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Description of mass spectrum of B_c-meson family

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Abstract

In the framework of potential models for heavy quarkonium, the mass spectrum for the system $(\bar{b}c)$ is considered. In particular, the spin-dependent splittings of S and P energy levels caused by spin-spin, spin-orbital and tensor interactions in the framework of the Breit-Fermi quasi-relativistic approximation with the screened quark interaction potential are calculated. The dependence of the energy splitting on the parameters of the interaction potential between the quarks is investigated. The calculated energy levels of the system $\bar{b}c$ can be used for experimental search for B_c mesons.

1 Introduction

Among the heavy quarkonia, the $\bar{b}c$ (B_c-meson family) consisting of two heavy quarks of different masses – a system with open flavors of quarks — occupies a special position. Unlike experimentally [1] and theoretically [2, 3, 4, 5, 6] sufficiently well-described the charmonium $c\bar{c}$ and bottomonium $b\bar{b}$ families with the hidden flavors, the properties of heavy quarkonium $\bar{b}c$ due to the specific mechanisms of formation and decay remain poorly understood [7].

From the point of spectroscopy view, the system $\bar{b}c$ occupies an intermediate position between the charmonium $c\bar{c}$ and bottomonium $b\bar{b}$ in terms of the mass of levels and average distances between heavy quarks. In contrast to $c\bar{c}$ and $b\bar{b}$, B_c-mesons (because they carry flavor) cannot annihilate into gluons and are consequently more stable with widths less than a hundred keV (see [8]). The excited B_c states lying below BD (and BD^* or B^*D) threshold can only undergo E1 or M1 transitions or hadronic cascades to the $^1S_0$ ground state B_c, which then decays weakly. Therefore, the widths of the electromagnetic radiative and hadronic transitions of the given excited state to other levels will be its total width. As a result of this, the total widths of the excited levels of the $\bar{b}c$ system are two orders of magnitude smaller than the total widths of the excited levels of charmonium and bottomonium, for which annihilation channels are significant. This results in the rich spectroscopy of narrow radial and orbital excitations below BD threshold

which are more stable than their charmonium and bottomonium analogs: there are two sets of S-wave states, as many as two P-wave multiplets ($1P$ state and some or all of the $2P$ one) and one D-wave multiplet below BD threshold. As well, the F-wave multiplet is sufficiently close to threshold that they may also be relatively narrow due to angular momentum barrier suppression of the Zweig allowed strong decays. Therefore, the methods used to study charmonium and bottomonium (non-relativistic potential models or QCD sum rules) can be extended to the study of the properties of B_c-mesons. The description of their spectrum can be as a test of self-consistency for potential models, the parameters of which (for example, masses of quarks, values of the constant of the strong interaction, potential parameters) were recorded by fitting the spectroscopic data of charmonium and bottomonium.

Thus, on the one hand, the theoretical methods used in the physics of heavy quarks can accurately determine the spectroscopic characteristics of the $\bar{b}c$-system to conduct a purposeful experimental search for this heavy quarkonium. On the other hand, the measurement of spectroscopic data in the B_c-mesons family will improve the methods and ways of finding the fundamental parameters of the Standard Model both in the physics of B_c-mesons and in other areas of heavy quark physics.

However, it should be noted that despite the fact that experimental techniques on existing detectors allow us to distinguish predicted events with the creation and decay of B_c-mesons, only the existence of the lowest singlet state $\bar{b}c$-system has been reliably experimentally established – $B_s^+(0^-)$ [1]: its mass $M(B_s^+(0^-)) = 6.2749 \pm 0.0008$ GeV, and the lifetime is $0.507 \pm 0.009) \cdot 10^{-12}$ s. There is a contender for the first excited state – $M(B_c(2S)^{\pm}) = 6.842 \pm 0.004$ GeV, but its quantum numbers are not precisely established yet [1].

Preliminary theoretical estimates of the masses of bound states of the heavy quarks system ($\bar{b}c$) were made in the papers [6, 9], devoted to the description of the properties of charmonium ($c\bar{c}$) and bottomonium $b\bar{b}$), as well as in [10]. In the framework of the potential approach and QCD sum rules, the more thorough analysis of the spectroscopy of B_c-mesons was carried out in [7, 8, 11, 12].

Thus, the aim of this work is to study the energy spectrum of the $\bar{b}c$-system within the framework of potential models of heavy quarkonium with a screened quark interaction potential. In the quasi-relativistic Brait-Fermi approximation, the spin-dependent splittings of energy S- and P-levels due to spin-spin, spin-orbital, and tensor interactions are investigated. The dependence of the splitting of energy levels on the parameters of the interaction potential between the quarks is also analyzed.

2 Description of the splitting of the levels of $\bar{b}c$-quarkonium caused by spins of quarks

2.1 Potential quark-antiquark interaction

In order to describe the mass spectrum $\bar{b}c$-system within the potential approach, it is better to use potentials that describe the mass spectra of both $c\bar{c}$ and

$b\bar{b}$ systems well, that is, potentials whose parameters do not depend on the flavors of quarks being a part of heavy quarkonium.

The complete effective quark-antiquark interaction following [5, 13] will be presented as a combination of the perturbative one-gluon exchange potential (Coulomb-like interaction) $V_{Coul}(r) = -\xi/r$ (where $\xi = 4/3\alpha_s$, α_s is the running coupling constant of QCD) with long-range scalar $S_{conf}(r) = (1-\lambda)v(r)$ and vector $V_{conf}(r) = \lambda v(r)$ confining potentials:

$$V(r) = V_{Coul}(r) + V_{conf}(r) = -\xi/r + \lambda v(r), \tag{1}$$

$$S(r) = S_{conf}(r) = (1-\lambda)v(r). \tag{2}$$

Here, λ is the coefficient of mixing between the vector and scalar confining potentials ($0 \leqslant \lambda \leqslant 1$). The value of α_s in the one-loop approximation on the momentum scale p^2 is determined by the expression

$$\alpha_s(p^2) = \frac{12\pi}{(33-2N_f)\ln(p^2/\Lambda^2)}, \tag{3}$$

where N_f is the number of quark flavors, and $\Lambda = 360$ MeV is the QCD parameter. Using the expression for the kinetic energy $\langle T\rangle = \langle p^2\rangle/(2\mu)$, where μ is the reduced mass of heavy quarks c and b, we obtain

$$\alpha_s(p^2) = \frac{12\pi}{(33-2N_f)\ln\left(2\langle T\rangle\mu/\Lambda^2\right)}. \tag{4}$$

As it is shown in [7, 11, 12], the kinetic energy of heavy quarks is almost constant, which does not depend on the flavors of heavy quarks and quantum numbers of the excited level at which they are in the system of heavy quarkonium. Therefore, the value of the effective constant α_s is mainly determined by the reduced mass of the heavy quarkonium and can be considered approximately constant in each family of mesons and such that it changes only during the transition from one family to another.

For practical calculations, it is necessary to specify the form of the confinement part of the interquark potential $v(r)$. The most commonly used within the potential models is the Cornell potential [6], which provides a satisfactory description of the spectrum of masses of heavy quarkonia [6, 8, 12] and heavy-light mesons [13, 14]. However, in [15] (in lattice QCD calculations) the transition of a static quark-antiquark string into a static meson-antimeson system i.e. the breaking of colored string between quarks was studied. It was shown that it can occur at interquark distances of the order of 1.25 Fm and lead to polarization of the QCD vacuum and screened by color forces. Observations of the nonlinear hadronic Regge trajectory are also considered to confirm the screened interquark potential [16]. In addition, quite rapid crossover from a linear rising to a flat potential is well established in SU(2) Yang-Mills theories [17].

The screened potential was successfully applied to describe the spin-averaged spectrum of masses of heavy mesons and baryons [18, 19]. In [20], a detailed description of the nucleon-nucleon interaction is obtained using the screened potential. In the papers [5, 21, 22, 23, 24], spin-spin splitting of energy levels, lepton

widths and radiation decays of heavy quarkonium were calculated by means of the screened potential. From the results of these works it follows that at the same quality of the calculated spectrum of masses, between the models [21] with screened potential and models [12, 14], which use nonscreened confinement, important differences arise. The most characteristics are: the screened potential leads to a limited spectrum of quark-antiquark bound states (a finite number of bound states) and the pattern of energy differences between the highly excited states. In particular, the predicted [22] number of states is in almost perfect agreement with the experimentally observed states, a fact that might shed new light on the so-called missing resonance problem (see Ref. [25]).

Given the above, in our calculations, we use the simple form of screened potential, proposed in [5, 22]:

$$V(r) = -\frac{\xi}{r} + \lambda\left(\frac{g^2}{6\pi}\frac{(1-e^{-\sigma r})}{\sigma} + V_0\right), \tag{5}$$

$$S(r) = (1-\lambda)\left(\frac{g^2}{6\pi}\frac{(1-e^{-\sigma r})}{\sigma} + V_0\right). \tag{6}$$

The parameters of the screened potential are equal to: $g^2/(6\pi) = 0,3$ GeV^2, $\sigma = 0,054$ GeV; V_0 is the constant of the additive shift of the binding energy.

2.2 Generalized Breit-Fermi Hamiltonian

In order to take into account the spin-spin, spin-orbital and tensor interactions which lead to the splitting of $(n_r\,{}^{2S+1}L)$-levels (where n_r is a radial quantum number, L is the orbital moment and S – total spin moment of two quarks), following [5, 26] we use the Breit-Fermi Hamiltonian (in the nuclear system of units $\hbar = c = 1$, $1\,\text{GeV} = 5.068\,\text{Fm}^{-1}$)

$$\hat{H} = \frac{\hat{\vec{p}}^2}{2\mu} + V(r) + S(r) + \hat{H}_{LS} + \hat{H}_{SS} + \hat{H}_T, \tag{7}$$

where μ is the reduced mass of heavy quarks c and b. The spin-dependent terms are of the form:

$$\hat{H}_{LS} = \frac{1}{4m_c^2 m_b^2}\frac{1}{r}\left\{\left[((m_c+m_b)^2 + 2m_c m_b)\,\vec{L}\vec{S}_+ + (m_b^2 - m_c^2)\,\vec{L}\vec{S}_-\right]\frac{dV(r)}{dr} - \right.$$
$$\left. - \left[(m_c^2+m_b^2)\,\vec{L}\vec{S}_+ + (m_b^2 - m_c^2)\,\vec{L}\vec{S}_-\right]\frac{dS(r)}{dr}\right\} \tag{8}$$

is the spin-orbit interaction, where $\vec{S}_+ \equiv \vec{S}_c + \vec{S}_b$, $\vec{S}_- \equiv \vec{S}_c - \vec{S}_b$,

$$(\vec{L}\vec{S}) = \frac{1}{2}[j(j+1) - l(l+1) - S(S+1)], \tag{9}$$

$$\hat{H}_{SS} = \frac{2}{3m_c m_b}\triangle V(r)\cdot\vec{S}_c\vec{S}_b \tag{10}$$

is the spin-spin interaction, where

$$\vec{S}_c\vec{S}_b = \frac{1}{2}S(S+1) - \frac{3}{4} = \begin{cases} -\dfrac{3}{4}, & S=0, \\ +\dfrac{1}{4}, & S=1, \end{cases} \tag{11}$$

$$\hat{H}_T = \frac{1}{12\, m_c m_b}\left(\frac{1}{r}\frac{dV(r)}{dr} - \frac{d^2V(r)}{dr^2}\right)S_{12}, \tag{12}$$

is the tensor interaction, where

$$S_{12} = \frac{4}{(2l+3)(2l-1)}\left[\vec{L}^2\vec{S}^2 - \frac{3}{2}\,\vec{L}\vec{S} - 3(\vec{L}\vec{S})^2\right]. \tag{13}$$

From the given formulas it follows that the spin-orbit interaction is the most sensitive to the Lorentz nature of the potential of interquark interaction. Only it contains additive contributions from both scalar ($-S'(r)$) and vector ($V'(r)$) potentials, and they come with opposite signs and partially compensate each other (see (8)). Thus, information on both position and fine-splitting of levels can already reveal the role of each of the potentials $S(r)$ and $V(r)$ separately.

In the system of two interacting quarks, the spin-spin interaction leads (in the LS coupling scheme) to the splitting of the level with $L=0$ into two sublevels ${}^3S_1$ and ${}^1S_0$ which correspond to the total spin of two quarks being equal to 1 and 0. For $L=1$, similarly, we obtain the singlet ${}^1P_1$ state corresponding to the total spin 0 and the triplet ${}^3P_J$ state (J is the total moment of the quark-antiquark system) with total spin 1. In turn, the ${}^3P_J$ level splits into three sublevels due to the spin-orbit interaction: ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$. The contribution to the splitting value of the ${}^3P_J$ level is also given by the tensor interaction, while contribution of states with $L=0$ or $S=0$ is zero. In systems consisting of quarks of unequal masses ($m_q \neq m_{\bar{q}}$), an additional contribution to the magnitude of splitting between the ${}^3P_J$ and ${}^1P_1$ levels is defined by the term in the spin-orbit interaction (see (8)), proportional to $\vec{L}\vec{S}_-$. In addition, it leads to mixing of states with the same angular momentum L but different total spins of the quark-antiquark system S (for example between ${}^3P_1$ and ${}^1P_1$ states or between ${}^3D_2$ and ${}^1D_2$ states) because in this case, the charge conjugation parity is no longer a good quantum number. Thus, the P state with total moment $J=1$ is a linear combination of the states ${}^3P_1$ and ${}^1P_1$:

$$\begin{aligned} P' &= {}^1P_1\cos\theta + {}^3P_1\sin\theta, \\ P &= -{}^1P_1\sin\theta + {}^3P_1\cos\theta, \end{aligned} \tag{14}$$

where θ is the mixing angle. Some authors [7, 12] prefer to use the jj coupling scheme, mixing states with different values of J_c ($\vec{J}_c = \vec{L} + \vec{S}_c$) for a given total moment J ($\vec{J} = \vec{J}_c + \vec{S}_b$). But since the operators (8)-(13) are written in the LS coupling, we use the notation of the equation (14). It turns out that the radiation transitions E1 are particularly sensitive to the ${}^3P_1$ $-{}^1P_1$ mixing angle. Since the

definition of the mixing angles is ambiguous, and sometimes different models give radically different results, measurement of radiation transitions can be a criterion for selecting models.

Thus, the Hamiltonian (7) we use to calculate the magnitude of the fine and hyperfine splitting of S and P levels of B_c-mesons with potentials (5), (6) takes the form:

$$\hat{H} = \hat{H}_0 + \hat{W}, \tag{15}$$

where

$$\hat{H}_0 = -\frac{1}{2\mu}\Delta + \left(-\frac{4}{3}\frac{\alpha_s}{r} + \frac{g^2}{6\pi}\frac{(1-e^{-\sigma r})}{\sigma}\right), \tag{16}$$

$$\hat{W} = \hat{H}_{LS} + \hat{H}_{SS} + \hat{H}_T, \tag{17}$$

and the operators $\hat{H}_{SS}$, $\hat{H}_{LS}$ and $\hat{H}_T$ have the form:

$$\hat{H}_{LS} = \frac{1}{4\,m_c^2 m_b^2}\frac{1}{r}\left\{\left[((m_c+m_b)^2 + 2m_c m_b)\,\vec{L}\vec{S}_+ + (m_b^2 - m_c^2)\,\vec{L}\vec{S}_-\right]\times\right.$$
$$\left.\times\left(\frac{4}{3}\frac{\alpha_s}{r^2} + \lambda\frac{g^2}{6\pi}e^{-\sigma r}\right) - \left[(m_c^2+m_b^2)\,\vec{L}\vec{S}_+ + (m_b^2-m_c^2)\,\vec{L}\vec{S}_-\right](1-\lambda)\frac{g^2}{6\pi}e^{-\sigma r}\right\}, \tag{18}$$

$$\hat{H}_{SS} = \frac{2}{3\,m_c m_b}\left[\frac{16}{3}\pi\alpha_s\delta(\vec{r}) + \lambda\frac{g^2}{6\pi}\left(\frac{2}{r} - \sigma\right)e^{-\sigma r}\right]\vec{S}_c\vec{S}_b, \tag{19}$$

$$\hat{H}_T = \frac{1}{12\,m_c m_b}\left[3\frac{\alpha_s}{r^3} + \lambda\left(\frac{1}{r} + \sigma\right)\frac{g^2}{6\pi}e^{-\sigma r}\right]S_{12}. \tag{20}$$

3 Calculation of splitting of S and P levels of $\bar{b}c$ quarkonia

In order to calculate the spin splitting of the S and P levels of $\bar{b}c$ quarkonium, let us consider the Schrödinger equation with the Hamiltonian (15)–(17):

$$\left(\hat{H}_0 + \hat{W}\right)\Psi(\vec{r}) = E\Psi(\vec{r}). \tag{21}$$

Let us represent the wave function $\Psi(\vec{r})$ expanded in the complete orthonormal set of eigenfunctions φ_n of the unperturbed Hamiltonian $\hat{H}_0$

$$\Psi(\vec{r}) = \sum_n a_n\varphi_n(\vec{r}), \quad \hat{H}_0\varphi_n(\vec{r}) = E_n^0\varphi_n(\vec{r}). \tag{22}$$

After substituting (22) into (21) and using the eigenvalue E_n^0, we obtain the system of linear algebraic equations for the coefficients of expansion a_n which have to be truncated for a reasonably large n:

$$\left.\begin{array}{c} a_1(E - E_1^0 - W_{11}) - a_2W_{12} - ... - a_nW_{1n} = 0 \\ -a_1W_{21} + a_2(E - E_2^0 - W_{22}) - ... - a_nW_{2n} = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -a_1W_{n1} - a_2W_{n2} - ... + a_n(E - E_n^0 - W_{nn}) = 0 \end{array}\right\} \tag{23}$$

where

$$W_{ij} = \langle \varphi_i | \hat{W} | \varphi_j \rangle.$$

Nontrivial solutions are derived by diagonalizing the matrix for E only if the determinant of this system is equal to zero

$$\begin{vmatrix} E + E_1^0 + W_{11} & W_{12} & \cdots & W_{1n} \\ W_{21} & E + E_2^0 + W_{22} & \cdots & W_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ W_{n1} & W_{n2} & \cdots & E + E_n^0 + W_{nn} \end{vmatrix} = 0. \qquad (24)$$

Both the basic functions φ_i and matrix elements W_{ij} are calculated numerically. The calculations take into account 6 configuration states in the expansion (22).

The preliminary results of calculations have shown (as well as results are presented in [12]) that the mixing angle θ between $^3P_1$ and $^1P_1$ states is small $\sim 2^\circ$, the P' state is almost pure $^1P_1$ one, and P state is a almost pure $^3P_1$ one (see formula (14)). Therefore, in what follows, in the tables 2 and 2 for the mixed states P' and P, we use the spectroscopic notation of the corresponding pure states $^1P_1$ and $^3P_1$.

When calculating the matrix elements of the spin-spin interaction, the integration of the first term in (19) originating from the Coulomb-like one-gluon exchange potential and containing the $\delta(\vec{r})$ function leads to $|\Psi(0)|^2$. Since $\Psi_{nP}(0) = 0$, the contribution from the Coulomb-like potential to the splitting of the P states is zero. Thus, in the splitting $^3P_J - ^1P_1$, which is due to the spin-spin interaction, only the vector part of the confinement screened potential is contributed. The known experimental value of the splitting $^3P_J - ^1P_1$ would make it possible to establish more accurately the relationship between the vector and scalar parts of the confinement potential.

As already mentioned in the Introduction, at present, only the mass of the main pseudoscalar $1^1S_0(0^-)$ state of the $\bar{b}c$ system has been experimentally established (see [1]), so in this paper, we have obtained the masses of other states being predictable and compared with the results on lattice calculations [27] as well as calculated in other approaches [8].

It is important be noted that the main parameters of potentials (5) (6) and of b and c quarks mass are taken from [5, 18] where the excellent description of bottomonium and charmonium spectra was obtained: $g^2/(6\pi) = 0.3$ GeV^2, $\sigma = 0.054$ GeV, $m_b = 5.05$ GeV, $m_c = 1.675$ GeV.

The results of calculations of the splitting of S and P states of the B_c-mesons family due to spin-spin, spin-orbital and tensor interactions are presented in table 2 (masses are given in GeV, and $\alpha_s = 0.343$, $V_0 = -0.770$ GeV, $\lambda = 0.3$).

The calculations of the B_c mass spectrum were performed with the coupling constant α_s both running during the transition from the ground state to the highly excited states and constant one equal to $\alpha_s = 0.343$ (see (4)) for all states. It was found that the difference between the obtained spectra is insignificant. Thus, we can conclude that the coupling constant α_s within the B_c-mesons family is varying very weakly and can be considered approximately constant. This, in turn, indicates that, as in the case of bottomonium and charmonium [7, 11, 12], the

Table 1: The B_c mass spectrum ($*$ – our results).

State	$*$	[8]	[27]
1^1S_0	6.200	6.271	6.280±190
1^3S_1	6.329	6.338	6.321±20
2^1S_0	6.918	6.855	6.960±80
2^3S_1	6.989	6.887	6.990±80
3^1S_0	7.389	7.250	
3^3S_1	7.444	7.272	
1^3P_0	6.713	6.706	6.727±30
1^3P_1	6.775	6.741	6.743±30
1^3P_2	6.814	6.768	6.783±30
1^1P_1	6.793	6.750	6.765±30
2^3P_0	7.216	7.122	
2^3P_1	7.265	7.145	
2^3P_2	7.299	7.164	
2^1P_1	7.282	7.150	
3^3P_0	7.612		
3^3P_1	7.655		
3^3P_2	7.686		
3^1P_1	7.607		

kinetic energy of heavy quarks in the B_c-mesons family is practically constant and does not depend on the quantum numbers of the their excited states.

Table 2 shows that our results, within the accuracy of potential models, are consistent with the results of [8] and [27]. However, there are some differences. In particular, as follows from the table 2, the splitting of states due to spin-spin interaction (between $^3S_1$ and $^1S_0$) in our model is approximately 2 times greater than the corresponding splitting obtained in [8, 27]. Our predictions of the value of the splitting of the states in the triplet ^3P_J due to spin-orbital and tensor interactions are 1.7 times greater than the corresponding results of [8]. This can be explained as follows: in our model, the spin-orbital (18), spin-spin (19), and tensor (20) interactions contain contributions from both the vector Coulomb-like potential and the vector confinement potential (see (5)), while the model [8] considers a purely scalar confinement. These additional contributions from the vector confinement potential lead also to an increase in the splitting of the states.

Summary

In the framework of potential models of heavy quarkonium, $\bar{b}c$ mass spectrum (B_c-mesons family) is investigated. In particular, in the quasi-relativistic Breit-Fermi approximation, the spin splittings of the S and P energy levels caused by the spin-spin, spin-orbital, and tensor interactions with a screened interaction potential are calculated.

Table 2: The splitting $\triangle E$ (in GeV) S and P states of the B_c-mesons family ($*$ – our results).

$n_rL_J - n_rL_{J'}$	$\triangle E$			$n_rL_J - n_rL_{J'}$	$\triangle E$		
	$*$	[8]	[27]		$*$	[8]	[27]
$1^3S_1 - 1^1S_0$	0.128	0.067	0.041	$1^3P_2 - 1^3P_1$	0.040	0.027	0.040
$2^3S_1 - 2^1S_0$	0.071	0.032	0.030	$1^3P_2 - 1^3P_0$	0.101	0.062	0.056
$3^3S_1 - 3^1S_0$	0.055	0.022		$1^3P_1 - 1^3P_0$	0.061	0.035	0.016
				$1^1P_1 - 1^3P_1$	0.019	0.009	0.022

$n_rL_J - n_rL_{J'}$	$\triangle E$		$n_rL_J - n_rL_{J'}$	$\triangle E$
	$*$	[8]		$*$
$2^3P_2 - 2^3P_1$	0.034	0.019	$3^3P_2 - 3^3P_1$	0.031
$23P_2 - 2^3P_0$	0.083	0.042	$3^3P_2 - 3^3P_0$	0.074
$2^3P_1 - 2^3P_0$	0.049	0.023	$3^3P_1 - 3^3P_0$	0.042
$2^1P_1 - 2^3P_1$	0.018	0.005	$3^1P_1 - 3^3P_1$	0.017

It is established that the energy states and their fine and hyperfine structure significantly depend on the coefficient λ of mixing of the long-range scalar and vector screened potentials. An increase in the contribution of the Lorentz vector potential to the spin-orbital (18), spin-spin (19), and tensor (20) interactions leads to an increase in the value of the splitting energy states. The value of strong-coupling constant α_s within B_c-mesons family can be considered constant.

The obtained value of $\lambda = 0.3$ indicates that the confinement screened potential are predominantly Lorentz-scalar ($\sim 70\%$), and the one-gluon exchange potential is purely a Lorentz-vector.

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