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Fine structure of meson spectrum and Lorenz nature of quark-antiquark potential from Dirac equation

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Abstract

The numerical solution of relativistic Dirac equation for mesons with both equal and unequal quark masses is carried out. The obtained results both for mass spectrum and for fine splitting are in quite resonable agreement with experiment. We also investigate the Lorentz structure of the potential, and obtain that most of the confinement part of the potential must be in the vector part of the interaction potential.

Quark potential models are still convenient tools for the calculation the most of the properties of hadrons. Acceptable results are provided by relativistic (semirelativistic) and nonrelativistic approaches. The idea is soaring in the air concerning the importance of relativistic effects even in the $c\bar{c}$ and $b\bar{b}$ systems, therefore it would be interesting to account for these effects by way of direct solving of Dirac equation. Recently exactly such approch was discussed in several papers [1-4]. All abovementioned attemps use a certain kind of "QCD-motivated" two-body interaction. Most frequently discussed is a potential which, consists of one-gluon exchange term $V_g(r)$ and confining long range part $V_c(r)$

$$V(r) = V_g(r) + V_c(r) = -\frac{\alpha}{r} + \lambda \cdot r + V_o. \tag{1}$$

Here $\alpha = 4/3\alpha_s$, where α_s is standerd strong coupling constant, V_o is used to fix the ground state of the spectrum.

For the relativistic models the Lorentz character of the potential has to be specified

$$V(r) = V_V(r) + V_S(r), \tag{2}$$

where the one-gluon exchange term is usually treated as the vector part of potential (V_V) and the confinement term is treated as the scalar part (V_S).

In general both relativistic as well as nonrelativistic approaches give rather good description of radial and orbital excitations of mesons. Spin-orbital effect in the quazyrelativistic models is obtained in the framework of the so called Generalized Breit-Fermi approach [1], where it has the form

$$V_{SL}(r) = \frac{1}{2m^2 r}\frac{dV(r)}{dr}\vec{S}\vec{L}.$$

For Coulomb-like potential it gives $1/r^3$ term which leads to the problem of particle falling to the force center and therefore the fine structure of mesons is calculated usually only in the perturbative approximation.

The main purpose of the present work is to estimate spin-orbital splitting of spectra of mesons, considered as bound $Q\bar{q}$-systems, in the frame of the one particle Dirac equation which should describe correctly this phenomena. Such model could be realistic for the hydrogen-like system of bounded light (q) and heavy (Q) quarks.

The application of one-particle Dirac equation to the two-quark system has been recently actively discussed [5,6]. We have chosen a simplest variant of reducing the relativistic two-body problem to single Dirac equation for particle of reduced mass $\mu = Mm/(M+m)$.

To understand why in the Dirac approach we do not encounter the problem of the particle falling to the force center as in the semirelativistic one let us consider the Dirac equation for the radial functions $f(r)$ and $g(r)$ of the Dirac spinor

$$\begin{aligned} f' - \frac{\kappa}{r} f - (E + m - V_V + V_S) g = 0 \\ g' + \frac{\kappa}{r} g + (E - m - V_V - V_S) f = 0, \end{aligned} \tag{3}$$

where $\kappa = \begin{cases} \ell+1, & j = \ell + 1/2 \\ -\ell, & j = \ell - 1/2. \end{cases}$

The system of differential equations of the first order (3) can be reduced to the differential equation of the second order

$$f'' - \frac{A'}{A} f' + \left(\frac{\kappa(1-\kappa)}{r^2} + \frac{\kappa A'}{Ar} + AB \right) f = 0, \tag{4}$$

where $A = (e + m - V_V + V_S)$ and $B = (E - m - V_V - V_S)$.

The substitution $f = \sqrt{A}\varphi$ transforms equation (4) to the Shrodinger-like one

$$\varphi'' - \left(\frac{A''}{2A} + \frac{A'}{2A^{3/2}} + \frac{\kappa(1-\kappa)}{r^2} + \frac{\kappa A'}{Ar} + AB \right) \varphi = 0. \tag{5}$$

One can see that the third term in brackets has the orbital origin

$$\frac{\kappa(1-\kappa)}{r^2} = -\frac{\ell(\ell+1)}{r^2}, \tag{6}$$

while taking into account the relation $\kappa = \vec{\sigma}\vec{\ell} + 1$, the fourth term can be interpreted as the spin-orbital part of interaction

$$\frac{A'}{Ar} \to \begin{cases} \frac{1}{2m^2 r} \frac{dV}{dr}, & m \gg E \gg V, \\ \frac{1}{rV} \frac{dV}{dr}, & V \gg m, \quad V \gg E, \quad (r \to 0), \end{cases} \tag{7}$$

where in nonrelativistic limit (7a) one obtains $1/r^3$ behavior for Coulomb-like potential $V(r)$ and $1/r^2$ one in relativistic approach (7b).

The radial functions $f(r)$ and $g(r)$ obey the boundary conditions

$$f(0) = 0, \quad f(\infty) = 0,$$

$$g(0) = 0, \quad g(\infty) = 0.$$

The first two boundary conditions are used as the starting conditions and the second pair of boundary conditions can be used for selection of eigenvalues of energy in equation (3) under the numerical solution by Runge-Kutta method.

We suggested the following Lorentz structure of potential [7]

$$V_V = V_g + \varepsilon V_c \quad \text{and} \quad V_S = (1-\varepsilon)V_c,$$

where ε is parameter which indicates the part of confining interaction in the vector sector of Lorentz structure of potential. The calculation show that to obtain the correct sign of spin-orbital splitting in Dirac equation (3) with potential (1) it is necessary to appropriate the confining part of potential mainly to the vector sector of interaction. The following parameters are used

$$\alpha = 0.5, \qquad \lambda = 0.19 \text{ GeV}^2,$$

which corresponds to the usually taken value for Cornell potential [1]. We have also taken the current quark masses $m_u = m_d = 5$ MeV, $m_s = 270$ MeV, $m_b = 4.7$ GeV. Resulting masses of mesons were calculated according to expresion $M = m_q + M_Q + E_B$, $E_B = E - \mu$. The best values of spin – orbital spliting are obtained with $\varepsilon = 0.65$. It is interesting to note that Deoghuria and Chakrabarty [7] have came to the similar conclusion concerning the Lorentz character, having in mind their definition of interaction potential, notwithstanding that they were considering hyperfine structure.

As we can see, the mass spectrum (Table 1) is obtained with quite reasonable accuracy [8].

As for fine splitting, if one believes that in case when $m_Q \to \infty$ the ${}^3P_0 + {}^1P_1 \to P_{1/2}$ and ${}^3P_1 + {}^3P_2 \to P_{3/2}$ [9] then for $s\bar{u}$-system (Table 2) we have splitting $\Delta_P = 124$ MeV (108 MeV exp.) and $\Delta_D = 72$ MeV (95 MeV exp.) for P- and D-waves correspondingly. Other data (Table 3) can be considered as a predictions.

Table 1. Radial excitation of $(q\bar{q})$-system (Masses are given in MeV).

	$b\bar{b}$		$c\bar{c}$		$c\bar{s}$		$c\bar{u}$		$s\bar{u}$	
	M_{th}	M_{exp}	M_{th}	M_{exp}	M_{th}	M_{exp}	M_{th}	M_{exp}	M_{th}	M_{exp}
1S	9460	9460	3106	3096	2094	2110	2016	2010	884	892
2S	10016	10023	3596	3686	2554	–	2448	–	1316	1410
3S	10344	10355	3942	4040	–	–	2787	–	–	–
4S	10603	10580	–	–	–	–	–	–	–	–
5S	10826	10865	–	–	–	–	–	–	–	–

Table 2. $(s\bar{u})$-system (Masses are given in MeV).

	M_{exp}	M_{th}	
$^1P_1$	1270	1113	$P_{1/2}$
$3P_0$	1350		
	1430?		
$^3P_1$	1408	1237	$P_{3/2}$
$^3P_2$	1430		
$^3D_1$	1670	1412	$D_{1/2}$
$^3D_2$	1770	1484	$D_{3/2}$
$^3D_3$	1780		

Table 3. $(c\bar{u})$ and $(c\bar{s})$-systems (Masses are given in MeV).

$c\bar{u}$			
	M_{exp}	M_{th}	
$^1P_1$	–	2232	$P_{1/2}$
$3P_0$	–		
$^3P_1$	2420	2355	$P_{3/2}$
$^3P_2$	2480		
$c\bar{s}$			
$^1P_1$	2536	2477	$P_{1/2}$
$^3P_0$	–		
$^3P_1$	–	2577	$P_{1/3}$
$^3D_2$	–		

The problem of two-quark systems with different masses was also examined in [9] and excellent revue of situation is given in [10]. If we compare our results with the results obtained in [9] we shall see the splitting in $s\bar{u}$-system are quite similar however the splitting in $c\bar{u}$-system obtained in [9] are much smaller. The absence of experimental data does not allow us to make the final conclusion. It is interesting to note that both our and Godfrey's and Kokoski's [9] results strongly suggest that the value of $^3P_0 s\bar{u}$-mass equal to 1430 MeV is too large and previous one of 1350 MeV [11] is preferable.

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