Fractal Time Series Analysis in Non-Stationary Environment

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***Abstract*—A fractal analysis of the Bitcoin time series for the period from 2012 to 2019 is carried out: Hurst exponents were calculated, the behavior of this indicator in dynamics was investigated, V-statistics were plotted. For the automatic determination of the average length of the nonperiodic cycle in the information system of time series analysis, the smoothing method of V-statistics based on the Kaufman's adaptive moving average and simple moving average with different periods is proposed. The results fractal analysis of the time series of the Bitcoin cryptocurrency price show that the Bitcoin market is characterized by an inescapable efficiency, i.e. periods of effectiveness are replaced periods of inefficiency. This is manifested by changing the type of time series of Bitcoin prices from persistence to random and antipersistence, especially during periods of intense price growth, due to the significant influence on the mechanism of generation of time series of random factors.**

Keywords—Hurst exponent; Bitcoin; time series; cryptocurrency; fractal analysis

I. Introduction

Bitcoin, as an electronic currency, was created more than 10 years ago. The feature of Bitcoin is that it does not have centralized management and issuers. Digitally signed transactions between two nodes are transmitted to all peer-to-peer nodes, and the migration data itself is stored in a distributed database. Cryptographic methods are used to protect information about executed transactions and to prevent fraud. [1]. Bitcoin is one of the most successful cryptographic trades, as the capitalization of 2019 is about $ 150 billion.

The actual task of research is fractal analysis of time series Bitcoin for calculating the Hurst exponent and determining the class, which the time series belongs (persistence, random, antipersistence), calculating the average length of the non-recurrent cycle time series and evaluating information market efficiency Bitcoin generally in terms of non-stationary environment.

Financial time series is a complex of different components: component of the function of the trend, cyclic or quasi-cyclical component, random fluctuations, etc. In addition, according to the theory of fractal markets, it is assumed that the financial time series have fractal properties [2]. These properties have been described in detail in the last two decades, and a method of fractal analysis was proposed, which is based on the calculation of the Hurst exponent. There are several methods for calculating the Hurst exponent, the main of which is the R/S analysis [2] and the detrended fluctuation analysis (DFA) [3, 4]. Empirical rules and guidelines for R/S analysis were developed, as well as features of visual analysis of V- statistics for determining the average length of the nonperiodic cycle [2]. The fractal and multifractal properties of time series were analyzed [5] and the effect of long-term memory in time series was investigated on the basis of the DFA method [6].

Detection of long-term memory and asymmetry in the Bitcoin market is described in [7]. It was found that the Bitcoin market demonstrates performance periods alternating with periods when price dynamics is anti-static. In [8], it is shown that the Bitcoin market is generally effective, but periodic inefficiency is quite high. The analysis concluded that Bitcoin is a relatively new investment assets for which can be characterized by periods of ineffectiveness. In [9, 10], it was investigated that in 2013 there was a low level of liquidity and Hurst exponent was less than 0.5, which indicates the antipersistence of the Bitcoin time series. It was identified the periods of growth of the Hurst exponent above 0.5, which increased the market's efficiency, but for a short time. In [11], the results of fractal analysis of time series Bitcoin, which established that they have self-similar and multifractal properties, are

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given. In [12], a method for determining the bandwidth of communication channels for the input stream of self-similarity of the network is proposed. In [13], the task of classifying fractal time series using the purpose of algorithms based on decision trees is considered. The results of the classification showed the superiority of machine learning methods over the traditional method of calculating the Hurst exponent, especially in the case of short-term time series.

Fractal analysis is an important tool for pre-predictive study the properties of time series. In [14-16], the description of various methods for forecasting fractal time series is described. In [17, 18], the approaches to the implementation of combined information technologies for predicting time series for various applications are presented. In [19, 20], the development of adaptive combined models of prediction of time series with allowance for self-similarity is described. The creation of a parametric model for forecasting and evaluating the effectiveness of educational institutions is described in [21, 22]. Also, fractality of time series can be used to predict contamination in environmental monitoring systems [23] and to determine similarities in different types data [24, 25].

The purpose of this work is to conducting a fractal analysis of the Bitcoin time series for the period from 2012 to 2019, the calculation of the Hurst exponent, the study of this indicator in dynamics, the plotting V-statistics to determine the average length of the nonperiodic cycle, if it exists.

II. A Method of Fractal Analysis

The fractal dimension of the time series in practice is determined by the Hurst exponent *H*, which is calculated:

$$\left(\frac{R}{S}\right)_{\upsilon} \approx \alpha \upsilon^{H},$$

where $\left(\frac{R}{S}\right)_{\upsilon}$ is normalized scale from the accumulated average, υ is the number of timescales or number of observations, $\alpha = const$, independent of υ. The Hurst exponent is a parameter $H = [0,1]$, which characterizes the ratio of the component of the trend function to white noise. In this study, the R/S analysis procedure is used to calculate the Hurst exponent.

Let be a discrete time series $Z = \{z_i\}_{i=1}^{n}$, where *n* is the length of the time series. For each of the initial segments of this time series $Z = \{z_i\}_{i=1}^{\tau}$ length $\tau = 3, 4, \dots, n$ the average values are calculated of the formula $z^{\tau} = \frac{1}{\tau}\sum_{i=1}^{\tau} z_i$, accumulated deviations are calculated $x^{\tau,t} = \sum_{i=1}^{t}\left(z_i - z^{\tau}\right)$, $\tau = 3, 4, \dots, n$, the range is $R_{\tau} = \max_{1 \le t \le \tau} x^{\tau,t} - \min_{1 \le t \le \tau} x^{\tau,t}$.

The mean square deviation for each of the segments is determined by the formula $S_{\tau} = \sqrt{\frac{1}{\tau}\sum_{i=1}^{\tau}\left(z_i - z^{i}\right)^2}$, $\tau = 3, 4, \dots, n$. The range of accumulated deviation is normalized by dividing by S_{τ} for each segment τ and the graph of the dependence of $\lg\left(\frac{R}{S}\right)_{\tau}$ on $\lg(\tau)$, which is called a *R/S* trajectory, is constructed. The next step is to construct a linear regression equation based on the least squares method. The coefficient for an independent variable of this equation will be Hurst exponent:

$$\lg\left(\frac{R}{S}\right)_{\tau} = \lg(\alpha) + H\lg(\tau),$$

where $\alpha = const$.

For constructing a trajectory *H* a graph of dependency considers: $H(\tau) = \dfrac{\lg\left(\frac{R}{S}\right)_{\tau}}{\lg\left(\frac{\tau}{2}\right)}$ on $\lg\left(\frac{\tau}{2}\right)$.

The intersection moment of trajectory *H* with the *R/S* trajectory can characterize the presence or absence of long-term memory in the time series. According to the Hurst exponent, it is possible to distinguish the following time series classifications:

1. If $0 < H < 0.5$ or $0 < H < E$ [26], then there isn\`t correlation between retrospective and predictive values in the time series. This time series is antipersistence. Score E is determined [26]:

$$E\left(\frac{R}{S}\right)_{\tau} = \frac{\Gamma\left(\frac{\tau-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{\tau}{2}\right)}\sum_{i=1}^{\tau-1}\sqrt{\frac{\tau-i}{i}} = \sqrt{\frac{2}{\pi(\tau-1)}}\sum_{i=1}^{\tau-1}\sqrt{\frac{\tau-i}{i}}.$$

2. If $E < H < 1$, then the time series is characterized by persistence behavior.

3. If $H \approx E$ or $H = 0.5$, then the time series is random and has no memory of its initial conditions.

For a detailed study of the Hurst exponent change in the dynamics, it is proposed to construct a time series that represents the Hurst performance, which is calculated for each time series of the set $F(Z)$. The set of time series $F(Z)$ is formed from the time series Z by a moving window method with length *m*, $m < n$.

In order to find the length of a nonperiodic cycle, a visual analysis of the V-curve is usually used. It is proposed to smooth the curve V-statistics, to more precisely identify the point reversal curve. This point indicates a significant effect on the generation of a time series of random factors that are associated with the uncertainty of market participants. This, in turn, is a sign of a possible change in the trend due to the loss of a time series of memory about the initial conditions and the change in its behavior from persistent to random or antiperspirant. The growth of V-statistics with an increase in the number of observations indicates the persistence of the time series current section, and stabilization - on the predominance of white noise. Vstatistic V_τ is a relation $\frac{R_\tau}{S_\tau}$ to $\sqrt{\tau}$. The ratio increases if the time series is persistent. The ratio decreases if the time series is random or antipersistent [2]. To detect the ratio behavior graph of dependence $V_\tau = \frac{R_\tau}{\sqrt{\tau} S_\tau}$ on $\lg(\tau)$ is useful. V-statistics trend change indicates an increase in random factors influence on the time series generation mechanism. The market uncertainty may cause it. The market instability in turn often leads to the time series main trend change and to the current non-periodic cycle completion and the new cycle emergence. Non-periodic cycle completion and the time series trend change detection let to increase the time series prediction efficiency. This is very useful information for traders.

Authors propose the method of non-periodic cycle calculation based on V-statistics curve smoothing and the identification the moments of the change in the curve initial trend, taking into account the smoothed values . Time series V_τ, $\tau = 3, 4, \ldots, n$ is smoothed out by means of simple moving average with a period p: $S_{\tau+p} = \frac{1}{p}\sum_{j=0}^{p-1} V_{\tau+p-j}$, $\tau = 3, 4, \ldots, n-p$ and Kaufman's adaptive moving average $a_t = c_t V_t + (1 - c_t) a_{t-1}$, where

$$c_t = \left(\left(\frac{V_t - V_{t-r}}{\sum_{i=0}^{r-1} \left| V_{t-i} - V_{t-i-1} \right|} \right) \left(\frac{2}{p_1 + 1} - \frac{2}{p_2 + 1} \right) + \frac{2}{p_2 + 1} \right)^2 ,$$

$p_1 < p_2$, r is a period, $t = \overline{\tau + r, n}$. The length of the nonperiodic cycle is calculated automatically in the condition that this criterion is used in information systems for the time series analysis.

The length of the nonperiodic cycle is k if the conditions are fulfilled at the k-time:

1. The monotonous fall of Kaufman's adaptive moving values is fixed from the moment k, that is $a_k > a_{k+1} > a_{k+2}$. Kaufman index has to increase monotonously to the k^{th} point.

2. The simple moving average does not exceed the Kauffman index value at this moment, that is $S_k < a_k$, $S_{k+1} < a_{k+1}$.

3. There is a sharp change in the trend of V-statistics from growth to fall: $V_{k-1} < V_k$, $V_k > V_{k+1}$, V-statistics value in the k[th] point reaches the local maximum, exceeding the simple moving average value and the Kauffman index: $V_k > a_k > S_k$.

It is necessary to find the moment of a sharp change in the trend of the H trajectory to find the length of the nonperiodic cycle. Usually, changes occur from declining to decline. The R/S trajectory should change its initial trend until the trend of the H trajectory changes. The point for which these conditions are fulfilled indicates the length of the nonperiodic cycle.

III. Input Data

In the study, fractal analysis was investigated for three the time series segments of Bitcoin cryptocurrency price (daily data in the amount of 800 points for each segment): a is the period from 2012-11-09 to 2015-01-18, b is the period from 2015-01-19 to 2017-03-29, c is the period from 2017-03-30 to 2019-06-08 (see. Fig. 1).

Fig. 2 shows these time series after conversion: $z_i^* = \frac{\lg(z_i)}{\lg(z_{i-1})}$, $i = \overline{1, n-1}$.

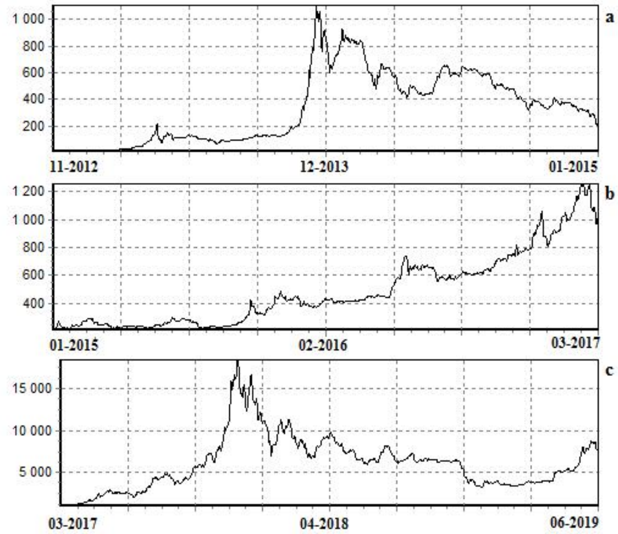

Fig. 1. Time series segments of Bitcoin cryptocurrency price (daily data): a – period from 2012-11-09 to 2015-01-18, b – period from 2015-01-19 to 2017-03-29, c – period from 2017-03-30 to 2019-06-08.

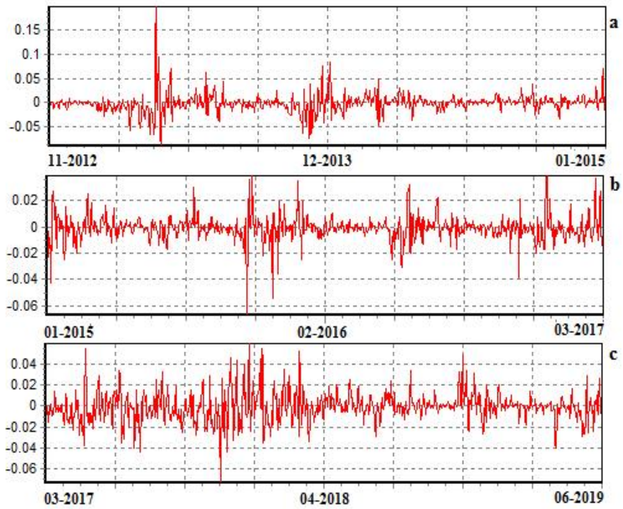

Fig. 2. Time series segments of Bitcoin cryptocurrency price after the conversion.

The fractal R/S analysis procedure was conducted separately for each of the specified time series: Hurst exponent was calculated, V-statistics, H- and R/S trajectories were plotted.

IV. Results of Fractal Analysis

The results of the R/S analysis for the three segments of the time series Bitcoin crypto currency price are as follows:

- *Time series a*. Hurst exponent $H = 0.770$. Time series is persistence everywhere.

- *Time series b*. Hurst exponent $H = 0.420$. The general Hurst exponent indicates that the time series is antipersistence. The research results of the change Hurst exponent in dynamics indicate that by mid-2015, the Bitcoin time series was persistence generally, and then, by the beginning of 2017, displaced below the 0.5 (see Fig.3.).

- *Time series c*. The general Hurst exponent $H = 0.738$, it indicates a generalized persistent behavior. But since April 2019 Hurst exponent has fallen below 0.5 or roughly 0.5, indicating a period of significant influence of uncertainty among market participants.

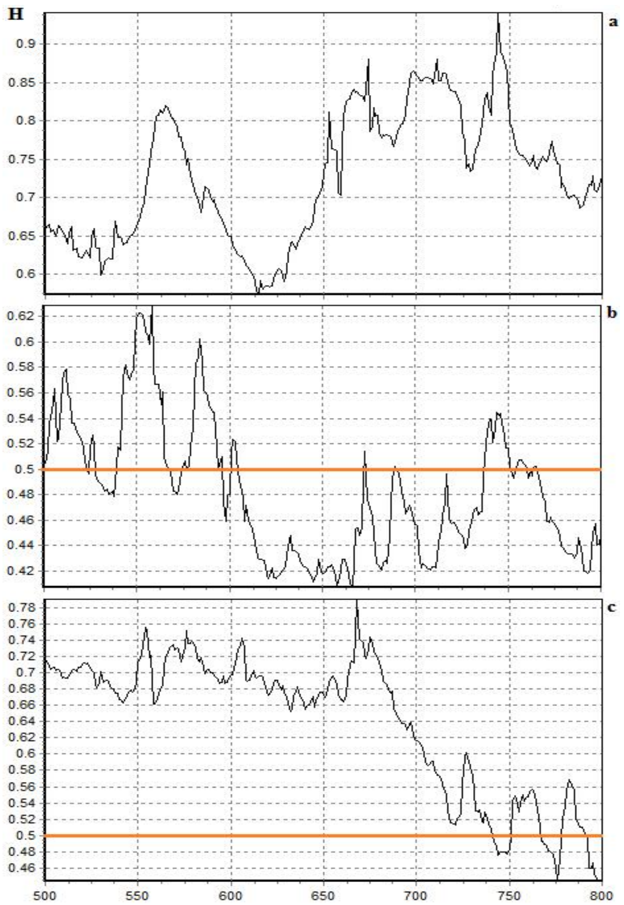

Fig. 3. Hurst exponent in dynamics. Length of moving window (m=500).

V-statistics was constructed for the time series «c» and was smoothed by Kaufman's adaptive moving average and simple moving average for $p = 7$, $r = 4$, $p_1 = 3$, $p_2 = 10$ (see Fig.4).

Also, the H- and R/S trajectories were plotted. Their intersection indicates the presence of insignificant memory in the time series «c» (see Fig.5).

V-statistics were constructed for each time series from the set. The average length of the nonperiodic cycle is calculated by determining the moment of change in the trend of the V-statistics curve. The curves of V-statistics were preliminary smoothed out with the help of the Kaufman's adaptive moving average and a simple moving average.

Fig. 4. V-statistics of the time series «c» smoothed by the Kaufman's adaptive moving average (red curve) and simple moving average (blue curve).

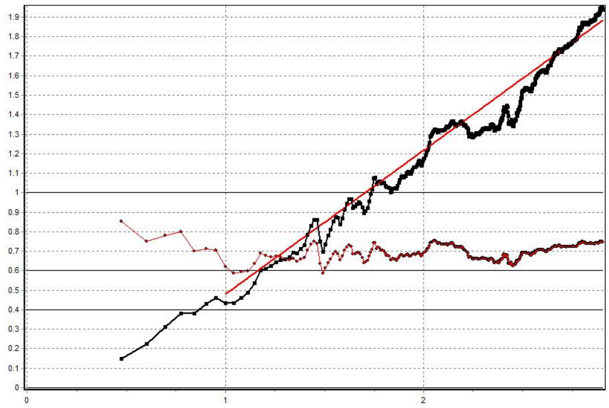

Fig. 5. H trajectory (red curve) and R/S trajectory (black curve) for time series «c».

Fig. 6 shows the histogram of the distribution the calculated average lengths nonperiodic cycle. Median of ranked series is equal 20. That is, the average length of the nonperiodic cycle for the time series «c» is approximately 20 points.

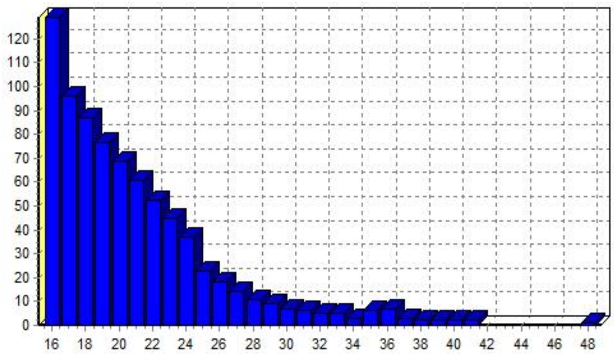

Fig. 6. Histogram of the calculated mean lengths distribution of the nonperiodic cycle for the time series «c».

In the study, all figures are the derivation of the developed information system for the time series analysis. The results of the study have shown that the time series Bitcoin price were obtained on the basis of the above information system.

V. Conclusion

In the work, Hurst exponents are determined and the average lengths of nonperiodic cycles are calculated on the fractal analysis results for three the time series segments of Bitcoin cryptocurrency price.

The results of the study have shown that the beginning of a significant increase the Bitcoin price since 2016 due to increased uncertainty, the

influence of random factors was amplified on the mechanism generating the time series. As a result, time series of Bitcoin cryptocurrency price in the period from 2016 to 2017 were in the zone of randomness and antipersistence.

According to the results of the study, the period from 2017 to 2019 is more trendstable, but the periodic transition to a random zone, especially since the beginning of 2019, indicates that the Bitcoin market is characterized by unsustainable efficiency.

It is important to keep track of these periods in time to make sound decisions on the market. The developed information system for the analysis of time series allows determining the periods of trend changes and identifies the values of Hurst exponent in dynamics.

Our future research will focus on study based on expansion the functional system and integration DFA method and other tools of time series fractal analysis.

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