УДК 519.21

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THE PROBABILITY OF NON-RUIN OF AN INSURANCE COMPANY WITH ADVERTISING EXPENSES AND INVESTING IN BANK TERM DEPOSIT BY MHULL INSURANCE OF 10 TOP INSURANCE COMPANIES OF UKRAINE. II.

The result presented in this paper is the second part of the paper [1], where was constructed an equation which allows calculating the probability of non-ruin for the classical model of risk when an insurance company has promotional costs. There is considered the case of investment in the financial (B,S)-market, precisely in riskless part of it, in the bank deposit, in this article. Some numerical results for the probability of non-ruin by MHull of 10 Top Insurance companies of Ukraine for the period 01/01/2015 to 06/30/2015 based on the developed formulae are provided. Some conclusions about the influence of advertising company and investing in the bank deposit on the probability of non-ruin are carried out.

Результат, поданий у даній роботі, є другою частиною статті [1], де було виведене рівняння, що дозволяє обчислити ймовірність небанкрутства для класичної моделі ризику за умови, коли страхова компанія має витрати на рекламу. Розглянуто випадок інвестицій у фінансовий (B,S)-ринок, а саме, у безризикову його частину. Було наведено деякі чисельні результати для ймовірності небанкрутства за період з 01/01/2015 по 06/30/2015 за страхуванням КАСКО та статистикою десяти провідних страхових компаній України. Було зроблено висновки щодо впливу проведення рекламних кампаній та одночасного інвестування у банківський депозит на ймовірність небанкрутства.

1. Introduction. Investing activities is important in the insurance company work because it can ensure the formation of the sufficient insurance fund to cover claims costs. However, the financial risk resulting from the process of investment activity can weaken the solvency of the insurance company no less than the claims payments. That is why last years a significant interest emerged in the analysis of an insurance company activity with the investing activities. In our article we are interested in the case when insurance company invests only in the riskless asset of financial (B,S)-market. In [2] was made the first attempt to consider the investing free capital problem. In suggested work the insurance company gets an additional profit investing the capital under the bank deposit interest rate r. This model was more particularly investigated in [3], [4] and generalized in [5]. In work [6] was generalized the Hipp's and Plum's model [7] by the riskless asset encompassing (the bank deposit) in the model and examined the optimal strategy in the case when claims have gamma, Pareto or exponential distribution. The authors of this article also studied such models without the promotional campaign assumption [8], [9] and with them [10]. The first part of the article [1] addressed the problem without the using bank deposit for the insurance company model illustrated by the example of MHull insurance. On the basis of the recorded facts and the resulting model, in this paper the work of 10 leading insurance companies in Ukraine during the period from 01.01.2015 on 06.30.2015 is analyzed and the effect of deductions for advertising and investing activities on the non-ruin probability of insurance companies is studied.

2. Classical model of risk with advertising expenses and bank deposit. Let consider the insurance company the surplus is modeled by Cramer-Lundberg model with the initial capital $x\,(x \geq 0)$, the premium rate c $(c > 0)$, the number of claims in $(0, t]$ is a Poisson process with rate $\lambda\,(\lambda > 0)$ and the claim sizes is a sequence of positive random variables independent of the number of claims with the distribution function $F\,(y)$. The insurance company invests all temporary free funds in the bank term deposit with interest rate r $(r(t) \equiv r > 0)$. Also, the insurance company starts advertising activities to attract new customers, some part of the income, exactly $\delta (0 < \delta < 1)$, is deducted on advertising. The intensity of premium income will increase by $j_1(\delta)$, at the same time the intensity of claims to the insurance company increases by the $j(\delta)$.

Thus, the capital of the company is described by the equation

$$\begin{aligned}&\xi_x(t+\Delta t) = \xi_x(t) + \\ &+(1-\delta)\left[\xi_x(t) r \Delta t + c(1+j_1(\delta))\Delta t - \textstyle\int_0^{+\infty}\int_t^{t+\Delta t} \alpha \nu_{\lambda(1+j(\delta))}(d\alpha, ds)\right].\end{aligned} \tag{1}$$

Equation (1) can be rewritten as

$$d\xi_x(t) = (1-\delta)\left[\xi_x(t) r dt + c(1+j_1(\delta))dt - \int_0^{+\infty} \alpha \nu_{\lambda(1+j(\delta))}(d\alpha, dt)\right]. \tag{2}$$

Теорема 1. *Let $\varphi(x)$ is the non-bankruptcy probability on an infinite time interval $t \in [0, \infty)$, that is*

$$\varphi(x) = P\left\{\xi_x(t) > 0,\ \forall t \in R_+\right\},$$

then, if the evolution of the insurance company capital is described by equation (2), the insurance company non-bankruptcy probability with the deductions on advertising and investing in bank deposit is a solution of the integro-differential equation on an infinite time interval $t \in [0, \infty)$:

$$\frac{(1-\delta)\left(xr + c\left(1+j_1(\delta)\right)\right)}{\lambda\left(1+j(\delta)\right)}\varphi'(x) = \varphi(x) - \int_0^x \varphi(x-y)dF(y) \tag{3}$$

with the limit condition $\lim\limits_{x \to +\infty} \varphi(x) = 1$.

Доведення. To prove this theorem we have used the methods represented in the book [11]. So, before the first jump of the process

$$dR(t) = \int_0^{+\infty} \alpha \nu_{\lambda(1+j(\delta))}(d\alpha, dt)$$

the capital of the company is described by the equation

$$d\xi_x(t) = (1-\delta)\left[\xi_x(t) r dt + c\left(1+j_1(\delta)\right)dt\right], \quad \xi_x(0) = x,$$

from which we derive

$$\xi_x(t) = \left[x + \frac{c\left(1+j_1(\delta)\right)}{r}\right] e^{(1-\delta)rt} - \frac{c\left(1+j_1(\delta)\right)}{r}.$$

Let the first jump of the process $R(t)$ is occurring at the time $\tau = s$, and its value is equal to y . Until the time τ_1 the ruin will not happen. In order to ruin of company will not happen from the time point τ_1 , it is necessary and sufficient the fulfillment of the condition

$$y \le \left[x + \frac{c\,(1 + j_1(\delta))}{r}\right] e^{(1-\delta)rt} - \frac{c\,(1 + j_1(\delta))}{r}.$$

The time point of the first jump has an exponential distribution with the parameter $\lambda\,(1 + j(\delta))$, by virtue of the fact that process $N(t)$ is Poisson with the parameter $\lambda\,(1 + j(\delta))$. Then we apply the formula of total probability and derive

$$\varphi(x) = \int_0^{+\infty} \lambda\,(1 + j(\delta))\, e^{-\lambda(1+j(\delta))s} \int_0^{A(s)} \varphi\,(A(s) - y)\, dF(y)ds, \qquad (4)$$

where $A(t) = \left[x + \frac{c(1+j_1(\delta))}{r}\right] e^{(1-\delta)rt} - \frac{c(1+j_1(\delta))}{r}$.

If we make the change of variable in integral (4), as follows $A(s) = u$, we obtain

$$\begin{aligned} &\varphi(x) = \tfrac{\lambda(1+j(\delta))}{(1-\delta)}\,(xr + c\,(1 + j_1(\delta)))^{\frac{\lambda(1+j(\delta))}{(1-\delta)r}} \times \\ &\times \textstyle\int_x^{+\infty} [ur + c\,(1 + j_1(\delta))]^{-\frac{\lambda(1+j(\delta))}{(1-\delta)r}-1} \int_0^u \varphi\,(u - y)\, dF(y)du. \end{aligned} \qquad (5)$$

Differentiating equation (5) with respect to x, we get

$$\begin{aligned} &\varphi'(x) = \left(\tfrac{\lambda(1+j(\delta))}{(1-\delta)}\right)^2 (xr + c\,(1 + j_1(\delta)))^{\frac{\lambda(1+j(\delta))}{(1-\delta)r}-1} \times \\ &\times \textstyle\int_x^{+\infty} [ur + c\,(1 + j_1(\delta))]^{-\frac{\lambda(1+j(\delta))}{(1-\delta)r}-1} \int_0^u \varphi\,(u - y)\, dF(y)du - \\ &- \tfrac{\lambda(1+j(\delta))}{(1-\delta)}\,[xr + c\,(1 + j_1(\delta))]^{-1} \textstyle\int_0^x \varphi\,(x - y)\, dF(y). \end{aligned} \qquad (6)$$

By substituting (6) to the (5), we have obtained the statement of the theorem.

Приклад 1. *Analytical solution of the integro-differential equation* (3) *in the case of exponentially distributed claims.*

Let claims to insurance company have exponential distribution with parameter a (a > 0), so $F(y) = 1 - e^{-ay}$. *We rewrite the integro-differential equation* (3) *as*

$$\frac{(1-\delta)\,(xr + c\,(1 + j_1(\delta)))}{\lambda\,(1 + j(\delta))}\varphi'(x) = \varphi(x) - \int_0^x \varphi(x - y)ae^{-ay}dy.$$

We write over this equation in the form

$$\begin{aligned} &(1-\delta)\,[rx + c\,(1 + j_1(\delta))]\,\varphi'(x) = \lambda\,(1 + j(\delta))\,\varphi(x) - \\ &-\lambda\,(1 + j(\delta)) \textstyle\int_0^x \varphi(x - y)ae^{-ay}dy. \end{aligned} \qquad (7)$$

Differentiating this equation (7) *with respect to x, we get*

$$\begin{aligned} &(1-\delta)\, r\varphi'(x) + (1-\delta)\,[rx + c\,(1 + j_1(\delta))]\,\varphi''(x) = \lambda\,(1 + j(\delta))\,\varphi'(x) + \\ &+\lambda(1 + j(\delta))\left[a \textstyle\int_0^x \varphi(x - y)ae^{-ay}dy - \varphi(x)a\right]. \end{aligned} \qquad (8)$$

Making the substitution from (7) *to* (8), *we get*

$$\begin{aligned} &(1-\delta)\,(rx + c\,(1 + j_1(\delta)))\,\varphi''(x) + (1-\delta)\, r\varphi'(x) - \lambda\,(1 + j(\delta))\,\varphi'(x) + \\ &+a\,(1-\delta)\,(rx + c\,(1 + j_1(\delta)))\,\varphi'(x) = 0. \end{aligned} \qquad (9)$$

The solution of the equation (9) *is*

$$\varphi(x) = 1 - A \int_x^{+\infty} e^{-av} \left(v + \frac{c\,(1 + j_1(\delta))}{r} \right)^{\frac{\lambda(1+j(\delta))}{(1-\delta)r} - 1} dv, \qquad (10)$$

or (10) *rewritten in another form*

$$\varphi(x) = 1 - Aa^{-\frac{\lambda(1+j(\delta))}{(1-\delta)r}} e^{\frac{ac(1+j_1(\delta))}{r}} \Gamma\left(\frac{\lambda\,(1 + j(\delta))}{(1-\delta)\,r}, \frac{ac\,(1 + j_1(\delta))}{r} + ax \right), \qquad (11)$$

where

$$A = \frac{1}{e^{\frac{ac(1+j_1(\delta))}{r}} a^{-\frac{\lambda(1+j(\delta))}{(1-\delta)r}} \Gamma\left(\frac{\lambda(1+j(\delta))}{(1-\delta)r}; \frac{ac(1+j_1(\delta))}{r} \right) + \frac{(1-\delta)r}{\lambda(1+j(\delta))} \left(\frac{c(1+j_1(\delta))}{r} \right)^{\frac{\lambda(1+j(\delta))}{(1-\delta)r}}}.$$

Constants were determined with the usage of boundary conditions:

$$\begin{cases} \lim\limits_{x \to +\infty} \varphi(x) = 1, \\ (1-\delta)\, c(1 + j_1(\delta))\varphi'(0) = \lambda(1 + j(\delta))\varphi(0). \end{cases}$$

3. Numerical results by Hull Insurance of 10 Top Ukrainian Insurance companies. We present numerical results by MHull of 10 Top Ukrainian Insurance companies using the developed formulae (11) for the probability of non-ruin. Motor Hull Insurance provides protection against risks, which lead to partial damage or total loss of the vehicle. Necessary data for analysis are presented in the following Table 1. These data must be considered as being approximate.

Таблиця 1.

The data of 10 Top Ukrainian Insurance companies by MHull

Company	$Exp(a)$	Premiums, c	Intensity of claims, λ
Oranta	$Exp(0,0000917)$	44337,78	1,944444
VUSO	$Exp(0,0000848)$	132218,3	4,872222
Illichevske	$Exp(0,0000824)$	58487,22	2,338889
Providna	$Exp(0,0000748)$	256393,3	10,51667
Arsenal - insurance	$Exp(0,0000742)$	459008,3	18,47778
UPSK	$Exp(0,0000721)$	113874,4	3,294444
Alfa-insurance	$Exp(0,0000718)$	170552,2	6,744444
Persha	$Exp(0,0000688)$	36177,78	1,138889
Aha-insurance	$Exp(0,0000684)$	1743217	62,09444
PZU-Ukraine	$Exp(0,0000683)$	442853,3	14,37778
The level of initial capital	x=1 500 000		
The period of modeling	1/2 of year		

According to the information represented on the web-site of Ministry of finance of Ukraine, the interest rates hesitated from from 16,25% to 19% per year. In accordance with regulatory requirements for the implementation of insurance activity in Ukraine the minimum authorized capital of the insurance company, engaged in types of insurance other than life insurance, is established in the amount of one million Euros,and for the insurance company, engaged in life insurance, it is equivalent 1.5 million EUR in UAH. The value of invested money in advertising should not exceed 1%. We consider the special case of advertising impact on the activities of companies, namely when $j(\delta) = j_1(\delta) = \sqrt{\delta}$ [12].

Let us turn to the study of the available data. We constructed graphs of the non-ruin probability as a function of the initial capital with fixed proportion of funds for advertising for each company in the case of investment in the riskless part of the financial (B,S) –market precisely in the bank deposit.

Let us consider the first company in the list, Oranta, and fix $\delta = 0,01$. The amount of initial capital varies from 0 to 180 thousand UAH in the Figure 1 and from 100 to 300 thousand UAH in the Figure 2. The blue color illustrates the non-ruin probability model of insurance companies without the investing, the red color is with the investing. We can note with the help of the Figure 1 and the Figure 2 that the model with the investing optimized before the model without the investing. Such small initial capital for the company was taken only with the aim to show the behavior of functions. Taking into account the regulatory requirements it also means that the company according to these models will not go bankrupt with all listed parameters above.

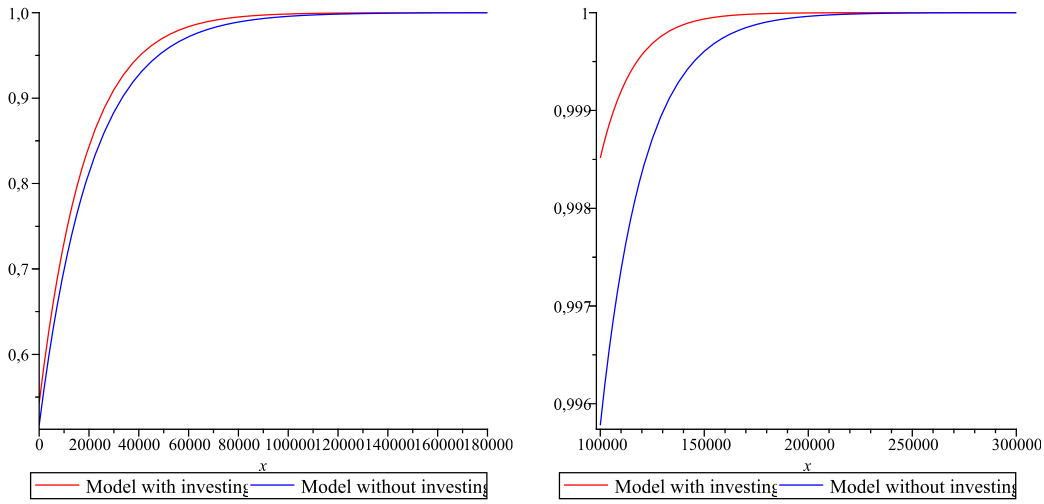

Рис. 1. The probabilities of non-ruin, Oranta

Let us turn to the study of the available data. We constructed graphs of the non-ruin probability as a function of the initial capital with fixed proportion of funds for advertising for each company in the case of investment in the riskless part of the financial (B,S)–market precisely in the bank deposit.

Let us consider the first company in the list, Oranta, and fix $\delta = 0,01$. The amount of initial capital varies from 0 to 180 thousand UAH in the first part of the Figure 1 and from 100 to 300 thousand UAH in the second part of the Figure 1. The blue color illustrates the non-ruin probability model of insurance companies without the investing, the red color is with the investing. We can note with the help of the

Figure 1 that the model with the investing optimized before the model without the investing. Such small initial capital for the company was taken only with the aim to show the behavior of functions. Taking into account the regulatory requirements it also means that the company according to these models will not go bankrupt with all listed parameters above.

The further analysis of model figures by the represented in the table companies allows us to draw the conclusion: the model with the investing in the bank deposit is optimized better than the same model but without the investment. All figures for companies are very similar and show bigger non-ruin probability for the model with investments. In the case of Aha-insurance the model without the investment is very close to the model with the investing which is confirmed by the Figure 2.

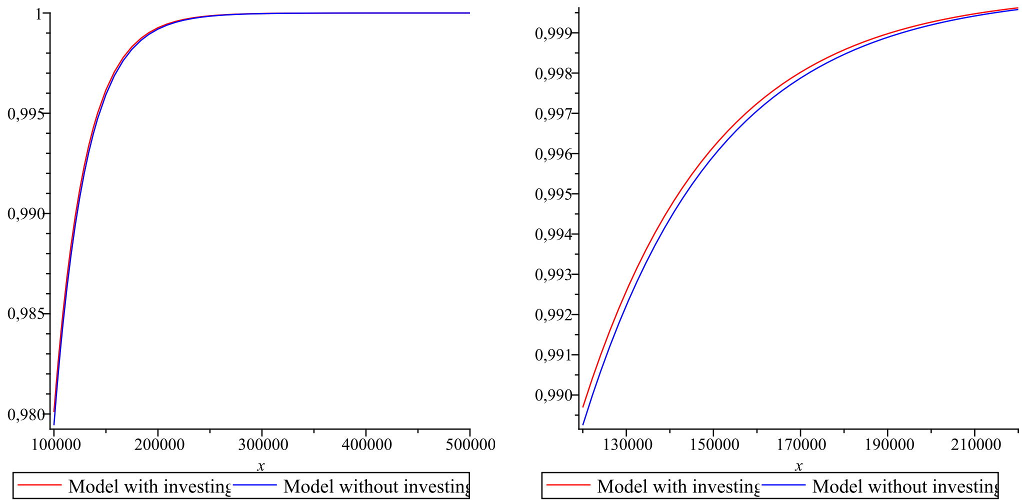

Рис. 2. The probabilities of non-ruin, Aha-insurance

4. Conclusions. There was considered a model of insurance company in the case of investing costs on bank deposit and using advertising in the article. An integro-differential equation for non-ruin probability of the insurance company for this model was derived. The analytic solution of the equation was found in the case when the sizes of insurance claims were described by an exponential distribution. The results have been applied in the analysis of the activities by MHull of 10 Top Insurance companies of Ukraine for the period 01/01/2015 to 06/30/2015. The results for the model of insurance company with investment were compared with the results for the model of insurance company without investment.

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Одержано 05.03.2018