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Let us suppose, that matrices $A_1^{-1}(z)B_1(z), A_1^{-1}(z)A_2(z), A_1^{-1}(z)B_2(z)$ are analytic in the domain D_{10} and have removable singularity at the point $z = 0$.

The system (2) can be written as:

$$Y_1' = P(z)Y_1 + F^*(z, Y_1, Y_2, Y_1', Y_2'), \tag{3}$$

where $P(z)$ is analytic matrix in the domain D_{10} and has removable singularity at the point $z = 0$. $F^*(z, Y_1, Y_2, Y_1', Y_2')$ is analytic vector-function in the domain $D_{10} \times G_{110} \times G_{120} \times G_{210} \times G_{220}, G_{jk0} = G_{jk} \backslash 0, j, k = 1, 2$.

Let us introduce the class of functions H_0^{n-p}. This class is a class of $(n-p)-$ dimensional analytic in the domain D_{10} functions that have removable singularity at the point $z = 0$.

In the case $Y_2 \in H_0^{n-p}$, let us study question about the existence of the analytic solutions of Cauchy's problem

$$\begin{cases} Y_1' = P(z)Y_1 + F^*(z, Y_1, Y_2, Y_1', Y_2'), \\ Y_1(z) \to 0 \text{ on the condition that } z \to 0, z \in D_{10}, \end{cases} \tag{4}$$

that satisfy additional condition

$$Y_1'(z) \to 0 \text{ for } z \to 0, z \in D_{10}. \tag{5}$$

Let us choose such vector-function $Y_2 \in H_0^{n-p}$, that after completing at the point $z = 0$, becomes analytic function in the domain D_1 and $Y_2(0) = 0$. In this case, the function F^* can be written as:

$$F^*(z, Y_1, Y_2, Y_1', Y_2') = F^*(z, Y_1, \sum_{k=1}^{\infty} S_k z^k, Y_1', \sum_{k=1}^{\infty} k \cdot S_k z^{k-1}) = F(z, Y_1, Y_1'), \tag{6}$$

where $F : D_1 \times G_{11} \times G_{21} \to C^p, S_k \subset \mathbb{C}^{n-p}, k = 1, 2, \ldots$, power series $\sum_{k=1}^{\infty} S_k z^k$ and $\sum_{k=1}^{\infty} k \cdot S_k z^{k-1}$ converge in the neighborhood of the point $z = 0$.

Thus problem (4) could be reduce to Cauchy's problem:

$$\begin{cases} Y_1' = P(z)Y_1 + F(z, Y_1, Y_1'), & (7) \\ Y_1(z) \to 0 \text{ on the condition that } z \to 0, z \in D_{10}. & (8) \end{cases}$$

The sufficient conditions have been found: For each arbitrary fixed function $Y_2 \in H_0^{n-p}, Y_2(0) = 0$, there exists at least one analytic solution of Cauchy's problem (7)-(8) with additional condition (5) in some subdomain of the domain D_{10} with point $z = 0$ at the domain boundary.

References

[1] D. Limanska, G. Samkova, "About behavior of solutions of some systems of differential equations, which is partially resolved relatively to the derivatives", *Bulletin of Mechnikov's Odessa National University*, **19**, Part 1 (21), 16–28 (2014).

[2] D. Limanska, "About behavior of solutions of some systems of differential equations, which is partially resolved relatively to the derivatives in case of pole", *Nonlinear Oscillations*, No. 20, 113–126 (2017).

On one antiperiodic boundary–value problem for fractional differential systems

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1. Problem Setting. Consider a boundary-value problem (BVP) for a fractional differential system (FDS) with antiperiodic boundary conditions:

$${}_0^c D_t^p x = f(t, x(t)), p \in (0, 1), \; x(0) = -x(T), \tag{1}$$

where ${}_0^cD_t^p$ is the generalized Caputo fractional derivative [1] with lower limit at 0, $t \in [0,T]$, $x : [0,T] \to D$, $f : G \to \mathbb{R}^n$ are continuous functions, $G := [0,T] \times D$ and $D \subset \mathbb{R}^n$ is a closed and bounded domain.

Assume that the BVP (1) satisfies conditions:

(i) function f is bounded by a constant vector $M \in \mathbb{R}^n$ and it satisfies the Lipschitz condition with a non–negative real matrix $K = (k_{ij})_{i,j=1}^n$, i.e., the following inequalities

$$|f(t,x)| \le M, \ |f(t,u) - f(t,v)| \le K|u - v| \tag{2}$$

are true for any $t \in [0,T]$, $x, u, v \in D$;

(ii) the set

$$D_\beta = \{\xi_0 \in D : \{u \in \mathbb{R}^n : |u - \xi_0| \le \beta\} \subset D\}, \tag{3}$$

is non-empty, where $\beta := \frac{MT^p}{2^{2p-1}\Gamma(p-1)}$;

(iii) the spectral radius $r(Q)$ of matrix

$$Q := \frac{KT^p}{2^{2p-1}\Gamma(p+1)} \tag{4}$$

satisfies the estimate:

$$r(Q) < 1. \tag{5}$$

All operations $|\cdot|$, $=$, $\le$, $\ge$, max, etc. between matrixes and vectors are understood componentwise.

The problem is to find a solution of the BVP (1) in the space of continuous vector–functions $x : [0,T] \to D$.

2. Construction of successive approximations. Using the main ideas of [2], let us connect with BVP (1) the sequence of functions $\{x_m\}_{m\in\mathbb{Z}_0}$, $\mathbb{Z}_0 = \{0,1,2,\ldots\}$, given by the iterative formula:

$$x_m(t,\xi_0) := \xi_0 + \frac{1}{\Gamma(p)}\left[\int_0^t (t-s)^{p-1} f(s, x_{m-1}(s,\xi_0))ds - \right.$$

$$\left. - \left(\frac{t}{T}\right)^p \int_0^T (T-s)^{p-1} f(s, x_{m-1}(s,\xi_0))ds\right] - 2\left(\frac{t}{T}\right)^p \xi_0, \tag{6}$$

where $t \in [0,T]$, $\xi_0 \in D_\beta$ and

$$x_0(t,\xi_0) = \left(1 - 2\left(\frac{t}{T}\right)^p\right)\xi_0$$

is a zero approximation. Here $\Gamma(p)$ is the Gamma function.

Theorem 1. *Assume that conditions (2)–(5) hold for the BVP (1).*

Then for all fixed $\xi_0 \in D_\beta$, it holds:

1. *Functions of the sequence (6) are continuous and satisfy antiperiodic boundary conditions:*

$$x_m(0,\xi_0) = -x_m(T,\xi_0), \ \forall m \in \mathbb{N}.$$

2. *The sequence of functions (6) for $t \in [0,T]$ converges uniformly as $m \to \infty$ to the limit function*

$$x_\infty(t,\xi_0) = \lim_{m\to\infty} x_m(t,\xi_0).$$

3. *The limit function x_∞ satisfies boundary condition:*

$$x_\infty(0,\xi_0) = -x_\infty(T,\xi_0).$$

4. *The limit function (7) is a unique continuous solution of the integral equation*

$$x(t) := \xi_0 + \frac{1}{\Gamma(p)}\left[\int_0^t (t-s)^{p-1} f(s,x(s))ds - \left(\frac{t}{T}\right)^p \int_0^T (T-s)^{p-1} f(s,x(s))ds\right] - 2\left(\frac{t}{T}\right)^p \xi_0.$$

i. e., it is the unique solution on $[0,T]$ of the Cauchy problem for the modified system of fractional differential equations:

$${}_0^cD_t^p = f(t,x) + \Delta(\xi_0), \ x(0) = \xi_0.$$

where

$$\Delta(\xi_0) := -\frac{p}{T^p}\int_0^T (T-s)^{p-1} f(s, x_\infty(s,\xi_0))ds - \frac{2\Gamma(p+1)}{T^p}\xi_0. \tag{8}$$

5. *The following error estimation holds:*

$$|x_\infty(t,\xi_0) - x_m(t,\xi_0)| \le \frac{T^p}{2^{2p-1}\Gamma(p+1)} Q^{m-1}(I_n - Q)^{-1} M,$$

where I_n is the unit n–dimension matrix, matrix Q and vector M are defined in formulae (4) and (2).

3. Connection between the limit function and the exact solution of the given BVP.

Consider the Cauchy problem

$${}_0^cD_t^p x = f(t,x) + \mu, \ t \in [0,T], \ x(0) = \xi_0, \tag{9}$$

where $\mu \in \mathbb{R}^n$ is a control parameter and $\xi_0 \in D_\beta$.

Theorem 2. *Let $\xi_0 \in D_\beta$ and $\mu \in \mathbb{R}^n$ be some given vectors. Suppose that for the BVP (1) all conditions of Theorem 1 hold.*

Then the solution $x = x(\cdot,\xi_0,\mu)$ of the initial-value problem (9) satisfies also boundary condition from (1) if and only if

$$\mu := -\frac{p}{T^p}\int_0^T (T-s)^{p-1} f(s, x_\infty(s,\xi_0))ds - \frac{2\Gamma(p+1)}{T^p}\xi_0,$$

where $x_\infty(\cdot,\xi_0)$ is a function from the assertion 2 of Theorem 1. In that case

$$x(t,\xi_0,\mu) = x_\infty(t,\xi_0) \ \text{ for } \ t \in [0,T].$$

Theorem 3. *Let the original BVP (1) satisfy conditions (2)–(5).*

Then $x_\infty(\cdot,\xi_0^*)$ is a solution of the FDS with antiperiodic boundary condition from (1) if and only if the point ξ_0^* is a solution of the determining system of algebraic or transcendental equations:

$$\Delta(\xi_0) = 0,$$

where Δ is given by (8).

References

[1] *J. Wang, Y. Zhou, M. Fečkan*, Alternative results and robustness for fractional evolution equations with periodic boundary conditions, *lectron. J. Qual. Theory Differ. Equ.* **97**, 1–15 (2011).

[2] *Michal Fečkan, Kateryna Marynets*, Approximation Approach to Periodic BVP for Fractional Differential Systems, *European Physical Journal.* (2017), (submitted).