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Discontinuous Cycles of Impulsive Autonomous Systems in the Plane

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The problem of existence of periodic solutions of linear two-dimensional impulsive autonomous systems is under investigation. The possible periods of discontinuous cycles were found and the coefficient conditions for the existence of periodic solutions for such periods were obtained.

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Key Words: autonomous impulsive system, discontinuous cycle, periodic solution.

1. Statement of the Problem

Let us consider the problem of existence of discontinuous cycles of two-dimensional linear autonomous impulsive system

$$\frac{dx}{dt} = Ax + f, \quad \Delta x|_{<a,x>=0} = Bx + g, \quad x, f, g, a \in R^2, \tag{1}$$

where $a, f, g \in R^2$ are constant vectors, $a = (a_1, a_2)$, $a_1, a_2 \neq 0$, A, B are constant matrices.

At first we will investigate the solutions of corresponding homogeneous system

$$\begin{aligned} &\frac{dx}{dt} = Ax, \quad \langle a, x\rangle \neq 0, \\ &\Delta x|_{\langle a,x\rangle=0} = Bx, \end{aligned} \tag{2}$$

Assume that all non-trivial solutions of autonomous differential system $dx/dt = Ax$ intersects the line l: $x \in R^2$: $\langle a, x\rangle = 0$ transversally i.e. the condition $\langle a, Ax\rangle \neq 0$ holds. From this it follows that there exists $x \in R^2$ such that $\langle (A^\top - \lambda E)a, x\rangle \neq 0$, i.e. vector a is not the eigenvector of the matrix $A^\top$.

The impulsive action takes place on the line $l: \; x_2 = -a_1x_1/a_2$. It means that getting on this line the point $x(t^*)$ jumps to the point $x^{(1)} = (E+B)x^{(0)}$ [1] of the same line at the moment of impulsive action $t = t^*$ (figure 1). Here E is an identity matrix. So, matrix B is such that $\langle a, Bx\rangle = 0$ for all x that satisfy the condition $\langle a, x\rangle = 0$. From this we get that matrix B is of the type

$$(3)\qquad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{11} + \frac{a_2}{a_1} b_{21} - \frac{a_1}{a_2} b_{12}. \end{pmatrix}.$$

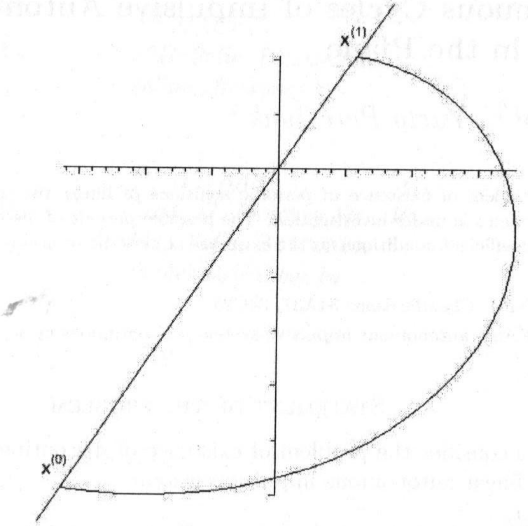

FIGURE 1. T_1-periodical discontinuous cycle

2. Existing of discontinuous cycles of the homogeneous system

If the system (2) has discontinuous periodical solution, then its trajectory should reach the line l. Let the point $x^{(0)}$, $\langle a, x^{(0)}\rangle = 0$ is a generative point of such solution. Then, leaving the line l at $t = 0$ after some time, the solution should reach it again. It means that the system

$$(4)\qquad \begin{aligned} \langle a, x^{(0)}\rangle &= 0, \\ \langle a, e^{At} x^{(0)}\rangle &= 0, \end{aligned}$$

should has a solution $(x^{(0)}, t^*)$, $t^* > 0$. It is equivalent to the system

$$\begin{aligned} \langle a, x^{(0)}\rangle &= 0, \\ \langle ((e^{At})^\top - e^{\lambda t} E) a, x^{(0)}\rangle &= 0, \end{aligned}$$

and therefore vector a is required to be the eigenvector of the matrix $\left(e^{At^*}\right)^\top$, which belongs to the eigenvalue $e^{\lambda t^*}$.

So, concerning system (2), we assume that the next conditions are executed:

- real vector a is not the eigenvector of the matrix $A^\top$;
- a is the eigenvector of the matrix $\left(e^{At^*}\right)^\top$ for some $t^* > 0$;
- a is the eigenvector of the matrix $B^\top$ of the type (3);

Denote by J the real Jordan form of the matrix A, then

$$\det\left(\left(e^{At^*}\right)^\top - e^{\lambda t^*} E\right) = \det\left(\left(e^{Jt^*}\right)^\top - e^{\lambda t^*} E\right),$$

i.e. the eigenvalues of the matrices $\left(e^{At^*}\right)^\top$ and $\left(e^{Jt^*}\right)^\top$ coincides. If matrix A has complex conjugate eigenvalues $\lambda = \alpha \pm i\beta$, then real eigenvalues of the matrix e^{Jt^*} are determined from the equation

$$\begin{vmatrix} e^{\alpha t^*} \cos\beta t^* - \lambda & e^{\alpha t^*} \sin\beta t^* \\ -e^{\alpha t^*} \sin\beta t^* & e^{\alpha t^*} \cos\beta t^* - \lambda \end{vmatrix} = 0.$$

This equation has real solutions if and only if

$$\begin{aligned} \lambda &= e^{\alpha t^*} \cos\beta t^*, \\ \sin\beta t^* &= 0. \end{aligned}$$

Taking into account that $t^* > 0$ is a moment of the first hit on the line l, the solutions of this system are

$$(5)\qquad (\lambda_2, t^*) = \left(e^{\frac{\alpha\pi}{\beta}}, \frac{\pi}{\beta}\right), \qquad (\lambda_2, t^*) = \left(e^{\frac{2\alpha\pi}{\beta}}, \frac{2\pi}{\beta}\right).$$

So the real eigenvalues of the matrix e^{Jt^*} are $e^{\lambda t^*}$ if λ is a real eigenvalue of the matrix A, or real eigenvalues of the type (5), which correspond to the couple of complex conjugate eigenvalues $\lambda = \alpha \pm i\beta$ of the matrix A. Matrices $A^\top$ and $\left(e^{At}\right)^\top$ has the same matrices of the transformation of similarity, which columns are also the real eigenvectors of the matrix $A^\top$. Therefore, if its eigenvalues are real, then the eigenvectors of the matrix $A^\top$ are the eigenvectors of the matrix $\left(e^{At^*}\right)^\top$. Vector a is not the eigenvector of the matrix $\left(e^{At^*}\right)^\top$ since a is not the eigenvector of the matrix $A^\top$. It means that $t^* > 0$ is the solution of the system (4) only in case that corresponds to complex eigenvalues of the matrix A, which means that we may rewrite system (4) as

$$\left\{ \begin{aligned} &\langle a, x^{(0)}\rangle = 0, \\ &\left\langle a, -e^{\frac{\alpha\pi}{\beta}} x^{(0)}\right\rangle = 0, \end{aligned} \right. \quad \cup \quad \left\{ \begin{aligned} &\langle a, x^{(0)}\rangle = 0, \\ &\left\langle a, -e^{\frac{2\alpha\pi}{\beta}} x^{(0)}\right\rangle = 0. \end{aligned} \right.$$

From this it follows that the system (2) may have periodic solutions only with period, which is multiple to $T_1 = \frac{\pi}{\beta}$. It means that matrix A is of the type

(6) $$A = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}, \quad \beta \neq 0.$$

Let us investigate the problem of existence of periodical cycles with period $T = T_1 = \frac{\pi}{\beta}$ for the system (2) . It can be if the point $x(t)$ moves in the following way (figure 1): after impulse action, point $x^{(0)}$ of the line l jumps to the point $x^{(1)} = (E+B)x^{(0)}$ of the same line and then, during the time $T = T_1$, the point moves along the phase curve to the starting position $x^{(0)}$. It means that the point $x^{(0)}$ is a solution of the system

(7) $$\begin{gathered} \langle a, x^{(0)} \rangle = 0, \\ -e^{\alpha \frac{\pi}{\beta}} (E+B) x^{(0)} = x^{(0)}. \end{gathered}$$

The second equation of this system may be rewritten in the form

(8) $$(E - F)x^{(0)} = 0,$$

where

$$F = -e^{\frac{\alpha\pi}{\beta}}(E+B) = \begin{pmatrix} -e^{\frac{\alpha\pi}{\beta}}(1+b_{11}) & -e^{\frac{\alpha\pi}{\beta}} b_{12} \\ -e^{\frac{\alpha\pi}{\beta}} b_{21} & -e^{\frac{\alpha\pi}{\beta}} \left(1 + b_{11} - \frac{a_1}{a_2} b_{12} + \frac{a_2}{a_1} b_{21}\right) \end{pmatrix}.$$

It has a nontrivial solutions $x^{(0)}$ in a two cases:

(9) $$e^{\frac{\alpha\pi}{\beta}} \left(1 + b_{11} - \frac{a_1}{a_2} b_{12}\right) = -1,$$

or

(10) $$e^{\frac{\alpha\pi}{\beta}} \left(1 + b_{11} + \frac{a_2}{a_1} b_{21}\right) = -1.$$

If the condition (9) holds, then the solution $x^{(0)} = (-a_2, a_1)$ of the equation (8) also satisfies the equation $\langle a, x^{(0)} \rangle = 0$. In a case when condition (10) fulfills, the solution of the equation (8) is $x^{(0)} = (a_1 b_{12}, a_2 b_{21})$, but this $x^{(0)}$ does not satisfy the first equation of (7).

Now let us investigate $T = T_2 = \frac{2\pi}{\beta}$-periodical trajectories of the system (2), which are not the T_1-periodical (figure 2). In this case, point $x(t)$ moves in the following way:

- point $x^{(0)}$ on the line l after impulse action jumps into the point $x^{(1)} = (E+B)x^{(0)}$ which also lies on the line l;
- during the time $T = T_1 = \frac{\pi}{\beta}$ the point moves along the phase curve to the point $x^{(2)} = e^{AT_1} x^{(1)}$ on the line l;

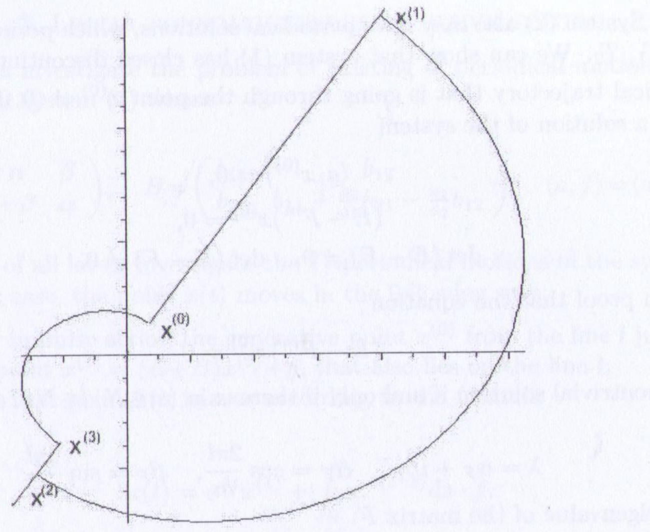

FIGURE 2. T_2-periodical discontinuous cycle

- from this point after impulse action it immediately jumps to the point $x^{(3)} = (E+B)x^{(2)}$;
- during the time $T_1 = \frac{\pi}{\beta}$ the point moves along the phase curve to the starting position $x^{(0)}$.

We can describe all this moves by the equations:

$$x^{(0)} = e^{AT_1} x^{(3)} = e^{AT_1}(E+B)x^{(2)} = e^{AT_1}(E+B)e^{AT_1}x^{(1)} = \left(e^{AT_1}(E+B)\right)^2 x^{(0)}.$$

It means that closed discontinuous $T_2 = \frac{2\pi}{\beta}$-periodical trajectory goes through the point $x^{(0)}$, that is a solution of the algebraic system

(11) $$\begin{gathered} \langle a, x^{(0)} \rangle = 0, \\ \left(E - F^2\right) x^{(0)} = 0, \\ \det\left(E - F\right) \neq 0. \end{gathered}$$

We can see that system (11) has a solution only in case when the parameters of the homogeneous system (2) fulfill the equality

(12) $$e^{\frac{\alpha\pi}{\beta}} \left(1 + b_{11} - \frac{a_1}{a_2} b_{12}\right) = 1.$$

System (2) also may have periodical solutions, which periods are different from T_1, T_2. We can show that system (1) has closed discontinuous $T_m = \frac{m\pi}{\beta}$-periodical trajectory that is going through the point $x^{(0)} \neq (0,0)$ if and only if $x^{(0)}$ is a solution of the system

$$\langle a, x^{(0)} \rangle = 0,$$
$$(E - F^m)x^{(0)} = 0,$$
$$\det(E - F) \neq 0, \quad \det(E - F) \neq 0.$$

We can proof that the equation

$$(E - F^m)x^{(0)} = 0 \tag{13}$$

has a nontrivial solution if and only if there exist $m \in N$, $l \in N$, $l \in [1, m-1]$ such that

$$\lambda = \alpha_F + i\beta_F, \quad \alpha_F = \cos\frac{2\pi l}{m}, \quad \beta_F = \sin\frac{2\pi l}{m}$$

is the eigenvalue of the matrix F.

Finally we obtain the statement about the existing of the closed discontinuous cycles of the homogeneous system (2), (3).

Theorem 1.

(1) *System (2), (3) has a closed discontinuous cycles if matrix A is of the form (6);*

(2) *system (2), (3), (6) has $T_1 = \frac{\pi}{\beta}$-periodical discontinuous cycles if and only if the condition (9) holds. In this case all points of the line $x_2 = -\frac{a_1}{a_2}x_1$ are the generative points of such cycles;*

(3) *system (2), (3), (6) has $T_2 = \frac{2\pi}{\beta}$-periodical discontinuous cycles if and only if the condition (12) holds. In this case all points of the line $x_2 = -\frac{a_1}{a_2}x_1$ are the generative points of such cycles;*

(4) *if the matrix F has only real eigenvalues, then linear homogeneous autonomous impulsive system (2), (3), (6) has no discontinuous cycles which are different from T_1- and T_2-periodicals;*

(5) *if $\lambda = \alpha_F + i\beta_F$ is a complex eigenvalue of the matrix F, then*

$$\frac{m}{2\pi}\arccos\alpha_F = \frac{m}{2\pi}\arcsin\beta_F = k,$$

where $k \in N$, is the necessary condition for the existence of $T_m = \frac{\pi m}{\beta_F}$-periodical discontinuous cycles of the system (2), (3), (6), and fulfilment of the condition $\langle a, x^{(0)} \rangle = 0$ by the nontrivial solution of the equation (13) is sufficient condition for the existence of $T_m = \frac{\pi m}{\beta_F}$-periodical discontinuous cycles of the system (2), (3), (6).

3. Linear nonhomogeneous impulsive system

Let us investigate the problem of existing of periodical motions of nonhomogeneous system (1) where

$$A = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{11} + \frac{a_2}{a_1}b_{21} - \frac{a_1}{a_2}b_{12} \end{pmatrix}, \quad \langle a, f \rangle = \langle a, g \rangle = 0. \tag{14}$$

First of all let us investigate the T_1-periodical motions of the system (1), (14). In this case, the point $x(t)$ moves in the following way:

- after impulse action the generative point $x^{(0)}$ from the line l jumps into the point $x^{(1)} = (E+B)x^{(0)} + g$, that also lies on the line l;
- after that point $x(t)$ moves according to the formula

$$x(t) = e^{At}x^{(1)} + \int_0^t e^{A(t-s)}ds \cdot f,$$

and during the time T_1 comes back to the point $x^{(2)} = x^{(0)}$.

Finally we have that point $x^{(0)}$ is a generative point of T_1-periodical discontinuous cycle if and only if $x^{(0)}$ is a solution of the algebraic system

$$\begin{aligned} \langle a, x^{(0)} \rangle &= 0, \\ (E - F)x^{(0)} &= r, \end{aligned} \tag{15}$$

where $r = -\left(e^{\frac{\alpha\pi}{\beta}}g + \left(e^{\frac{\alpha\pi}{\beta}} + 1\right)A^{-1}f\right)$.

From the condition $\langle a, f \rangle = \langle a, g \rangle = 0$ we have that

$$f = k_f\begin{pmatrix} a_2 \\ -a_1 \end{pmatrix}, \quad g = k_g\begin{pmatrix} a_2 \\ -a_1 \end{pmatrix},$$

where k_f, k_g are real constants and

$$r = -\overline{r}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \widehat{r}\begin{pmatrix} a_2 \\ -a_1 \end{pmatrix}, \quad \overline{r} = \frac{(e^{\frac{\alpha\pi}{\beta}}+1)\beta k_f}{\alpha^2+\beta^2}, \quad \widehat{r} = e^{\frac{\alpha\pi}{\beta}}k_g + \frac{(e^{\frac{\alpha\pi}{\beta}}+1)\alpha k_f}{\alpha^2+\beta^2}.$$

After thorough investigations of all possible solutions of the system (15) we have obtained the next statement.

Theorem 2.

1) *If* $\det(E-F)\neq 0$ *and* $f=0$, *then for all* g *the linear nonhomogeneous impulsive system* (1), (14) *has a unique discontinuous* T_1*-periodical cycle and its generative point is*

$$x^{(0)}=\frac{e^{\frac{\alpha\pi}{\beta}}}{1+e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}-\frac{a_1}{a_2}b_{12}\right)}g;$$

2) *if the relations*

$$e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}-\frac{a_1}{a_2}b_{12}\right)\neq-1,\qquad e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}+\frac{a_2}{a_1}b_{21}\right)=-1$$

fulfill, then for all f,g *linear nonhomogeneous impulsive system* (1), (14) *has a unique discontinuous* T_1*-periodical cycle, and its generative point is*

$$x^{(0)}=\frac{a_1a_2k_g}{a_1^2b_{12}^2+a_2^2b_{21}^2}\left(\frac{a_1a_2(b_{12}-b_{21})}{a_1^2b_{12}+a_2^2b_{21}}\begin{pmatrix}a_1b_{12}\\ a_2b_{21}\end{pmatrix}+\begin{pmatrix}a_2b_{21}\\ -a_1b_{12}\end{pmatrix}\right);$$

3) *in all other cases linear nonhomogeneous impulsive system* (1), (14) *has no* T_1*-periodical discontinuous cycles for all* f, g.

Now let us investigate periodical motions with the smallest period T_2 for the system (1), (14) (figure 2). After analysing such motions we can see that a generative point $x^{(0)}$ of such motion is a solution of the equation

$$(E-F^2)x^{(0)}=(E+F)r, \tag{16}$$

i.e. $x^{(0)}$ is a generative point for T_2-periodical discontinuous cycle if and only if it satisfies the system of equations

$$\begin{aligned}&\langle a,x^{(0)}\rangle=0,\\ &(E+F)\Big((E-F)x^{(0)}-r\Big)=0.\end{aligned} \tag{17}$$

If $\det(E+F)\neq 0$, i.e. the corresponding homogeneous system (2) has no T_2-periodical discontinuous cycles, then from (16) we obtain system (15). So the next statement is true.

Theorem 3. *If* $\det(E+F)\neq 0$, *then system* (1), (14) *has no* T_2*-periodical discontinuous cycles which differs from* T_1*-periodical.*

So, system (2) can have T_2-periodical cycles only in case if $(E+F)$ is a singular matrix. So we have two cases:

$$\det(E+F)=0,\quad \det(E-F)\neq 0. \tag{18}$$

or

$$\det(E+F)=\det(E-F)=0. \tag{19}$$

When the relationships (18) fulfill, then the second equation of the system (17) is solvable if and only if

$$(E-F)x^{(0)}-r=P_{(E+F)}\cdot c,$$

where matrix $P_{(E+F)}$ is the orthogonal projector to the matrix $(E+F)$ [2], $c\in R^{2-rank(E+F)}$ is an arbitrary constant. So if (18) fulfills then (17) is equivalent to the condition

$$\langle a,(E-F)^{-1}\Big(P_{(E+F)}\cdot c+r\Big)\rangle=0. \tag{20}$$

If we can find c from this equation, then system (1) has T_2-periodical discontinuous cycles with the generative point

$$x^{(0)}=(E-F)^{-1}\Big(P_{(E+F)}\cdot c+r\Big). \tag{21}$$

System (18) takes place when one eigenvalue of matrix F is equal to one and at the same time the another eigenvalue is differs from -1.

Furthermore, the relationships (19) take place when one eigenvalue of matrix F is equal to 1 and another is equal to -1.

After analysing all this cases we can formulate the next statement about the relations between the parameters of the system (1), (14) .

Theorem 4.

1) *If conditions*

$$e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}+\frac{a_2}{a_1}b_{21}\right)=1,\quad e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}-\frac{a_1}{a_2}b_{12}\right)\neq\pm1$$

hold, then for arbitrary f, g *system* (1), (14) *has a unique* T_2*-periodical discontinuous cycle. Its generative point is*

$$x^{(0)}=(E-F_4)^{-1}\left(\left(\begin{pmatrix}a_1b_{12}\\ a_2b_{21}\end{pmatrix}\frac{a_1^2+a_2^2}{a)1^2b_{12}+a_2^2b_{21}}-\begin{pmatrix}a_1\\ a2\end{pmatrix}\right)\overline{r}-\begin{pmatrix}a_2\\ a_1\end{pmatrix}\widehat{r}\right);$$

2) *if there holds one of the conditions:*

2.1) $f=0$, $e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}-\frac{a_1}{a_2}b_{12}\right)=1$, $e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}+\frac{a_2}{a_1}b_{21}\right)\neq\pm1$;

2.2) $f=0$, $e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}-\frac{a_1}{a_2}b_{12}\right)=1$, $e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}+\frac{a_2}{a_1}b_{21}\right)=1$;

2.3) $e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}+\frac{a_2}{a_1}b_{21}\right)=1$, $e^{\frac{\alpha\pi}{\beta}}\left(1+b_{11}-\frac{a_1}{a_2}b_{12}\right)=-1$ *and*

$$\left(2a_1a_2-(a_1^2+a_2^2)e^{\frac{\alpha\pi}{\beta}}b_{12}\right)\frac{(e^{\frac{\alpha\pi}{\beta}}+1)\beta k_f}{\alpha^2+\beta^2}+2a_2^2\left(e^{\frac{\alpha\pi}{\beta}}k_g+\frac{(e^{\frac{\alpha\pi}{\beta}}+1)\alpha k_f}{\alpha^2+\beta^2}\right)=0;$$

2.4) $b_{12} \neq 0$, $b_{21} \neq 0$, $f = 0$ *and*

$$(22) \qquad e^{\frac{\alpha\pi}{\beta}}\left(1 + b_{11} - \frac{a_1}{a_2}b_{12}\right) = -1, \quad e^{\frac{\alpha\pi}{\beta}}\left(1 + b_{11} + \frac{a_2}{a_1}b_{21}\right) = 1;$$

2.5) $e^{\frac{\alpha\pi}{\beta}}(1 + b_{11}) = 1$, *and* $b_{12} = b_{21} = 0$, *then all points of the line* $x_2 = -\frac{a_1}{a_2}x_1$ *are the generative points for* T_2-*periodical discontinuous cycles of the system* (1), (14);

3) *in all other cases for an arbitrary* f, g *system* (1), (14) *has no* T_2-*periodical discontinuous cycles.*

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A certain class of discontinuous dynamical systems in the plane

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A certain class of linear dynamical systems in the plane with discontinuous trajectories is investigated. The necessary and sufficient conditions for the existence of one-impulsive and two-impulsive discontinuous cycles, as well as conditions for asymptotic stability of the zero equilibrium point of impulsive system are obtained.

AMS Subj. Classification: 34C05

Key Words: Location of integral curves, singular points, limit cycles.

In the monographs [1-3], a theory of impulsive differential equations is built. Mainly, the mathematical models of evolutionary processes that undergo impulsive perturbations at fixed moments of time or at the moments, when the moving point meets the given hypersurfaces in the extended phase space are considered. However, in the monographs [1-3], the importance of studying of systems with impulsive perturbations that occur at the moments, when the phase point meets the given sets in the phase space is emphasized. In this paper, we investigate a linear differential systems in the plane that are subjected to impulsive perturbations on the given line.

Without loss of generality, we assume that the matrix of linear differential system in the plane is in real Jordan form. Thus, the object of our study is a linear discontinuous dynamical system in the plane

$$(1) \qquad \frac{dx}{dt} = Jx, \quad < a, x > \neq 0; \quad \Delta x\Big|_{<a,x>=0} = Bx,$$

where $x = coll(x_1, x_2)$, J is a real Jordan block; the given line

$$< a, x >= 0, \qquad a_1x_1 + a_2x_2 = 0$$

is not a coordinate axis and

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

is a constant matrix.