Международная конференция «ТЕОРИЯ ВЕРОЯТНОСТЕЙ И ЕЕ ПРИЛОЖЕНИЯ», посвященная 100-летию со дня рождения Б. В. Гнеденко

Москва, 26–30 июня 2012 года

Тезисы докладов

"PROBABILITY THEORY and its APPLICATIONS" in Commemoration of the Centennial of B. V. Gnedenko

Abstracts

URSS

МЕЖДУНАРОДНАЯ КОНФЕРЕНЦИЯ «ТЕОРИЯ ВЕРОЯТНОСТЕЙ И ЕЕ ПРИЛОЖЕНИЯ», ПОСВЯЩЕННАЯ 100-ЛЕТИЮ СО ДНЯ РОЖДЕНИЯ Б. В. ГНЕДЕНКО (МОСКВА, 26–30 ИЮНЯ 2012 ГОДА)

Тезисы докладов

INTERNATIONAL CONFERENCE "PROBABILITY THEORY and its APPLICATIONS" in Commemoration of the Centennial of B. V. Gnedenko (Moscow, June 26–30, 2012)

Abstracts

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Международная конференция «Теория вероятностей и ее приложения», посвященная 100-летию со дня рождения Б. В. Гнеденко (Москва, 26–30 июня 2012 года): Тезисы докладов / Под ред. А. Н. Ширяева, А. В. Лебедева. — М.: ЛЕНАНД, 2012. — 400 с.

В сборнике представлены тезисы докладов Международной конференции «Теория вероятностей и ее приложения», посвященной 100-летию со дня рождения великого русского ученого Бориса Владимировича Гнеденко (01.01.1912 – 27.12.1995), прошедшей на механико-математическом факультете Московского государственного университета имени М. В. Ломоносова 26–30 июня 2012 года.

Семь основных секций конференции отражают научные интересы и достижения Б. В. Гнеденко в различных областях теории вероятностей и ее приложений, истории математики и математического образования. В докладах конференции представлены итоги развития его идей и результатов.

Сборник будет полезен преподавателям школ и вузов, научным сотрудникам, студентам и всем, кто интересуется современной математикой, историей математики или ее преподаванием.

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Define the set Λ_Φ of all probability measures $Q \in \Lambda$ such that $\int_S |f| dQ < \infty$ for all $f \in \Phi$. Denote $\Lambda_{0\Phi}$ the set of all charges $G = Q - R$ with $Q, R \in \Lambda_\Phi$. Define the τ_Φ-topology as the coarsest topology in $\Lambda_{0\Phi}$ that makes continuous for all $f \in \Phi$ the map $\Lambda_{0\Phi} \ni G \to \int_S f\,dG$.
B1. The functional $T(P)$ is continuous in τ_Φ-topology.

B2. For any pm $Q, Q << P_0, q = \frac{dQ}{dP_0}, ||q||_2 < \infty$ and any sequence pms $Q_n, Q_n << P_0, q_n = \frac{dQ_n}{dP_0}, ||q_n - q|| \to 0$ as $n \to \infty$, there holds

$$\lim_{n\to\infty} h_n^{-1} \left(T(P_0 + h_n(Q_n - P_0)) - T(P_0) - h_n \int_S g d(Q_n - P_0) \right) = 0 \tag{2}$$

with $h_n \to 0$ as $n \to \infty$.

Lemma 1. *Assume* A *and* B1,B2. *Then*

$$\lim_{n\to\infty} (nb_n^2)^{-1} \ln V_n = -\frac{1}{2}\sigma_g^2 \tag{3}$$

with $\sigma_g^2 = \int_S g^2 dP_0$.

Hence, using $\mathrm{Var}_{Q_n} \hat{V}_n \geq 0$, for each importance sampling procedure, we get

$$\liminf_{n\to\infty} (nb_n^2)^{-1} \ln \mathbf{E}_{Q_n}[V_n^2] \geq -\sigma_g^{-2} \tag{4}$$

We say that importance sampling procedure is asymptotically efficient if the equality in (4) is attained. Thus, if importance sampling procedure is asymptotically efficient, one can hope that an amount of simulation does not have exponential growth with increasing accuracy.

Theorem 1. *Assume* A *and* B1, B2. *Let we implement the importance sampling procedure based on the pms* Q_n *with the densities* $q_{1n}(x) = \lambda_n + b_n h(x)\chi(h(x) > -\delta b_n^{-1})$ *or* $q_{2n}(x) = c_n \exp\{b_n h(x)\}\chi(|h(x)| < \delta b_n^{-1})$ *where* λ_n, c_n *are normalizing constants,* $0 < \delta < 1$ *and* $\mathbf{E}[h(X)] = 0, \mathbf{E}|h(X)| < \infty, \mathbf{E}[h^2(X)] < \infty$. *Then the importance sampling procedure is asymptotically efficient iff* $h = \sigma_g^{-2} g$.

We prove also other versions of Theorem. We apply these results to the problems of estimation of moderate deviation probabilities of rank statistics, L and M estimators, the Kaplan-Meier estimator, the empirical quantile process and the empirical copular function.

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Integrals calculation by Monte Carlo method with a given accuracy and reliability

Yuriy Yu. Mlavets[1]

Definition 1. A continuous even convex function $U = \{U(x), x \in \mathcal{R}\}$ is called a $\mathcal{C}$-function if $U(x)$ is monotonically increasing function for $x > 0$ and $U(0) = 0$.

Definition 2. Let U be an arbitrary $\mathcal{C}$-function. The Orlicz space of random variables $L_U(\Omega)$ is defined as a family of random variables, where for each $\xi \in L_U(\Omega)$ there exists a constant $r_\xi > 0$ such that $\mathsf{E}U\left(\frac{\xi}{r_\xi}\right) < \infty$.

The space $L_U(\Omega)$ is a Banach space with respect to the norm $\|\xi\|_U = \inf\{r > 0;\ \mathsf{E}U\left(\frac{\xi}{r}\right) \leq 1\}$ (Luxemburg norm).

Definition 3. A Orlicz space $L_U(\Omega)$ has the property $\mathbf{H}$ if for any centered independent random variables $\xi_1, \xi_2, \dots, \xi_n$ from $L_U(\Omega)$ the following inequality holds $\|\sum_{k=1}^n \xi_i\|_U^2 \leq C_U \sum_{k=1}^n \|\xi_i\|_U^2$, where C_U is some absolute constant.

Theorem 1. *Let* $Y = \{Y(t), t \in \mathbf{T}\}$ *be a random process.* Y *belongs to an Orlicz space* $L_U(\Omega)$ *such that the condition* g *holds for* U *and* $L_U(\Omega)$ *has the property* $\mathbf{H}$ *with the constant* C_U*. Let* $(\mathbf{T}, w)$ *be a compact metric space and* Y *be separable process on* $(\mathbf{T}, w)$*,* $N_w(u)$ *be the metric massiveness. There exist a continuous function* $\sigma = \{\sigma(h),\ 0 \leq h \leq \sup_{t,s\in\mathbf{T}} \rho(t,s)\}$*, such that* $\sigma(h) \to 0$*, when* $h \to 0$*, that* $\sup_{\rho(t,s)\leq h} \left\|Y(t) - Y(s)\right\|_U \leq \sigma(h)$ *and* $\int_0^{\delta_0} U^{(-1)}(N_w(\sigma^{(-1)}(u)))du < \infty$.

Let $X(t) = Y(t) - m(t)$*, where* $m(t) = \mathsf{E}X(t)$ *and* $X_k(t)$ *be independent copies of* $X(t)$*,* $S_n(t) = \frac{1}{\sqrt{n}} \sum_{k=1}^n X_k(t)$*. Then for all* $\varepsilon > 0$ *the following inequality holds* $P\{\sup_{t\in\mathbf{T}} |S_n(t)| > \varepsilon\} \leq \frac{1}{U\left(\frac{\varepsilon}{B(t_0,\theta)}\right)}$*, where* t_0 *is any point from* $\mathbf{T}$*,* $0 < \theta < 1$*,* $B(t_0, \theta) = \|X(t_0)\|_U + \frac{1}{\theta(1-\theta)} \int_0^{\delta_0\theta} \nu(N_w(\sigma_1^{(-1)}(u))du$*, where* $\sigma_1(h) = \left(1 + \frac{d_U}{U^{(-1)}(1)}\right)\sigma(h)$*,* $\delta_0 = \sigma_1\left(\sup_{t,s\in\mathbf{T}} \rho(t,s)\right)$*,* $\nu(n)$ *- majorant characteristic of* $L_U(\Omega)$ *space.*

This theorem is used for calculation of the integral $\int \dots \int_{R^d} f(\vec{x})d\vec{x}$ by Monte Carlo method with a given reliability and accuracy in $\mathbf{C}(\mathbf{T})$ spase.

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