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The Tensor Forces and the Quarkonia Properties

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Based on the numeric solution of a system of coupled channels for S- and D -waves a mass spectrum and wave functions of a family of vector mesons cc and ud in triplet states are obtained. The calculations are performed using a well known Cornell potential with a mixed Lorentz-structure of the confinement term. The spin-dependent part of the potential is taken from the Breit-Fermi approach. The effect of singular terms of potential is considered in the framework of the perturbation theory and by a configuration interaction approach (CIA), modified for a system of coupled equations. It is shown that even a small contribution of the D-wave to be very important at the calculation of certain characteristics of the meson states.

1 Introduction

Meson states, which are considered as the bound states of a quark-antiquark system, are convenient objects for studying both the strong interaction effects and various characteristics of weak interaction [1,2]. For the description of the low-energy properties of hadrons the following approaches are used: Bethe-Salpeter approach, lattice calculation technique and potential models. Each of these methods has its own advantages and shortcomings. The potential models are the simplest from the point of view of mathematics, what is essential for practical calculations. In the framework of the present model the averaged mass spectrum [1], spin effects [3] and the decay widths of heavy quarkonia [4] can be well described. As concerning the light-quark systems, the sit-uation is rather contradictory. Using the same parameters, some of the effects (spin

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effects, decay, averaged mass spectrum) can be described [5], but not all the effects together. The reason for this is the fact that the ligh-quark systems being explicitly relativistic, and relativistic potential models should be applied for them.

When the spin effects are considered in potential models, singular terms of the interquark interaction potential of the form are arised. This is a serious problem at the calculations of the meson characteristics. As a rule, in such case perturbation theory is used. It was in fact the first method, used to investigate the characteristics of mesons, considered as two-quark systems. But it has certain shortcomings. The main disadvantage is that the theory supposes small interaction, but in hadron physics it is the perturbation of the order 30-500 MeV, what is comparable with the distance between the unperturbed energy levels, hence, the condition of small perturbation not being fulfilled. In the triplet state configurations with uncertain orbital moment $J = L + 1$, $J = L - 1$ occur. For example, mixing of S and D-waves occurs in the state 1^- (we use the spectroscopic notations J^P where the total moment and the system parity are indicated). In most papers where the triplet states are considered (See Ref. 3 and references therein) the authors neglect the coupled channels or introduce an additional parameter - the mixing angle [6,7].

In the present paper the effect of the singular terms of the interquark interaction potential for the quarkonium state 1, described by a system of coupled equations, is considered, the comparison of perturbation methods and configuration interaction, modified for the coupled equation system, is performed. While choosing the structure, potential parameters and numeric solution scheme of the coupled differential equations were followed Ref. 8 where the contribution of the D-wave component upon the vector meson energy spectrum and decay width are considered, and the calculation version of the configuration interaction approach is taken from Ref. 3 where the hyperfine splitting is considered based on the Schroedinger equation. In such approach the mixing angle for S and D waves is determined by the system dynamics and does not require any additional experimental data.

2 Configuration interaction approach for a system of coupled equations

In the quasirelativistic Breit-Fermi approach the triplet states of two-quark systems are described by the following coupled Schroedinger equations [8]

$$\left\{ \begin{array}{c} (-\frac{1}{m_q}\frac{d^2}{dr^2} - E +^3 V_c)u + \sqrt{8}V_T w = 0 \\ (-\frac{1}{m_q}\frac{d^2}{dr^2} - E +^3 V_c - 2V_T - 3V_{LS})w + \sqrt{8}V_T u = 0 \end{array} \right\} \tag{1}$$

where ${}^3V_c = V_V + V_S + \frac{1}{4}V_{SS}$. We use the Cornell potential with a mixed Lorentz-structure where the confinement-related term contains vector and scalar parts:

$$V_{q\bar{q}}(r) = V_V(r) + V_S(r) = -\frac{\alpha_S}{r} + \beta_V r + \beta_S r$$

and the corresponding spin corrections are equal to

$$\widehat{V_{LS}} = \frac{1}{2m_q m_{\bar{q}}}\left(3\frac{\alpha_S}{r^3} + 3\frac{\beta_V}{r} - \frac{\beta_S}{r}\right)(LS)$$

$$\widehat{V_{SS}} = \frac{4}{3m_q m_{\bar{q}}}\left(\frac{\beta_V}{r} - 2\pi\alpha_S\delta(r)\right)(S_1 S_2)$$

$$\widehat{V_T} = \frac{1}{m_q m_{\bar{q}}}\left(\frac{\alpha_S}{r^3} + \frac{\beta_V}{r}\right)\widehat{S_{12}}$$

The Schroedinger equations are linked due to the presence of a tensor component V_T in the interaction potential. In Ref. 8 the system (1) is solved numerically when the singular terms $\frac{1}{r^3}$ and $\delta(\vec{r})$ in the potential are neglected. Here in after we study the contribution of the singular terms in the framework of the perturbation theory and CIA. We generalize these methods to the system of equations: for this purpose the radial wave function of the system can be conveniently written in the matrix form

$$\Psi = \begin{pmatrix} u \\ w \end{pmatrix} \tag{2}$$

where $u(r)$ is the radial function of the S -wave component and $w(r)$ is that of the D-wave component. Then the Schroedinger equation in the matrix form is given by

$$\hat{H}\begin{pmatrix} u \\ w \end{pmatrix} = E\begin{pmatrix} u \\ w \end{pmatrix} \tag{3}$$

where in the Hamiltonian $\hat{H}$ nonsingular and singular terms can be separated $\hat{H} = \widehat{H_0} + \widehat{W}$, namely

$$\widehat{H_0} =$$

$$= \begin{pmatrix} -\frac{1}{m}\frac{d^2}{dr^2} - \frac{\alpha}{r} + kr + \frac{\beta_V}{3m^2 r}; & \sqrt{8}\frac{\beta_V}{m^2 r}; \\ \sqrt{8}\frac{\beta_V}{m^2 r}; & -\frac{1}{m}\frac{d^2}{dr^2} - \frac{\alpha}{r} + kr + \frac{\beta_V}{3m^2 r} + \frac{6}{mr^2} - \frac{2\beta_V}{m^2 r} - \frac{3}{m^2}\left(\frac{3\beta_V - \beta_S}{r}\right); \end{pmatrix} \tag{4}$$

and

$$\widehat{W} = \begin{pmatrix} \frac{2\pi\alpha}{3m^2}\delta(r); & \sqrt{8}\frac{3\alpha}{m^2 r^3}; \\ \sqrt{8}\frac{3\alpha}{m^2 r^3}; & \sqrt{8}\frac{3\alpha}{m^2 r^3} - \frac{6\alpha}{m^2 r^3} - \frac{9\alpha}{2m^2 r^3}; \end{pmatrix} = \begin{matrix} \Delta V_{SS} & \Delta V_T \\ \Delta V_T & \Delta V_T - \Delta V_{SS} - \Delta V_{LS} \end{matrix} \tag{5}$$

Then the matrix elements

$$\Delta W_{mn} = \int \Psi_m^* \widehat{W} \Psi_n d\vec{r} \tag{6}$$

will be given by

$$\Delta W_{mn} = \int u_m(-\frac{2\pi\alpha}{3m^2}\delta(\vec{r}))u_n dr + \int w_m(\sqrt{8}\frac{3\alpha}{m^2r^3})u_n dr + \int u_m(\sqrt{8}\frac{3\alpha}{m^2r^3})w_n dr +$$

$$+ \int w_m(-\frac{2\pi\alpha}{3m^2}\delta(\vec{r}) - \frac{6\alpha}{m^2r^3} - \frac{39\alpha}{2m^2r^3})w_n =$$

$$= \Delta W_{SS}^{u_m u_n} + \Delta W_{ST}^{w_m u_n} + \Delta W_{ST}^{u_m w_n} + \Delta W_{SS+LS+ST}^{w_m w_n} \tag{7}$$

and in the framework of the perturbation theory the correction to the energy spectrum is equal to

$$\Delta E_{nn} = \Delta W_{SS}^{u_n u_n} + 2\Delta W_{ST}^{w_n u_n} + \Delta W_{SS+LS+ST}^{w_n w_n} \tag{8}$$

As it is seen from Eq. (7), the S-wave gives pure correction only for the spin-spin interaction (the first term), the intereferential SD term (the second and third terms) contains the spin-tensor correction, and pure D -wave (the fourth term) includes all the spin-dependent components of the interaction potential (that is reflected in Eq. (8)). In the case of the configuration interaction method we used an algorithm, developed in Ref. 2, namely: the corrected energies are solution of the following system of linear algebraic equations:

$$\begin{array}{c} a_1(E - E_1^0 - W_{11}) - a_2W_{12} - a_3W_{13} - \ldots - a_nW_{1n} = 0 \\ -a_1W_{21} + a_2(E - E_2^0 - W_{22}) - a_3W_{23} - \ldots - a_nW_{2n} = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -a_1W_{n1} - a_2W_{n2} - a_3W_{n3} - \ldots + a_n(E - E_n^0 - W_{nn}) = 0 \end{array} \tag{9}$$

where E_m^0- are eigenvalues of Eq. (1) solved with nonsingular part of the potential, and W_{im}- are the correspondent matrix elements. The system wave function is presented through the eigenfunctions Ψ_i of Eq. (1)

$$\Phi = \sum a_i \Psi_i \tag{10}$$

3 Numerical results and discussion

Below the numerical results for the mass spectrum of $u\bar{u}$ and $c\bar{c}$ systems are given, as well as the values of the radial wave function in origin, related to the widths of leptonic decays of vector states [9].

The numeric results are given for the cases of from one (corresponding to the perturbation theory) to three nonperturbed functions (Eq. (10)) being taken as the base functions. The following values of the quark masses are taken for the calculations: $m_c = 1.4$GeV, $m_u = 0.33$ GeV. The strong interaction constant is calculated according to the asymptotic freedom formula

$$\alpha_S(q) = \frac{12\pi}{(33 - 2n_f)\ln(\frac{q^2}{\Lambda^2})}$$

where $\Lambda = \Lambda_{QCD} = 140$ MeV; $n_f = 3$ for light and mixed mesons;$n_f = 4$ for $c\bar{c}$ and $b\bar{b}$ -quarkonia. The transferred momentum q is chosen in the form $q = 2\mu$, μ is the reduced mass of the system. The Lorentz structure of the interaction potential is taken in accordance with Ref. 8, namely $\beta_V = 0.04$GeV, $\beta_S = 0.14$GeV.

The results of the mass spectrum calculations are given in Table 1 (nonperturbed eigenvalues of energy, correction according to the perturbation theory, energies according to CIA with two and three basis functions, respectively), and the experimental data are taken from Ref. 10. It is seen from the table that the series of Eq. (10) converges rapidly for heavy systems and already the perturbation theory gives more than 90% contribution of the singular terms, but for light mesons the extension beyond the perturbation theory is essential.

Table 1. Vector meson mass spectrum

States	M_{TH}, MeV non-perturb.	M_{TH}, MeV 1st approx.	M_{TH}, MeV 2nd approx.	M_{TH}, MeV 3rd approx.	M_{EXP}, MeV
$J/\psi(1S)$	3043.45	3098.58	3097.11	3096.87	3096.87
$\psi(2S)$	3649.04	3674.02	3675.50	3674.87	3685.96
$\psi(4040)$	4098.04	4113.11	− − −	4113.97	4040
$\rho(770)$	685.5	778	771	768.5	768.5
$\rho(1450)$	1574.5	1643	1650	1643	1465
$\rho(1700)$	2270.5	2332	− − −	2341	1700

The absolute values of corrections due to certain singular terms of the potential in the framework of the perturbation theory, are rather interesting. The correction values for the spin-spin, spin-tensor and spin-orbit components of the singular part of the interaction for charmonium and ρ meson are given in Tables 2 and 3, respectively. It is seen that for heavy systems the spin-tensor part of the correction value by order of magnitude exceeds the spin-spin part and is totally determined by the presence of the D-wave admixture. For light systems the spin-spin and spin-tensor correction values are of the same order. Hence, when the spin effects are considered, one should take into account the orbital structure of the meson states, especially for the systems of heavy quarks. However, we note that the D-wave admixture for the light and heavy systems is less than 1 % and about 4% , respectively [8]. In comparison with Ref. 7, we note that for the $\Psi(2S+D)$ state the D-wave admixture in our case is $P_D = 0.008$ [8], which corresponds to the mixing angle of $\varphi = 5^o$. Note, however, that in Ref. 7 the mixing angle of pure triplet states $2S$ and $1D$ $\varphi = 12^o$ is quoted.

Table 2.Perturbation theory for a J/ψ meson

	$\Delta W^{u_n u_n}_{SS}$, MeV	$2\Delta W^{w_n u_n}_{ST}$, MeV	$\Delta W^{w_n w_n}_{SS+ST+LS}$, MeV
ΔE_{11}, MeV	6.04	49.6	0.00050
ΔE_{22}, MeV	4.18	21.16	0.00036
ΔE_{33}, MeV	3.65	11.95	0.00052

Table 3.Perturbation theory for a ρ meson

	$\Delta W^{u_n u_n}_{SS}$, MeV	$2\Delta W^{w_n u_n}_{ST}$, MeV	$\Delta W^{w_n w_n}_{SS+ST+LS}$, MeV
ΔE_{11}, MeV	60.98	31.67	−0.19
ΔE_{22}, MeV	50.19	19.02	−0.28
ΔE_{33}, MeV	46.64	14.69	−0.075

But small integral contribution of the D-wave is particularly noticeable in the origin of the radial variable. The value of the radial wave function in the origin is related to the width of the vector meson leptonic decays [9]

$$\Gamma = (^3S_1 \to e^+e^-) = \frac{4\alpha_{em}Q_q^2}{M_V}[R_S(0)]^2$$

The values of the squared radial wave function in the origin for the non-perturbed and the perturbed case along with the CIA calculations with two and three basic sets are listed in Table 4. It is seen that, contrary to the energy spectra, the consideration of the decay widths beyond the framework of the perturbation theory results in the correction value of 20-30% . Our results are very close to those of Ref. 9 where the same quark systems were considered on the base of Schroedinger equation with the generalized Breit-Fermi potential. The difference from our work is in the choice of the Lorentz structure of the quark-quark interaction and neglecting of the D-wave admixture.

Table 4. Wave functions in origin $|R_S(0)|^2$ in GeV^3.

State	non-perturb.	1st approx.	2nd approx.	3rd approx.	[4]	[11]	[9]
$J/\psi(1S)$	0.77	0.69	0.83	.85	1.05	1.45	0.81
$\psi(2S)$	0.53	0.56	0.47	0.50	0.63	0.93	0.53
$\psi(4040)$	0.47	0.53	− − −	0.41	0.52	0.79	0.46
$\rho(770)$	0.11	0.086	0.126	0.136	− − −	− − −	− − −
$\rho(1450)$	0.091	0.081	0.073	0.089	− − −	− − −	− − −
$\rho(1700)$	0.084	0.12	− − −	0.061	− − −	− − −	− − −

4 Conclusions

Thus, in the present work the studies of the influence of the quark-quark interaction potential structure on the vector meson structure are carried out. The non-singular part of the potential is taken into account by numerical solution of a coupled differential equation system, reflecting the mixing of the S- and D-waves, and the singular part is taken into account in the framework of the perturbation theory and CI approach. It is shown that, in spite of a small admixture of the D-wave (less than 1%) this component of the wave function plays an essential role at the account of the singular part of the interaction potential, and namely the singular terms are considered by most authors while the spin effects being studied. The presence of the *D* -wave essentially enhances the contribution of the spin-tensor component into the mass spectrum. As concerning the technique of the account of the singular part of the interaction it should be noted that in case the mass spectrum of the systems being considered, one can restrict oneself by the first correction of the perturbation theory, but at the analysis of the decay widths the CIA results essentially improve the perturbation theory.

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