USE OF THE CONFIGURATION INTERACTION METHOD TO DESCRIBE "FINE"-SPLITTING IN THE BOUND TWO-QUARK SYSTEMS

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The screened quasi-relativistic potential is used for describing the spin-orbit splitting in ^3P_J waves in a quark-antiquark system. The Fermi—Breit equation is numerically solved in the configuration interaction approximation. This approximation takes the mixing of wave functions into account up to the fifth order and corrects substantially perturbative calculations. We research the Lorentz nature of the potential. The good quantitative results for $b\overline{b}$ and $c\overline{c}$ quarkonia and the quite acceptable qualitative characteristics for unequal quark masses are obtained.

It seems evident that the quark potential model gives a rather good description of the spin-averaged mass spectrum of hadrons, considered as a system of quarks (see, e.g., [1] and references therein). Authors used usually the nonrelativistic Cornell as well as oscillator potentials or other power-law confinement terms with Coulumb-like one-gluon exchange. In this work, we try to include the second order spin-dependent terms in the two-quark Fermi—Breit equation to this picture for obtaining the spin-orbit splitting.

Our main goal is to clarify some aspects of the interaction potential form in the framework of the configuration interaction (CI) approach [2]. This method does not need the assumption that the coupling constant is to be small, the assumption which is required by the perturbation method. The configuration interaction approach is based on the Galerkin method, and it is the next stage of development of this method.

Let us suggest that the static quark potential has vector and scalar properties of the Lorentz transformation:

$$V_{\mathrm{NR}}(r) = V_v(r) + V_s(r). \tag{1}$$

The index v means that the potential is a 4th component of the operator $\widehat{p}_\mu$, and the index s means that the potential has scalar nature.

Following many authors, we assume the admixture of a vector-scalar screened potential

$$V_v(r) = V_{\mathrm{OGE}} + \varepsilon V_{\mathrm{conf}}, \tag{2}$$

$$V_s(r) = (1-\varepsilon) V_{\mathrm{conf}}, \tag{3}$$

where V_{OGE} is the one gluon exchange (OGE) term, V_{conf} is the confinement part of the potential, where ε is the mixing constant. Here, the Lorentz nature of the one-gluon and confinement potentials is different, the one-gluon potential being totally vector, while the confinement one is a vector-scalar mixture. This choice seems to be reasonable since non-perturbative vertex corrections are important at small q^2. A very interesting rewiew was done in [3] concerning the choice of interaction potential.

Our approach is based on the model of a non-perturbative gluon propagator, which was proposed by Chikovani, Jenkovszky, and Paccanoni (CJP) [4]. In this model, the potential has the form

$$V(r) = \frac{g^2}{6\pi\mu}\left(1 - \exp\left[-\mu r\right]\right) - \\ -\frac{16\pi}{25}\frac{\exp\left[-Mr\right]}{r\ln\left(b + 1/\left(\Lambda r\right)^2\right)}, \tag{4}$$

where Λ is a the constant of renormalization group.

In fact, for accelerating numerical calculations, we use the simple form α_s/r for the one-gluon-exchange term with asymptotic freedom and take

$$\alpha_s(r) = \frac{12\pi}{33-2N}\frac{1}{\ln\left(1/\left(\widetilde{\Lambda} r\right)^2\right)}, \tag{5}$$

where $\widetilde{\Lambda}$ is taken to be equal to 0.14 GeV. The initial value of α_s was defined via calculating the $b\overline{b}$-masses.

So we choose the scalar and vector parts of potential (2), (3) in the form

$$V_{\mathrm{OGE}}(r) = -\frac{\alpha_s}{r}, \tag{6}$$

$$V_{\mathrm{conf}}(r) = \frac{g^2}{6\pi}\frac{\left(1 - e^{-\mu r}\right)}{\mu}. \tag{7}$$

Recently, a screened potential of type (6), (7) was successfully applied to the description of spin-averaged mass-spectra of mesons and baryons [5]. E. Laermann et al. [6] showed that the quark confinement potential goes lower than the linear confinement potential, when **r**, the distance between quarks, becomes larger in studing the loop-diagrams of fermions in the lattice gauge calculation. It reflects that the screening between valence quarks and quark sea enhances. Consequently, it shows the strength of color confinement between valance quarks.

In addition, (see, e.g., [7]) the spin-orbit term has to be of short range, as indicated by quantum chromodynamics (QCD). This condition is evidently satisfied by the CJP potential.

In what follows, we shall use the screened potential (4), which proved to be very good in description of the spin-averaged mass-spectrum of both mesons and baryons as quark systems [4,8].

Let us start with the two-body Fermi—Breit equation. We shall use the nuclear system of units $\hbar = c = 1$, 1 GeV $= \frac{5.068}{1\ \mathrm{Fm}}$. The Hamiltonian of the system has the form:

$$\widehat{H} = \widehat{H}_0 + \widehat{W}, \tag{8}$$

where

$$\widehat{H}_0 = -\frac{1}{2m}\triangle + \left(-\frac{\alpha_s}{r} + \frac{g^2}{6\pi}\frac{(1-e^{-\mu r})}{\mu}\right), \tag{9}$$

m is the reduced mass,

$$\widehat{W} = \widehat{H}_{\mathrm{LS}} + \widehat{H}_{\mathrm{ST}}, \tag{10}$$

where spin-dependent potentials are given in [1]:

- Spin-orbit interaction

$$\widehat{H}_{\mathrm{LS}} = \frac{1}{4m_1^2m_2^2}\frac{1}{r}\left\{\left[((m_1+m_2)^2+2m_1m_2)\mathbf{L}\cdot\mathbf{S}_+ + \right.\right.$$

$$\left. +(m_2^2-m_1^2)\mathbf{L}\cdot\mathbf{S}_-\right]\frac{dV_v}{dr} -$$

$$\left. -\left[\left(m_1^2+m_2^2\right)\mathbf{L}\cdot\mathbf{S}_+ + (m_2^2-m_1^2)\mathbf{L}\cdot\mathbf{S}_-\right]\frac{dV_s}{dr}\right\},$$

$$\mathbf{L} = \mathbf{r}\times\mathbf{p}, \qquad \mathbf{S}_+ \equiv \mathbf{S}_1+\mathbf{S}_2, \qquad \mathbf{S}_- \equiv \mathbf{S}_1-\mathbf{S}_2,$$

$$\mathbf{L}\cdot\mathbf{S} = \frac{1}{2}\left[j\,(j+1) - l\,(l+1) - S\,(S+1)\right]. \tag{11}$$

- Tensor terms

$$\widehat{H}_T = \frac{1}{12m_1m_2}\left[\frac{1}{r}\frac{dV_v}{dr} - \frac{d^2V_v}{dr^2}\right]S_{12},$$

$$S_{12} = \frac{4}{(2l+3)\,(2l-1)}\left[\mathbf{S}^2\cdot\mathbf{L}^2 - \frac{3}{2}\mathbf{L}\cdot\mathbf{S} - 3\,(\mathbf{L}\cdot\mathbf{S})^2\right]. \tag{12}$$

All notations are considered to be familiar and coincide with those that were used in [1].

Lucha and Schoeberl [1] has indicated that the term $\frac{1}{r}\frac{dV_v}{dr}$ causes the serious dimensional trouble because of particle falling on the center. But, in a more consistent approach based on the Dirac equation, we obtained the previous term as $\frac{1}{E-V+mc^2}\frac{1}{r}\frac{dV_v}{dr}$, which will behave like $1/r^2$ for $r \to 0$, and the problem is removed. We show that there are no essential differences between these two results in the matrix elements, which we were able to calculate. Previously, we have studied these questions in [9].

The either possibility to solve this problem is the regularization of the potential [1]. The same idea is based on the use of $V_{\mathrm{reg}} \sim \frac{1}{r+r_0}$ in the meaning of the regularized potential; where r_0 is the free parameter, which has been obtained from the comparison with experimental data. So, for $r \to 0$, the problem of the particle falling to a center is gone.

However, there are more "exact" QCD-motivated potentials, which are based on two-loop back diagrams [10], but their disadvantage is the large number of free parameters.

In addition to the above, the interaction terms presented in the Fermi—Breit Hamiltonian are the $\mathbf{S}_1\cdot\mathbf{S}_2$ (spin-spin) and relativistic correction terms (of p^4-order). Some authors (like [1]) show that these terms are important for calculating mass spectra. Other authors (like [11]) argue that these results are purely constant. In our case, the relativistic correction terms for LS-mass differences will play no significant role.

Now, we consider the Fermi—Breit equation

$$\left(-\frac{1}{2m}\triangle + \left(-\frac{\alpha_s}{r} + \frac{g^2}{6\pi}\frac{(1-e^{-\mu r})}{\mu}\right) + \widehat{W}\right)\Psi(\mathbf{r}) =$$

$$= \widehat{E}\Psi(\mathbf{r}). \tag{13}$$

There are certain difficulties in solving Eq. (13) in the case of spin-orbit-coupling. Potential (10) leads us to the term of $1/r^3$ order. Usually, authors use one of the two possibilities:

a) perturbation method;

b) numerical computation.

Both of these possibilities are inferior: the first one is unacceptable because, in some cases, the "fine" splitting

turns out to be not so "fine" at all, being up to a half contribution to the final mass, the second one needs introducing the cut-off parameter, which is highly undesirable.

Here, we suggest to use the CI approach that was previously very successfully applied in atomic physics [2]. The essence of this approximation is that the total wave function $\Psi(\mathbf{r})$ is expanded in the set of eigenfunctions φ_n of the unperturbed Hamiltonian $\widehat{H}_0$, that is,

$$\Psi(\mathbf{r}) = \sum_n a_n \varphi_n(\mathbf{r}). \tag{14}$$

After substituting (14) into (13) and using the eigenvalue E_n^0, we obtain the system of linear equations for a_n, which have to be truncated for a reasonably large n:

$$\begin{aligned}
&a_1\left(E - E_1^0 - W_{11}\right) - \\
&\qquad -a_2 W_{12} - a_3 W_{13} - \ldots - a_n W_{1n} = 0 \\
&-a_1 W_{21} + a_2\left(E - E_2^0 - \right. \\
&\qquad \left. -W_{22}\right) - a_3 W_{23} - \ldots - a_n W_{2n} = 0 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&-a_1 W_{n1} - a_2 W_{n2} - \\
&\qquad -a_3 W_{n3} - \ldots + a_n\left(E - E_n^0 - W_{nn}\right) = 0,
\end{aligned} \tag{15}$$

where

$$W_{ij} = \langle \varphi_i | \widehat{W} | \varphi_j \rangle. \tag{16}$$

Both the basic functions φ_i and matrix elements W_{ij} are calculated numerically. Nontrivial solutions will be derived by diagonalizing of the matrix for E only if the determinant of this system vanishes. System (15) is called CI. This procedure goes far outside of the perturbation method.

CIA allowed one to increase the precision of calculating energy levels by one order in calculating the atomic structure. In scattering processes, it allowed one to reveal a fine resonance structure in scattering cross-sections due to the formation of autoionizing states. So we expect that its applications will be even more important for strong interaction, where the perturbation method is evidently not correct. The technique of application of CIA is quite complicate, since it needs to handle the matrices of large dimensions.

In this work, we have applied the above-described method for calculating P-wave "fine"-splitting of $b\overline{b}$-, $c\overline{c}$-, and $u\overline{u}$-systems to ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$ levels. In our case, the corresponding operators $\widehat{H}_{\rm LS}$, $\widehat{H}_T$ have the form:

$$\widehat{W}_{\rm LS} = \frac{1}{2m^2}\frac{1}{r}\left[3\frac{\alpha_s}{r^2} + (4\varepsilon - 1)\cdot\frac{g^2}{6\pi}e^{-\mu r}\right]\mathbf{L}\cdot\mathbf{S}, \tag{17}$$

$$\widehat{W}_T = \frac{1}{12m^2}\left[3\frac{\alpha_s}{r^3} + \left(\frac{1}{r} + \mu\right)\varepsilon\cdot\frac{g^2}{6\pi}e^{-\mu r}\right]S_{12}. \tag{18}$$

It is important that all parameters except ε are taken from [4,8], where the excellent description of bottomonium and charmonium spectra was obtained. Moreover, as shown in [8], the same parameters give good ρ-meson trajectories. Actually, the values $g^2/6\pi = 0.3$ GeV^2, $\mu = 0.054$ GeV were taken in accordance with QCD. The only adjustable parameter was ε. The experimental values are taken from [12]. As mentioned above, all calculations were carried out numerically. A special code was constructed for this purpose. The calculations were extended to the fifth order in (14) (see Tables I — VIII), i.e., until the differences between the results did not go below the several MeV level.

It is interesting to note that, in the same approach with the same mixing parameter ε, very good results were obtained for spin-spin mass differences in [13].

Let us make the following conclusions:

1. The results for heavy quarkonium are quite good for values $\varepsilon = 0.3 \div 0.45$, which coincides with the results obtained in [14]. For light quarkonium, the results are worse, which means that relativistic effects have to be taken into account more carefully. We believe that the difference between ε in case of $b\overline{b}$ and $u\overline{u}$ systems exactly reflects this fact. The results introduced in Tables VII—VIII play the role of a prediction, as the experimental data concerning to system $s\overline{u}$ are contradictory.

2. As follows from (17) at $\varepsilon = 0.25$ $(4\varepsilon - 1 = 0)$, the contribution of confinement vanishes totally. May be, exactly this circumstance was the reason that some authors stated the pure one-gluon character of LS-splitting.

3. The first column in tables corresponds to the pure perturbative approach. It is clearly seen that this approach gives only a rough qualitative estimate, but the results improve drastically with switch on the

CIA expansion. Very convenient for the fine-splitting characteristic is the coefficient

$$R = \frac{M\left({}^3P_2\right) - M\left({}^3P_1\right)}{M\left({}^3P_1\right) - M\left({}^3P_0\right)}. \qquad (19)$$

As cited in [1], the experimental values of this parameter are $R = 0.66$ for $b\overline{b}$, $R = 0.48$ for $c\overline{c}$, and $R = 0.21$ for $u\overline{u}$. It is interesting to note that firstly this parameter was introduced in atomic physics [15], where its value was established for atoms with two external electrons (like Mg) as being $R = 0.5$. Since the fine mass splitting in atoms is totally due to one-photon exchange, this value gives good indication as to the nature of the Lorentz character of the $q\overline{q}$-potential. The value of ε indicates that the confinement part of the potential has a mixing character. This conclusion does not contradict other authors [16]. As our calculations show, the one-gluon term is totally a vector, while the nonperturbative confinement term is a mixture of two contributions (75—80% has a scalar character, while 25—30% has a vector character).

T a b l e I. $b\overline{b}$-system, $\alpha_s = 0.3$, $\varepsilon = 0.5$, $m_b = 5.05$ GeV

State	ΔM_{TH}	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	Order 4 (MeV)	ΔM_{exp}[12] (MeV)
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	21.32	21.23	21.20	21.19	21.3 ± 1.3
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	24.25	25.51	26.08	26.44	32.1 ± 2
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	45.57	46.74	47.29	47.63	53.4 ± 1.9
R		0.88	0.83	0.81	0.8	0.66
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$2^3P_2 - 2^3P_1$	–	17.72	17.61	17.58	13.3 ± 0.9
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_1 - 2^3P_0$	–	19.09	20.37	21	23.1 ± 1.1
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_2 - 2^3P_0$	–	36.81	37.98	38.58	36.4 ± 1.0
R		–	0.93	0.86	0.84	0.57

T a b l e II. $b\overline{b}$-system, $\alpha_s = 0.3$, $\varepsilon = 0.45$, $m_b = 5.05$ GeV

State	ΔM_{TH}	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	Order 4 (MeV)	ΔM_{exp}[12] (MeV)
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	20.00	19.92	19.89	19.87	21.3 ± 1.3
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	23.37	24.57	25.13	25.48	32.1 ± 2
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	43.37	44.49	45.02	45.36	53.4 ± 1.9
R		0.85	0.81	0.79	0.78	0.66
e $\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b1}\left(0^+\left(1^{++}\right)\right)$	$2^3P_2 - 2^3P_1$	–	16.68	16.76	16.65	13.3 ± 0.9
$\chi_{b1}\left(0^+\left(1^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_1 - 2^3P_0$	–	18.49	19.73	20.34	23.1 ± 1.1
$\chi_{b2}\left(0^+\left(2^{++}\right)\right) - \chi_{b0}\left(0^+\left(0^{++}\right)\right)$	$2^3P_2 - 2^3P_0$	–	35.25	36.38	36.95	36.4 ± 1.0
R		–	0.90	0.85	0.82	0.57

T a b l e III. $c\overline{c}$-system, $\alpha_s = 0.386$, $\varepsilon = 0.3$, $m_c = 1.675$ GeV

State	ΔM_{TH}	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	Order 4 (MeV)	Order 5 (MeV)	Order 6 (MeV)	ΔM_{exp}[12] (MeV)
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	52.20	51.66	51.49	51.41	51.37	51.33	45.64 ± 0.25
$\chi_{c1}\left(0^+\left(1^{++}\right)\right) - \chi_{c0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	69.31	77.27	81.55	84.51	86.72	88.48	95.43 ± 1.12
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c2}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	121.51	128.93	133.04	135.93	138.09	139.82	141.07 ± 1.13
R		0.75	0.66	0.63	0.61	0.59	0.58	0.48

T a b l e IV. $c\overline{c}$-system, $\alpha_s = 0.386$, $\varepsilon = 0.4$, $m_c = 1.675$ GeV

State	ΔM_{TH}	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	Order 4 (MeV)	ΔM_{exp}[12] (MeV)
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c1}\left(0^+\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	66.81	66.17	65.99	65.92	45.64 ± 0.25
$\chi_{c1}\left(0^+\left(1^{++}\right)\right) - \chi_{c0}\left(0^+\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	79.36	88.57	93.45	96.75	95.43 ± 1.12
$\chi_{c2}\left(0^+\left(2^{++}\right)\right) - \chi_{c2}\left(0^+\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	146.17	154.74	159.45	162.67	141.07 ± 1.13
R		0.84	0.75	0.70	0.68	0.48

T a b l e V. $u\overline{u}$-system, $\alpha_s = 0.52$, $\varepsilon = 0.14$, $m_u = 0.33$ GeV

State	$\Delta M_{\rm TH}$	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	$\Delta M_{\rm exp}$[12] (MeV)
$a_2\left(1^-\left(2^{++}\right)\right) - a_1\left(1^-\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	15.34	10.26	8.57	88.2 ± 40.7
$a_1\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	241.19	316.91	374.09	246.5 ± 40.9
$a_2\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	256.53	327.17	382.66	334.6 ± 1.6
R		0.063	0.032	0.023	0.357

T a b l e VI. $u\overline{u}$-system, $\alpha_s = 0.52$, $\varepsilon = 0.145$, $m_u = 0.33$ GeV

State	$\Delta M_{\rm TH}$	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	$\Delta M_{\rm exp}$[12] (MeV)
$a_2\left(1^-\left(2^{++}\right)\right) - a_1\left(1^-\left(1^{++}\right)\right)$	$1^3P_2 - 1^3P_1$	24.41	19.15	17.45	88.2 ± 40.7
$a_1\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_1 - 1^3P_0$	247.52	324.96	383.24	246.5 ± 40.9
$a_2\left(1^-\left(2^{++}\right)\right) - a_0\left(1^-\left(0^{++}\right)\right)$	$1^3P_2 - 1^3P_0$	271.92	344.10	400.69	334.6 ± 1.6
R		0.098	0.059	0.046	0.357

T a b l e VII. $s\overline{u}$-system, $\alpha_s = 0.421$, $\varepsilon = 0.1875$, $m_s = 0.5$ GeV, $m_u = 0.33$ GeV

State	$\Delta M_{\rm TH}$	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	Order 4 (MeV)	$\Delta M_{\rm exp}$[12] (MeV)
?	$1^3P_2 - 1^3P_1$	43.16	40.23	39.29	38.87	–
?	$1^3P_1 - 1^3P_0$	187.78	230.62	259.86	282.63	–
?	$1^3P_2 - 1^3P_0$	230.94	270.85	299.15	321.50	–
R		0.23	0.17	0.15	0.14	–

T a b l e VIII. $s\overline{u}$-system, $\alpha_s = 0.421$, $\varepsilon = 0.2$, $m_s = 0.5$ GeV, $m_u = 0.33$ GeV

State	$\Delta M_{\rm TH}$	Order 1 (MeV)	Order 2 (MeV)	Order 3 (MeV)	$\Delta M_{\rm exp}$[12] (MeV)
?	$1^3P_2 - 1^3P_1$	60.13	56.95	56.00	–
?	$1^3P_1 - 1^3P_0$	199.43	244.92	331.74	–
?	$1^3P_2 - 1^3P_0$	259.56	301.84	387.75	–
R		0.30	0.23	0.17	–

Confinement gives 20—25% plus to the results obtained with using only the one gluon exchange term for the description of the LS-, ST-interactions. Finally, we have obtained the description of fine-effects for some quark systems with a precision of 80—90%.

These conclusions are consistent with the qualitative estimation of Lucha and Schoeberl [1]. Their results approximately confirm the conclusion of Franzini [11] in the part that the confinement is a pure scalar. We showed that the results are very sensitive to the exact value of ε, and a small change of ε can destroy the agreement with the experimental data, which actually occurred in the case of Franzini.

We believe that exactly this feature will be successfully exploited in the near future not only for the hadronic structure, but also in explaining the $\pi\pi$- or πN-resonances.

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НАКЛАДАННЯ КОНФІГУРАЦІЙ
ДО ОПИСУ "ТОНКОГО" РОЗЩЕПЛЕННЯ
РІВНІВ У ДВОКВАРКОВИХ СИСТЕМАХ

В. Лендьел, *В. Рубіш, О. Шпеник*

Р е з ю м е

Використано екранований квазірелятивістський потенціал для опису спін-орбітального розщеплення ^3P_J – рівнів у кварк-антикваркових системах. Рівняння Фермі — Брейта розв'язано чисельно, методом накладання конфігурацій. У розкладі використано хвильові функції до п'ятого наближення включно, що поліпшує непертурбативні розрахунки. Досліджується лоренц-структура потенціалу. Отримано хороші результати для $b\bar{b}$- і $c\bar{c}$-кварконіїв і задовільні для систем з різними масами.

ПРИМЕНЕНИЕ МЕТОДА
НАЛОЖЕНИЯ КОНФИГУРАЦИЙ
ДЛЯ ОПИСАНИЯ "ТОНКОГО" РАСЩЕПЛЕНИЯ
УРОВНЕЙ В ДВУКВАРКОВЫХ СИСТЕМАХ

В. Лендьел, *В. Рубиш, А. Шпеник*

Р е з ю м е

Используется экранированный квазирелятивистский потенциал для описания спин-орбитального расщепления ^3P_J-уровней в кварк-антикварковых системах. Уравнение Ферми — Брейта решается численно методом наложения конфигураций. В разложении используются волновые функции вплоть до пятого приближения, что улучшает пертурбативные расчеты. Исследуется лоренц-структура потенциала. Получены хорошие результаты для $b\bar{b}$- и $c\bar{c}$-кваркониев и удовлетворительные для систем с разными массами.