Hermitian position/momentum operators, and deformed oscillators</i>	117
Gokhman O., Terentyev D., Kondrea M., <i>Investigation of the post-irradiation annealing on the defect structure of tungsten</i>	117

Golovnya B.P., <i>Some mistakes in near-wall turbulence modeling and possible way to overcome them</i>	118
Grushka Ya.I., <i>Theorem of non returning for universal kinematics</i>	119
Grytsay V.I., <i>Spectral analysis and invariant measure in the study of a nonlinear dynamics of the metabolic process in a cells</i>	120
Hassan A.R., <i>Thermal radiation effects on a reactive hydromagnetic internal heat generating fluid flow within parallel porous plates</i>	121
Ivanov M.A., Naumuk A.Yu., <i>Description of motion of the front of phase decomposition with simultaneous account of the bulk and boundary diffusion</i>	122
Kornienko S.V., <i>Modeling of void formation during the process of reaction diffusion in a binary system</i>	123
Kulinich M.V., Kosintsev S.G., Ustinov A.I., Bezpachuk V.M., Gusak A.M., Zaporozhets T.V., <i>Theoretical and experimental research of thermal fields during local heating in the process of brazing</i>	124
Levchuk K., <i>Frees stuck pipe strings by means mechanical transverse oscillator</i>	126
Lyashenko Yu.O., Gonda A., <i>Entropy production as a regularization factor in solution of inverse diffusion problem using spline functions</i>	128
Marchenko S.V., Bogatyrev A.O., Gusak A.M., <i>Atomistic simulation of phase separation in a binary alloy under high current density</i>	129
Mishchenko Yu.A., Gavrilik A.M., Kachurik I.I., <i>Properties of Bose gases trapped in potential $U(\mathbf{r}) = Ar^n + Br^{-m}$ in D-dimensions</i>	130
Mohammad M., <i>Gibbs errors in B-spline tight framelet expansions for different generators</i>	131
Oliinyk O.V., Tatarenko V.A., <i>Post-irradiation modulation of distribution of interacting vacancies in the elastically anisotropic b.c.c. crystals</i>	131
Pantelyat M.G., Yeloiev A.K., <i>Application of the finite element method to computer simulation of electromagnetic and thermal processes in induction cookers and heated dishes</i>	133
Pasichnyy M., Gusak A., Bezpachuk V., Erdelyi Z., Toman J., <i>Stochastic kinetic mean-field (SKMF) - almost new method of simulation</i>	135
Radchenko T.M., Lazorenko M.O., Tatarenko V.A., <i>Implementation of numerical algorithm for calculation of electron transport properties within the Kubo formalism</i>	136
Sandrakov G.V., <i>Homogenized multiphase models for composite materials</i>	138
Sytnyk D., <i>Parallel numerical method for nonlocal-in-time Schrödinger equation</i>	139
Tatarenko V.A., Oliinyk O.V., <i>The dissipative modulated structures in a spatial distribution of interacting vacancies or nanovoids in the elastically anisotropic crystals under an isothermal irradiation</i>	140
Ustinov A.I., Demchenkov S.O., <i>Initial stages of phase transformations in multilayer Al/Ni foils produced by vacuum deposition</i>	142
Tsotsko V.I., Peleshenko B.G., <i>Simulation of crystallization of the surface layer of track castings in the region of holes</i>	144
Tsotsko V.I., Peleshenko B.G., Denysenko A.I., <i>Heating of metal surface layer with partial melting</i>	145
Vynogradov A.G., <i>Mathematical modeling of hydrodynamic parameters of fire water curtains</i>	147
Zhukovskiy V., Gokhman O., <i>The aging of duralumin according to the rules of "Life"</i>	147
Zolotaryuk Ya., <i>Discrete embedded solitons in nonlinear Klein-Gordon lattices</i>	148

Let us suppose, that matrices $A_1^{-1}(z)B_1(z), A_1^{-1}(z)A_2(z), A_1^{-1}(z)B_2(z)$ are analytic in the domain D_{10} and have removable singularity at the point $z = 0$.

The system (2) can be written as:

$$Y_1' = P(z)Y_1 + F^*(z, Y_1, Y_2, Y_1', Y_2'), \quad (3)$$

where $P(z)$ is analytic matrix in the domain D_{10} and has removable singularity at the point $z = 0$. $F^*(z, Y_1, Y_2, Y_1', Y_2')$ is analytic vector-function in the domain $D_{10} \times G_{110} \times G_{120} \times G_{210} \times G_{220}$, $G_{j k 0} = G_{j k} \setminus 0, j, k = 1, 2$.

Let us introduce the class of functions H_0^{n-p} . This class is a class of $(n - p)$ - dimensional analytic in the domain D_{10} functions that have removable singularity at the point $z = 0$.

In the case $Y_2 \in H_0^{n-p}$, let us study question about the existence of the analytic solutions of Cauchy's problem

$$\begin{cases} Y_1' = P(z)Y_1 + F^*(z, Y_1, Y_2, Y_1', Y_2'), \\ Y_1(z) \rightarrow 0 \text{ on the condition that } z \rightarrow 0, z \in D_{10}, \end{cases} \quad (4)$$

that satisfy additional condition

$$Y_1'(z) \rightarrow 0 \text{ for } z \rightarrow 0, z \in D_{10}. \quad (5)$$

Let us choose such vector-function $Y_2 \in H_0^{n-p}$, that after completing at the point $z = 0$, becomes analytic function in the domain D_1 and $Y_2(0) = 0$. In this case, the function F^* can be written as:

$$F^*(z, Y_1, Y_2, Y_1', Y_2') = F^*(z, Y_1, \sum_{k=1}^{\infty} S_k z^k, Y_1', \sum_{k=1}^{\infty} k \cdot S_k z^{k-1}) = F(z, Y_1, Y_1'), \quad (6)$$

where $F : D_1 \times G_{11} \times G_{21} \rightarrow C^p, S_k \subset \mathbb{C}^{n-p}, k = 1, 2, \dots$ power series $\sum_{k=1}^{\infty} S_k z^k$ and $\sum_{k=1}^{\infty} k \cdot S_k z^{k-1}$ converge in the neighborhood of the point $z = 0$.

Thus problem (4) could be reduce to Cauchy's problem:

$$\begin{cases} Y_1' = P(z)Y_1 + F(z, Y_1, Y_1'). \\ Y_1(z) \rightarrow 0 \text{ on the condition that } z \rightarrow 0, z \in D_{10}. \end{cases} \quad (7)$$

$$(8)$$

The sufficient conditions have been found: For each arbitrary fixed function $Y_2 \in H_0^{n-p}, Y_2(0) = 0$, there exists at least one analytic solution of Cauchy's problem (7)-(8) with additional condition (5) in some subdomain of the domain D_{10} with point $z = 0$ at the domain boundary.

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On one antiperiodic boundary–value problem for fractional differential systems

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1. Problem Setting. Consider a boundary-value problem (BVP) for a fractional differential system (FDS) with antiperiodic boundary conditions:

$${}^c D_t^p x = f(t, x(t)), p \in (0, 1), x(0) = -x(T), \quad (1)$$

where ${}^c D_t^p$ is the generalized Caputo fractional derivative [1] with lower limit at 0, $t \in [0, T]$, $x : [0, T] \rightarrow D$, $f : G \rightarrow \mathbb{R}^n$ are continuous functions, $G := [0, T] \times D$ and $D \subset \mathbb{R}^n$ is a closed and bounded domain.

Assume that the BVP (1) satisfies conditions:

(i) function f is bounded by a constant vector $M \in \mathbb{R}^n$ and it satisfies the Lipschitz condition with a non-negative real matrix $K = (k_{ij})_{i,j=1}^n$, i.e., the following inequalities

$$|f(t, x)| \leq M, \quad |f(t, u) - f(t, v)| \leq K|u - v| \quad (2)$$

are true for any $t \in [0, T]$, $x, u, v \in D$:

(ii) the set

$$D_\beta = \{\xi_0 \in D : \{u \in \mathbb{R}^n : |u - \xi_0| \leq \beta\} \subset D\}, \quad (3)$$

is non-empty, where $\beta := \frac{MT^p}{2^{2p-1}\Gamma(p-1)}$;

(iii) the spectral radius $r(Q)$ of matrix

$$Q := \frac{KT^p}{2^{2p-1}\Gamma(p+1)} \quad (4)$$

satisfies the estimate:

$$r(Q) < 1. \quad (5)$$

All operations $|\cdot|$, $=$, \leq , \geq , \max , etc. between matrixes and vectors are understood componentwise.

The problem is to find a solution of the BVP (1) in the space of continuous vector-functions $x : [0, T] \rightarrow D$.

2. Construction of successive approximations. Using the main ideas of [2], let us connect with BVP (1) the sequence of functions $\{x_m\}_{m \in \mathbb{Z}_0}$, $\mathbb{Z}_0 = \{0, 1, 2, \dots\}$, given by the iterative formula:

$$\begin{aligned} x_m(t, \xi_0) := & \xi_0 + \frac{1}{\Gamma(p)} \left[\int_0^t (t-s)^{p-1} f(s, x_{m-1}(s, \xi_0)) ds - \right. \\ & \left. - \left(\frac{t}{T}\right)^p \int_0^T (T-s)^{p-1} f(s, x_{m-1}(s, \xi_0)) ds \right] - 2 \left(\frac{t}{T}\right)^p \xi_0. \end{aligned} \quad (6)$$

where $t \in [0, T]$, $\xi_0 \in D_\beta$ and

$$x_0(t, \xi_0) = \left(1 - 2 \left(\frac{t}{T}\right)^p\right) \xi_0$$

is a zero approximation. Here $\Gamma(p)$ is the Gamma function.

Theorem 1. Assume that conditions (2)–(5) hold for the BVP (1).

Then for all fixed $\xi_0 \in D_\beta$, it holds:

1. Functions of the sequence (6) are continuous and satisfy antiperiodic boundary conditions:

$$x_m(0, \xi_0) = -x_m(T, \xi_0), \quad \forall m \in \mathbb{N}.$$

2. The sequence of functions (6) for $t \in [0, T]$ converges uniformly as $m \rightarrow \infty$ to the limit function

$$x_\infty(t, \xi_0) = \lim_{m \rightarrow \infty} x_m(t, \xi_0). \quad (7)$$

3. The limit function x_∞ satisfies boundary condition:

$$x_\infty(0, \xi_0) = -x_\infty(T, \xi_0).$$

4. The limit function (7) is a unique continuous solution of the integral equation

$$x(t) := \xi_0 + \frac{1}{\Gamma(p)} \left[\int_0^t (t-s)^{p-1} f(s, x(s)) ds - \left(\frac{t}{T} \right)^p \int_0^T (T-s)^{p-1} f(s, x(s)) ds \right] - 2 \left(\frac{t}{T} \right)^p \xi_0,$$

i. e., it is the unique solution on $[0, T]$ of the Cauchy problem for the modified system of fractional differential equations:

$${}_0^c D_t^p = f(t, x) + \Delta(\xi_0), \quad x(0) = \xi_0.$$

where

$$\Delta(\xi_0) := -\frac{p}{T^p} \int_0^T (T-s)^{p-1} f(s, x_\infty(s, \xi_0)) ds - \frac{2\Gamma(p+1)}{T^p} \xi_0. \quad (8)$$

5. The following error estimation holds:

$$|x_\infty(t, \xi_0) - x_m(t, \xi_0)| \leq \frac{T^p}{2^{2p-1}\Gamma(p+1)} Q^{m-1} (I_n - Q)^{-1} M,$$

where I_n is the unit n -dimension matrix, matrix Q and vector M are defined in formulae (4) and (2).

3. Connection between the limit function and the exact solution of the given BVP.

Consider the Cauchy problem

$${}_0^c D_t^p x = f(t, x) + \mu, \quad t \in [0, T], \quad x(0) = \xi_0, \quad (9)$$

where $\mu \in \mathbb{R}^n$ is a control parameter and $\xi_0 \in D_\beta$.

Theorem 2. Let $\xi_0 \in D_\beta$ and $\mu \in \mathbb{R}^n$ be some given vectors. Suppose that for the BVP (1) all conditions of Theorem 1 hold.

Then the solution $x = x(\cdot, \xi_0, \mu)$ of the initial-value problem (9) satisfies also boundary condition from (1) if and only if

$$\mu := -\frac{p}{T^p} \int_0^T (T-s)^{p-1} f(s, x_\infty(s, \xi_0)) ds - \frac{2\Gamma(p+1)}{T^p} \xi_0,$$

where $x_\infty(\cdot, \xi_0)$ is a function from the assertion 2 of Theorem 1. In that case

$$x(t, \xi_0, \mu) = x_\infty(t, \xi_0) \quad \text{for } t \in [0, T].$$

Theorem 3. Let the original BVP (1) satisfy conditions (2)–(5).

Then $x_\infty(\cdot, \xi_0^*)$ is a solution of the FDS with antiperiodic boundary condition from (1) if and only if the point ξ_0^* is a solution of the determining system of algebraic or transcendental equations:

$$\Delta(\xi_0) = 0,$$

where Δ is given by (8).

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