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Research Article

The estimation of the object position on the digital CCD frame using the pixel model of the object image

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Abstract: The estimation methods of the objects image on a digital CCD frame have been developed. On the capacity of the form model the pixel Gaussian model has been offered. The pixel Gaussian model is considered on two types of objects: objects of compact groups and objects with the images blurred by a proper motion. The methods are based on the object position estimation by a criterion of the minimum residual squares between experimental and model images. The minimization of the sum of the residual squares is carried out using the Levenberg-Marquardt algorithm.
Keywords: Close objects; objects of the compact group; statistically dependent images; trailed objects; object images, blurred by a proper motion; image parameter estimation; asteroids; CCD frame.

INTRODUCTION

Studying of asteroids is generally carried out using optical telescopes. The task of asteroids monitoring and their registration becomes significantly complicated because of a set of factors. They include: a low level of signals from asteroids; the existence of an unevenly distributed background on frames which distribution form can very on one series of frames; the blurring of the asteroid image because of the atmosphere turbulence; the distortion of the asteroid image owing to the aberrations of the optical system; the lack of differences between the asteroids image and the stars image on a separate frame. The requirements to the accuracy of the coordinates' determination of the asteroids are rather high – the mistake must be less than 1 arc second [1].

The permeable ability of the optical systems has significantly increased that has also risen the density of the observed objects. Taking into consideration a great density of the images positions of the celestial objects on a CCD frame [2], and also the apparition of the objects of compact groups with statistically dependent images, the complexity of achievement the demanding accuracy of the close objects monitoring increases considerably.

A great interest is also caused by objects with a blurred image. Because of a long period of time of the image objects exposition, they become blurred by a proper motion. It, in its turn, complicates the determination of such objects location with the necessary accuracy.

In that way, the development of computing methods of a high **-** accuracy estimation of the objects position is actual. In such methods it is necessary to consider the main conditions of the monitoring of the studied objects and the formation features of their discrete images.

I. PUBLICATION ANALYSIS

There are many methods for determining asteroids` position with a CCD camera (charge coupled device). In one of them, the analytical analysis of the objects images on the CCD frame is used, for example, FWHM approach (Full Width Half Magnitude) [3]. In the others as object coordinates the brightness maximum is accepted in its image [4].

In general for the majority of determining methods of the object position on the CCD frame is the application while its image approximation of point spread function (PSF-fitting) [5-10]. Thus the information on regularities of distribution of the registered photons on the image of object is used. Most often as model of the photons distribution various options two - dimensional Gaussian are applied [10,11,7]: the Moffat's models [12,13] or the Lorentz's one [10, 13].

In the existing methods there is often not much attention paid to the background noise of the image. It is considered that its account and compensation are made at the preliminary stage of the image processing [6] or that the object image is exempted from noise according to the accepted model of images and hindrances [9,13]. At the same time the mistake brought by the operation of the background noise exception of the object image isn't considered in the subsequent procedure of the coordinates' determination.

The lack of the majority of the estimation parameters methods of the object position on a CCD frame is that they don't use the real law of the coordinate distribution of photons falling from object in the picture plane of the optical device, or don't take into consideration the existence of statistically dependent images from close objects, or they don't consider existence of the blur as a result of a long exposition, or don't take into consideration the main features of the process functioning of the image formation, or ignore the existence and coordinates distribution of noise photons falling on the image of statistically dependent objects and in its vicinity. The simultaneous accounting of all the listed factors and the conditions is the necessary condition for the estimation procedures creation of the close and trailed objects positions possessing the accuracy close to the potential one.

II. PROBLEM STATEMENT

It is considered that Q of celestial objects is found. The image of Q objects is really presented in the frame and is in area of intraframe processing (AIFP) – a great number of Ω_{AIFP} pixels where the existence of the objects image of the compact group Q is supposed to be. Objects images in the compact group can be crossed and, in this sense, objects are considered to be statistically dependent on each other. The number of the pixels belonging to AIFP is considered equal to N_{AIFP} . Alternatively AIFP can be set by the coordinates and the sizes of the corresponding area.

The monitoring is available for the brightness of A^*_{ikt} pixels from the Ω_{AIFP} multitude. The objects images on a CCD frame are formed due to the hit of alarm photons in CCD matrix pixels with the set borders:

$$x_{bi} = x_{it} - \frac{\Delta_{CCD}}{2},\ x_{ei} = x_{it} + \frac{\Delta_{CCD}}{2};\ (1)$$

$$y_{bk} = y_{kt} - \frac{\Delta_{CCD}}{2},\ y_{ek} = y_{kt} + \frac{\Delta_{CCD}}{2},\ (2)$$

where x_{bi}, x_{ei} - the borders of (the beginning and the end respectively) of the *ik*-pixel of the CCD matrix on the coordinate; y_{bk}, y_{ek} - the borders of the *ik*-pixel of the CCD matrix of the y coordinate; x_{it}, y_{kt} - the coordinates of the *ik*-pixel *t* frame of the CCD matrix.

Tension is available to the observation at the exit of the N_{CCD} pixels of the CCD matrix can be given to skilled relative frequencies of the photons hit in the *ik*-pixel of the CCD matrix in the *t* frame (relative brightness of the corresponding pixels):

$$\upsilon^*_{ikt} = A^*_{ikt} / \sum_{i,k}^{N_{IFPA}} A^*_{ikt},\ (3)$$

The images of celestial objects on a frame have a point form, and, taking into account atmospheric turbulence, objects images become blurred.

The interfering photons form a noise substrate in all pixels of AIFP. The main characteristics for each of the Q hypothetical celestial objects on the image of the frame are the following: the position of X_{jt}, Y_{jt}, the theoretical brightness of the A_G; image, the parameter of the form of the object the σ_G image. The set of the above mentioned parameters is unique for the image of each object and is the subject to the estimation. Also the parameters of the A_N, B_N, C_N background noise are subject to estimation. In the case of its preliminary subtraction from the objects image the residual background noise of the image C_R is estimated. The mentioned objects parameters make the Θ_{gen} vector of the estimated parameters.

At a considerable amount of time of the objects image exposition Δ_τ the object images can be blurred by a proper motion. In this regard the object image has a form of the trailed hill lengthways the directions of its apparent motion. In this case, the effect of object movement during the exposition $\Delta\tau$ while estimating the object positions on a frame can't be neglected. It is possible to call such object images trailed, sometimes, in this context, the objects can be called trailed.

The trailed objects in the picture plane of the telescope move evenly along each coordinate:

$$x_{j\tau}(\Theta^{sub}_{\tau noise}) = x_{j\tau t}(\Theta^{sub}_{\tau noise}) + V_{xijt}(\tau - \tau_t);$$
$$y_{j\tau}(\Theta^{sub}_{\tau noise}) = y_{j\tau t}(\Theta^{sub}_{\tau noise}) + V_{ykjt}(\tau - \tau_t), \quad (4)$$

where τ - time; $x_{j\tau}(\Theta^{sub}_{\tau noise}), y_{j\tau}(\Theta^{sub}_{\tau noise})$ - the coordinates of the *j* trailed object at time; $x_{j\tau t}(\Theta^{sub}_{\tau noise}), y_{j\tau t}(\Theta^{sub}_{\tau noise})$ - the coordinates of the *j* trailed object at the time of the τ_t frame binding (as a rule, a half of the exposition time); V_{xijt} V_{ykjt} - the speed of the *j* trailed object on the coordinates x and y accordingly.

It is considered [14] that on any time point τ coordinates of the alarm photons falling on a CCD matrix has a normal circular distribution. Thereby, instant images of celestial objects on any time point τ have a point form, and, taking into account atmospheric turbulence, object images on a frame becomes blurred.

The initial coordinate approximations of celestial objects on a t shaped frame x_{jt}, y_{jt} (where $j = \overline{1,Q}$) are known and correspond to the coordinates of the binding of the peak pixels of the image hypothetically corresponding to the object positions. The image peak of the j object hypothetical image, as a rule, is in the pixel of the CCD matrix which brightness is more than one of any of adjacent to it [15].

As a model of the object image form the pixel Gaussian model [11, 10, 7] is used.

It is necessary to develop computing methods of the parameters estimation of the CCD image of each of Q hypothetical objects stated above. Thus the objects of two types are considered: trailed and close. As a criterion of the best estimation of the object positions on the CCD frame the minimum of the sum of squares of residuals between the experimental A^*_{ikt} and model brightness $A_{ikt}(\Theta_{gen})$ of the AIEP pixels are the following:

$$F_{\Delta A}(\Theta_{gen}) = \sum_{i,k}^{N_{IFPA}} \left(A^*_{ikt} - A_{ikt}(\Theta_{gen})\right)^2 \xrightarrow[\Theta gen]{} \min, \quad (5)$$

where $A_{ikt}(\Theta_{gen})$ – model (relative) brightness of the pixel (in a general view).

III. THE PIXEL GAUSSIAN MODEL OF THE DIGITAL IMAGE OF THE OBJECTS OF THE COMPACT GROUP

The pixel Gaussian model describes the distribution of the degree belonging to the studied AIFP. Thus it is considered that the image of each hypothetical object in AIFP has a form of a Gaussian. Thus the offered pixel Gaussian model considers the existence of a noise substrate.

In practice the pixel Gaussian model of the object images in the studied AIFP is presented by the following expression:

$$A_{ikt}(\Theta_{noise}^{over}) = A_{noise}x_{it} + B_{noise}y_{kt} + C_{noise} + \sum_{j=1}^{Q}\frac{A_{Gjt}}{2\pi\sigma_{Gjt}^{2}}\times$$
$$\times\exp\left\{-\frac{1}{2\sigma_{Gjt}^{2}}\left[(x_{it} - x_{jt}(\Theta_{noise}^{over}))^{2} + (y_{kt} - y_{jt}(\Theta_{noise}^{over}))^{2}\right]\right\}, \quad (7)$$

where $A_{ikt}(\Theta_{noise}^{over})$ – the model brightness of the *ik* pixel of the CCD matrix, on the *t* CCD frame; $A_{noise}, B_{noise}, C_{noise}$ – the model integral parameters of the background noise; A_{Gjt} – the theoretical brightness of the *j* hypothetical object on the *t* CCD frame; $x_{jt}(\Theta_{noise}^{over}), y_{jt}(\Theta_{noise}^{over})$ – the coordinates of the *j* hypothetical object on the *t* CCD frame; σ_{Gjt} – the parameter of the model image on the *j* hypothetical object on the *t* CCD frame; Q – the number of the AIFP studied objects; x_{it}, y_{kt} – the coordinates of the center reference of the *ik* pixel of the CCD matrix frame;

$$\Theta_{noise}^{over} = (A_{noise}, B_{noise}, C_{noise}, x_{1t}(\Theta_{noise}^{over}), y_{1t}(\Theta_{noise}^{over}), A_{G1t}, \sigma_{G1t}, \ldots$$
$$\ldots x_{nt}(\Theta_{noise}^{over}), y_{nt}(\Theta_{noise}^{over}), A_{Gnt}, \sigma_{Gnt}, \ldots x_{Qt}(\Theta_{noise}^{over}), y_{Qt}(\Theta_{noise}^{over}), A_{GQt}, \sigma_{GQt}) =$$

$= (\theta_1, \ldots \theta_n, \ldots \theta_{4Q+3}),$ – the vector of the estimated parameters of the object images; θ_n – the *n* estimated parameter.

IV. THE PIXEL GAUSSIAN MODEL OF THE DIGITAL IMAGE OF THE OBJECTS WITH THE IMAGES BLURRED BY A PROPER MOTION

The object images during the exposition Δ_τ are blurred by a motion. In this regard the object image has a hill form. To estimate the object position, it is expedient to approximate the object image blurred with proper motion, mix of Gaussians. This approximation is explained by the fact that all the time the exposition formally with the sufficient accuracy for practice, can be divided on the N moments, the Gaussians quantity will also be equal to N.

Within the used pixel model of the Gaussian image of this mix are located along the direction of the object motion as equals distances between the centers of the neighbor Gaussian. The bigger way the objects pass during the exposition, the bigger Gaussian quantity is necessary for adequate object presentation. The function of image pixel model (theoretical brightness of the pixels image) of the objects with trailed images is possible to present in the form the sums of the final quantity of final two-dimension Gaussian:

$$A_{ikt\tau}(x_{it}, y_{kt}, \Theta_{\tau noise}^{over}) = \Delta_\tau(A_{\tau noise}x_{it} + B_{\tau noise}y_{kt} + C_{\tau noise}) + \sum_{j=1}^{Q}A_{G\tau jt}\times$$
$$\times\sum_{n=0}^{N_G}\exp\left\{-\frac{1}{2\sigma_{G\tau jt}^{2}}\left[(x_{it} - x_{j\tau t}(\Theta_{\tau noise}^{over}) - \lambda_n\cos\omega_j)^2 + \right.\right. \quad (9)$$
$$\left.\left. + (y_{kt} - y_{j\tau t}(\Theta_{\tau noise}^{over}) - \lambda_n\sin\omega_j)^2\right]\right\},$$

where $A_{\tau noise}, B_{\tau noise}, C_{\tau noise}$ – the parameters of the flat noise base of the pixel image model; $\Theta_{\tau noise}^{over}$ – the vector of the estimated parameters of object images; $x_{j\tau t}(\Theta_{\tau noise}^{over})$, $y_{j\tau t}(\Theta_{\tau noise}^{over})$ - the coordinates of the *j* at the moment of the frame reference; $A_{G\tau jt}$ – the average brightness of the hypothetical *j* object on the *t* CCD frame; $\sigma_{G\tau jt}$ – the parameter of a form of pixel model of the *j* object image on a CCD frame.

The offered pixel Gaussian model (9) due to the summand $\Delta_\tau (A_{\tau noise} x_{it} + B_{\tau noise} y_{kt} + C_{\tau noise})$ also includes the existence of a background noise of the image pixel model.

The vector of the estimated parameters $\Theta_{\tau noise}^{over}$ taking into account parameters of the background noise consists of the $5Q+3$ elements and includes a set of the objects coordinates, the average brightness of the $A_{G\tau jt}$ image, the parameter $\sigma_{G\tau jt}$ of the form of the pixel image model of a hypothetical object and the corner between its direction of the movement and the abscissa axis:

$$\begin{gathered}\Theta_{\tau noise}^{over} = (A_{\tau noise}, B_{\tau noise}, C_{\tau noise}, x_{1\tau t}(\Theta_{\tau noise}^{over}), y_{1\tau t}(\Theta_{\tau noise}^{over}), A_{G\tau 1t}, \sigma_{G\tau 1t}, \omega_1, \ldots \\ \ldots x_{n\tau t}(\Theta_{\tau noise}^{over}), y_{n\tau t}(\Theta_{\tau noise}^{over}), A_{G\tau nt}, \sigma_{G\tau nt}, \omega_n, \ldots \\ \ldots x_{Q\tau t}(\Theta_{\tau noise}^{over}), y_{Q\tau t}(\Theta_{\tau noise}^{over}), A_{G\tau Qt}, \sigma_{G\tau Qt}, \omega_Q) = (\theta_1, \ldots \theta_n, \ldots \theta_{5Q+3}).\end{gathered} \quad (10)$$

V. THE COMPUTING METHOD OF THE CLOSE OBJECT POSITION ESTIMATION ON A DISCRETE IMAGE

The minimization of a quadratic form (5) for close objects is carried out with the help of the Levenberg–Marquardt method [12]. This method is the most widespread method of quadratic forms minimization.

For the realization of the Levenberg–Marquardt method the Jacobian matrix (11). The elements of the $J_{\Delta AG1}$ matrix are partial derivatives of the residuals $\Delta A_{m(i,k)}$ between experimental and model pixel brightness according to the estimating parameters (the vector elements of the estimated Θ_{noise}^{over} parameters) in each pixel of each Ω_{AIFP} set. The residual $\Delta A_{m(i,k)}$ is counted for a set of pixels, belonging to the studied AIFP. At the same time, the m line of Jacobi matrix corresponds to the definition of the derivative according to all the estimated parameters Θ_{noise}^{over} to the residuals in the pixel of the Ω_{AIFP} set.

Then it is considered that the number of the m in the Ω_{AIFP} set is the $m(i,k)$ function from the pixel numbers in the frame.

Based on the information given, the Jacobi matrix is determined in the expression:

$$J_{\Delta AG1} = \begin{bmatrix} \dfrac{\partial \Delta A_{1(1,1)}}{\partial \theta_1} & \Lambda & \dfrac{\partial \Delta A_{1(1,1)}}{\partial \theta_n} & \Lambda & \dfrac{\partial \Delta A_{1(1,1)}}{\partial \theta_{4Q+3}} \\ \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\ \dfrac{\partial \Delta A_{m(i,k)}}{\partial \theta_1} & \Lambda & \dfrac{\partial \Delta A_{m(i,k)}}{\partial \theta_n} & \Lambda & \dfrac{\partial \Delta A_{m(i,k)}}{\partial \theta_{4Q+3}} \\ \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\ \dfrac{\partial \Delta A_{N_{SIFP}(i,k)}}{\partial \theta_1} & \Lambda & \dfrac{\partial \Delta A_{N_{SIFP}(i,k)}}{\partial \theta_n} & \Lambda & \dfrac{\partial \Delta A_{N_{SIFP}(i,k)}}{\partial \theta_{4Q+3}} \end{bmatrix}. \quad (11)$$

The expression for private derivatives of the $\Delta A_{m(i,k)}$ residual between the model and the experimental pixel brightness according to: the theoretical brightness image of the A_{Gjt} object, the σ_{Gjt} parameter of the model form of the object image, the parameters $x_{jt}(\Theta^{over}_{noise}), y_{jt}(\Theta^{over}_{noise})$ (the coordinates of the *j* hypothetical object on the *t* CCD frame) is represented as the following:

$$\frac{\partial \Delta A_{m(i,k)}}{\partial A_{Gjt}} = -\frac{1}{2\pi\sigma^2_{Gjt}} \exp\left\{ -\frac{(x_{it} - x_{jt}(\Theta^{over}_{noise}))^2 + (y_{kt} - y_{jt}(\Theta^{over}_{noise}))^2}{2\sigma^2_{Gjt}} \right\}, \quad (12)$$

$$\begin{aligned} \frac{\partial \Delta A_{m(i,k)}}{\partial \sigma_{Gjt}} = & -\frac{1}{2\pi} \frac{A_{Gjt}}{\sigma^5_{Gjt}} \exp\left\{ -\frac{(x_{it} - x_{jt}(\Theta^{over}_{noise}))^2 + (y_{kt} - y_{jt}(\Theta^{over}_{noise}))^2}{2\sigma^2_{Gjt}} \right\} \cdot \times \\ & \times \left((x_{it} - x_{jt}(\Theta^{over}_{noise}))^2 + (y_{kt} - y_{jt}(\Theta^{over}_{noise}))^2 \right) + \\ & + \frac{A_{Gjt}}{\pi\sigma^3_{Gjt}} \exp\left\{ -\frac{(x_{it} - x_{jt}(\Theta^{over}_{noise}))^2 + (y_{kt} - y_{jt}(\Theta^{over}_{noise}))^2}{2\sigma^2_{Gjt}} \right\}; \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{\partial \Delta A_{m(i,k)}}{\partial x_{jt}(\Theta^{over}_{noise})} = & -\frac{1}{4\pi} \frac{A_{Gjt}\left(2x_{it} - 2x_{jt}\right)}{\sigma^4_{Gjt}} \times \\ & \times \exp\left\{ -\frac{(x_{it} - x_{jt}(\Theta^{over}_{noise}))^2 + (y_{kt} - y_{jt}(\Theta^{over}_{noise}))^2}{2\sigma^2_{Gjt}} \right\}; \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{\partial \Delta A_{m(i,k)}}{\partial y_{jt}(\Theta^{over}_{noise})} = & -\frac{1}{4\pi} \frac{A_{Gjt}\left(2y_{kt} - 2y_{jt}\right)}{\sigma^4_{Gjt}} \times \\ & \times \exp\left\{ -\frac{(x_{it} - x_{jt}(\Theta^{over}_{noise}))^2 + (y_{kt} - y_{jt}(\Theta^{over}_{noise}))^2}{2\sigma^2_{Gjt}} \right\}. \end{aligned} \quad (15)$$

The expressions for private derivatives according to the parameters of the background noise for the residual (5) between the model and experimental pixel brightness can be presented as the following:

$$\frac{\partial \Delta_{SH} A_{m(i,k)}}{\partial A_{noise}} = -x_{it}; \quad (16)$$

$$\frac{\partial \Delta_{SH} A_{m(i,k)}}{\partial B_{noise}} = -y_{kt}; \quad (17)$$

$$\frac{\partial \Delta_{SH} A_{m(i,k)}}{\partial C_{noise}} = -1. \quad (18)$$

Similar to the Jacobi matrix (11) the Jacobi matrix for the sum of squares of residuals at the previously subtracted background noise is formed. At the same time the residual model between the experimental and model pixel brightness is represented as the following:

$$\Delta A_{m(i,k)} = A^{*}_{ikt} - C^{residual}_{noise} - \sum_{j=1}^{Q} \frac{A_{Gjt}}{2\pi\sigma^{2}_{Gjt}} \times$$
$$\times \exp\left\{ -\frac{1}{2\sigma^{2}_{Gjt}} \left[(x_{it} - x_{jt}(\Theta))^2 + (y_{kt} - y_{jt}(\Theta))^2 \right] \right\}. \quad (19)$$

Taking into consideration the background noise (7), and also without its (19) private derivative from the sum of residuals according to the estimated parameters look identically and correspond to the expressions (12) ÷ (15).

The private derivative of the $\Delta A_{m(i,k)}$ residual between the experimental and the model pixel brightness according to the parameter $C^{residual}_{noise}$ (residual component of the background noise) looks as follows:

$$\frac{\partial \Delta A_{m(i,k)}}{\partial C^{residual}_{noise}} = -1. \quad (20)$$

VI. THE COMPUTING METHOD OF THE ESTIMATION OF THE TRAILED OBJECTS POSITION ON THE DISCRETE IMAGE

As in the case with close objects, minimization of (5) is carried out by means of the Levenberg–Marquardt algorithm**.** Thus the Jacobi matrix is formed similarly (11), considering the vector of the estimated parameters (10).

The private derivative of the $\Delta A_{\tau m(i,k)}$ residual between the theoretical and the experimental pixel brightness according to the parameter $\sigma_{G\tau jt}$ the forms of the model image of the object on the *t*-CCD frame in the general case looks as follows:

$$\frac{\partial \Delta A_{\tau m(i,k)}}{\partial \sigma_{G\tau jt}} = A_{G\tau jt} \times$$
$$\times \sum_{n=0}^{N} \exp\left\{-\frac{1}{2\sigma_{G\tau jt}^{2}}\left[(x_{it} - x_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \cos\,\omega_j)^2 + \right.\right.$$
$$\left.\left. + (y_{kt} - y_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \sin\omega_j)^2\right]\right\} \times \tag{21}$$
$$\times \frac{1}{\sigma_{G\tau jt}^{3}}\left[(x_{it} - x_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \cos\,\omega_j)^2\right.$$
$$\left. + (y_{kt} - y_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \sin\omega_j)^2\right]$$

The general view of private derivative of the $\Delta A_{\tau m(i,k)}$ residual between the theoretical and experimental pixel brightness according to the parameters $x_{j\tau_t}(\Theta_{\tau noise}^{over}), y_{j\tau_t}(\Theta_{\tau noise}^{over})$ (the coordinates of the j hypothetical object on the t CCD frame) is represented by the following expressions (22) and (23) accordingly:

$$\frac{\partial \Delta A_{\tau m(i,k)}}{\partial x_{j\tau_t}(\Theta_{\tau noise}^{over})} = A_{G\tau jt} \times$$
$$\times \sum_{n=0}^{N} \exp\left\{-\frac{1}{2\sigma_{G\tau jt}^{2}}\left[(x_{it} - x_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \cos\,\omega_j)^2 + \right.\right. \tag{22}$$
$$\left.\left. + (y_{kt} - y_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \sin\omega_j)^2\right]\right\} \times$$
$$\times \frac{1}{2\sigma_{G\tau jt}^{2}}\left[(2x_{it} - 2x_{j\tau_t}(\Theta_{\tau noise}^{over}) + 2\lambda_n \cos\,\omega_j)\right];$$

$$\frac{\partial \Delta A_{\tau m(i,k)}}{\partial y_{j\tau_t}(\Theta_{\tau noise}^{over})} = A_{G\tau jt} \times$$
$$\times \sum_{n=0}^{N} \exp\left\{-\frac{1}{2\sigma_{G\tau jt}^{2}}\left[(x_{it} - x_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \cos\,\omega_j)^2 + \right.\right. \tag{23}$$
$$\left.\left. + (y_{kt} - y_{j\tau_t}(\Theta_{\tau noise}^{over}) + \lambda_n \sin\omega_j)^2\right]\right\} \times$$
$$\times \frac{1}{2\sigma_{G\tau jt}^{2}}\left[(2y_{it} - 2y_{j\tau_t}(\Theta_{\tau noise}^{over}) + 2\lambda_n \sin\omega_j)\right].$$

The private derivatives of the $\Delta A_{\tau m(i,k)}$ residual between the theoretical and the experimental pixel brightness according to the parameters $A_{G\tau jt}$ (the average image brightness of the *j* hypothetical object on the *t* CCD frame) and ω_j (the angle between the directions of the *j* object motion and the axis abscissa in the coordinate system of the CCD frame) in the general case the expressions (10) and (11) are represented accordingly:

$$\frac{\partial \Delta A_{\tau m(i,k)}}{\partial A_{G\tau jt}} = \sum_{n=0}^{N} \exp\left\{ -\frac{1}{2\sigma^2_{G\tau jt}} \left[(x_{it} - x_{j\tau_t}(\Theta^{over}_{\tau noise}) + \lambda_n \cos \omega_j)^2 + \right.\right.$$
$$\left.\left. + (y_{kt} - y_{j\tau_t}(\Theta^{over}_{\tau noise}) + \lambda_n \sin \omega_j)^2 \right] \right\}. \quad (24)$$

$$\frac{\partial \Delta A_{\tau m(i,k)}}{\partial \omega_j} = A_{G\tau jt} \times$$
$$\times \sum_{n=0}^{N} \exp\left\{ -\frac{1}{2\sigma^2_{G\tau jt}} \left[(x_{it} - x_{j\tau_t}(\Theta^{over}_{\tau noise}) + \lambda_n \cos \omega_j)^2 + \right.\right.$$
$$\left.\left. + (y_{kt} - y_{j\tau_t}(\Theta^{over}_{\tau noise}) + \lambda_n \sin \omega_j)^2 \right] \right\} \frac{1}{2\sigma^2_{G\tau jt}} \times \quad (25)$$
$$\times \left[2\lambda_n (\sin \omega_j (x_{j\tau_t}(\Theta^{over}_{\tau noise}) - x_{it}) - \cos \omega_j (y_{j\tau_t}(\Theta^{over}_{\tau noise}) - y_{kt}) \right]$$

The expression for the private derivative of the $\Delta A_{\tau m(i,k)}$ residual between the theoretical and experimental pixel brightness according to the parameter (the residual component of a background noise) looks as the following:

$$\frac{\partial \Delta A_{\tau m(i,k)}}{\partial C^{residual}_{\tau noise}} = \Delta_\tau \quad (26)$$

Thus, the minimization of the (5) using the Levenberg–Marquardt algorithm can be carried out having the Jacobi matrix (11), which elements will be the values of the private derivative of the residuals between the theoretical and the experimental brightness according to the parameters of the estimated vector parameters $\Theta^{over}_{\tau noise}$, shown in the expressions (21)÷(25).

VII. CARRYING OUT THE RESEARCHES OF THE ESTIMATION ACCURACY OF THE NUMBERED ASTEROIDS POSITIONS

The developed method is introduced and is used within the program of the automatic asteroids search on a series of the CCD frames of CoLiTec software.

For researches of the accuracy estimation of the numbered asteroids positions 36 series on 4 frames were picked up (starting from January, 2014 prior to the beginning of April, 2014), each of them contains not less than 35 measurements of the numbered asteroids on a frame, the objects of compact groups (close objects) and objects with the blurred images (trailed objects) were chosen from the total of measurements.

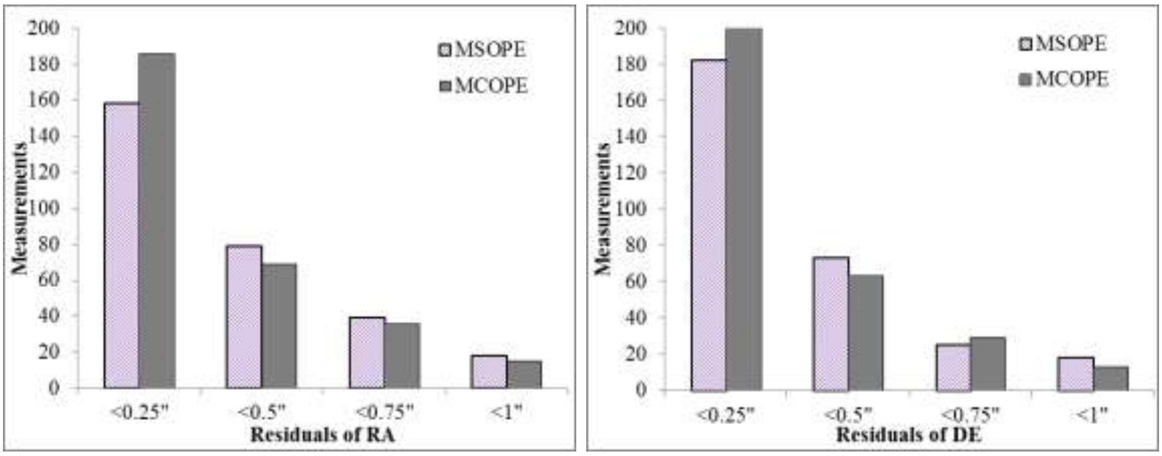
As a source of the coordinates reference values of the numbered asteroids the NASA HORIZONS service [14] was used. The set of frames which had been mentioned earlier was processed with two versions CoLiTec software using the following methods:

- The method of the single object position estimation on a CCD frame (MSOPE) - includes the use of the one-target model and the method of the maximum likelihood for the object parameters estimation;
- The method of the close object position estimation on a CCD frame (MCOPE) – includes the use of the pixel Gaussian model of the object image (for a multi-purpose case), the Least-squares criterion for the object parameter estimation and the Levenberg–Marquardt algorithm for the minimization of the quadratic form.

The main lack of the method of the single object position estimation on a CCD frame [2] is that it didn't take into consideration the possibility of the images crossing of the several nearby objects (the statistical dependence of the close objects images).

The research of the estimation accuracy of the close objects position.

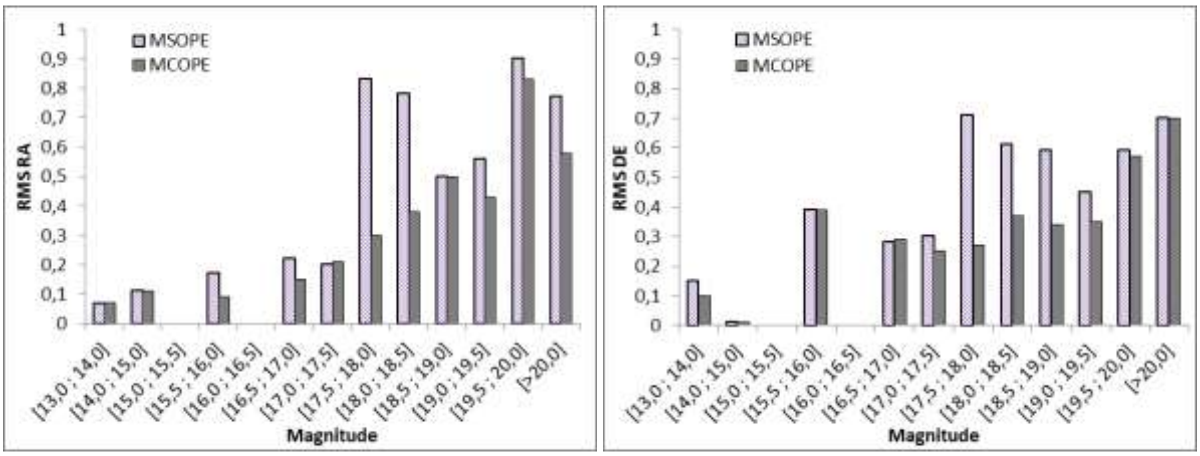
On the specified set of frames, the selection on close objects makes 324 measurements. The statistical properties of the residuals of the equatorial coordinates of the close objects were investigated (Fig. 1). The results of the comparison of the studied methods show that the use of the method of the close object position estimation considerably reduces the residuals on the right ascension and declination (Fig. 1). The number of measurements with the residual is less than 0.25´´ increased by 15% on the right ascension and for 10% on the declination.



a) b)

Fig-1: The distribution of the residuals of the equatorial coordinates for close objects (MSOPE and MCOPE): a) on the right ascension; b) on the declination.

At RMS (root-mean-square deviation) research of the equatorial coordinates estimation of the objects in various ranges of visible brightness, the method of the close object position estimation has also showed the best results. It is necessary to notice that this method works especially well at average ranges of brightness (Figure 2). Thus, on the specified range, the RMS definition on the right ascension has been decreased on average by 57% (Picture 2a); on the declination it has been decreased on average by 51% (Fig. 2b).

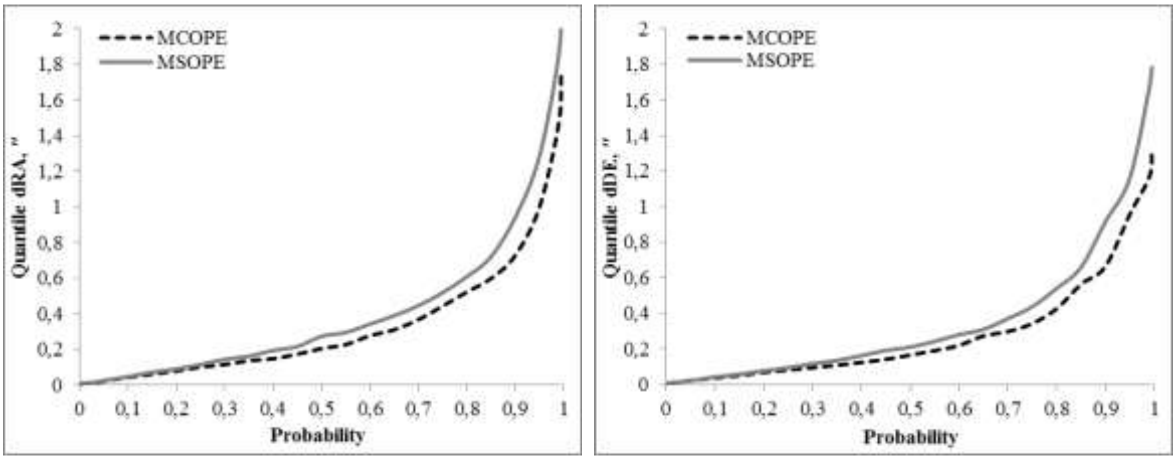


a) b)

Fig-2: RMS of the equatorial coordinates estimates of the objects in various ranges of visible brightness

(magnitude) for close objects (MSOPE and MCOPE): a) on the right ascension, b) on the declination.

The advantages of the method of the close object position estimation on accuracy are also shown by the means of the quantiles distribution of the modules residuals of the equatorial coordinates (Fig. 3). The value of quantiles of modules residuals in the developed method is much lower both on the right ascension (Fig. 3a), and on the declination (Fig. 3b).



a) b)

Fig- 3: Quantiles of the module residuals of the equatorial coordinates for close objects (MSOPE and MCOPE): a) on the right ascension, b) on the declination

The schemes of the residuals dependency of the equatorial coordinates of the brightness estimation are given (Figure 4, 5) below. While creating the schemes 2% of the worst measurements were discarded according to the module of the vector residuals. The designations in the schemes are the following. Round markers - the average value of the residual on an interval (quantity of intervals =20). The line - the polynom approximating the average values on graphics. Triangular markers - a confidential interval = 3*RMS of the residual.

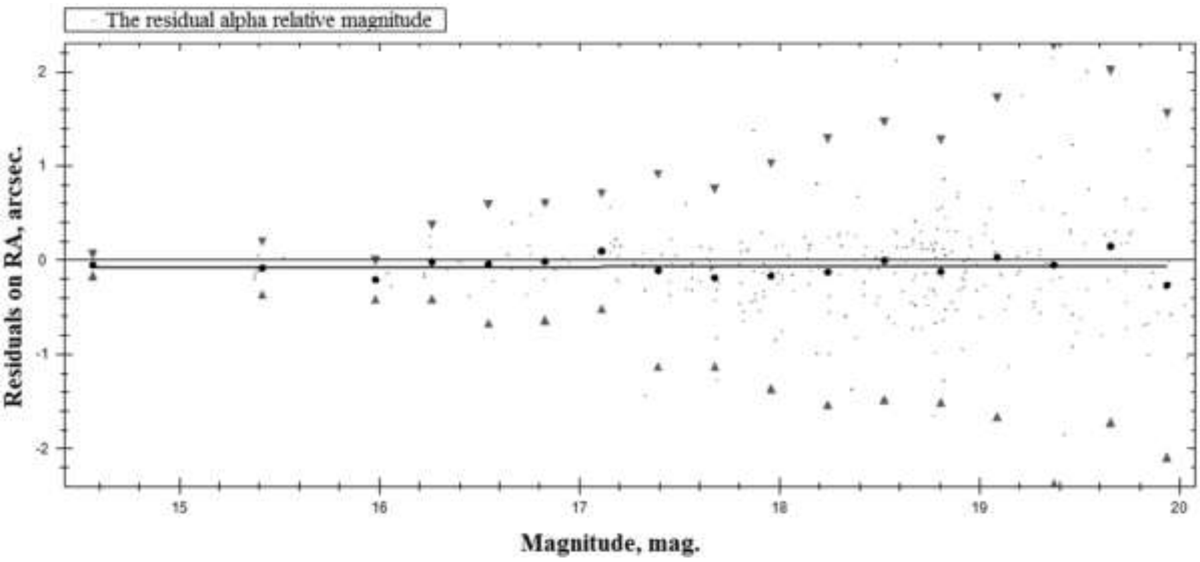


Fig-4: The residual dependence of on the right ascension from the brightness estimation (HORIZONT)

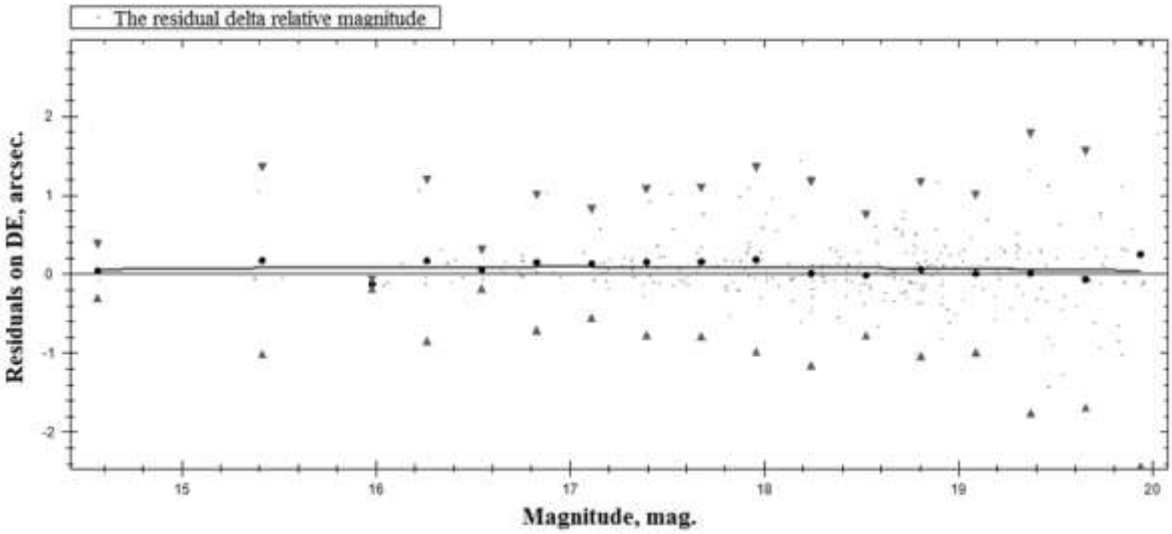


Fig-5: Dependence of the residuals on the declination from the brightness estimation (HORIZONT)

The comparative analysis of statistical characteristics of MSOPE and MCOPE of the parameters estimation of the object position on the CCD frame is displayed in a table 1. Also when using the MCOPE method RMS of the discrepancies of the equatorial coordinates of RA and DE provision of objects has been decreased. So the value of RMS of the RA residuals″ (MSOPE) has been decreased from 0.66 till 0.48″ (MCOPE), for DE from 0.57″ (MSOPE) has been decreased till 0.41″ (MCOPE).

Table-1: The key parameters of the residuals measurements of the numbered asteroids (close objects, MSOPE and MCOPE)

All the measurements						
Measurements processed	324		308		292	
The percent of the worst measurements screening according to the residual vector module	0%		5%		10%	
Methods	MSOPE	MCOPE	MSOPE	MCOPE	MSOPE	MCOPE
The average of the RA residuals, arc seconds	-0,09	-0,08	-0,06	-0,09	-0,04	-0,07
The average of the DE residuals, arc seconds	0,06	0,08	0,05	0,04	0,02	0,03
RMS of the RA discrepancies, arc seconds	0,66	0,48	0,44	0,34	0,37	0,29
RMS of the DE residuals, arc seconds	0,57	0,41	0,40	0,31	0,33	0,26

The developed method of the image parameters estimation of the objects shows the equally high precision, both for relatives, and for trailed objects.

CONCLUSIONS

New iterative methods of the asteroid coordinates' estimation on the digital image has been developed. The method of the close object position estimation (MCOPE) on the CCD frame as a model of the object image form the pixel Gaussian model is used. The pixel Gaussian model (7) of the object image describes the distribution of the pixels brightness of the intra frame processing studied area.

In order to find the estimates of the objects parameters, the criterion of the smallest squares and the Levenberg–Marquardt algorithm (LMA) for its minimization is used. Using LMA allows quickly and with the smallest expenses to put into practice the offered methods.

The method of the objects position estimation, blurred with proper motion works on the CCD frame in the same way as MCOPE. The difference from the above mentioned method is that the fact of the object motion during the exposition is considered. The model of the object image of the specified class is set (9) of Gaussians, located evenly along the direction of the object motion.

The results of the research show that in the average ranges of asteroids brightness the RMS value estimates the object position on the right ascension decreased by 57%, and on declination for 51% in comparison with the method of the single object position estimation (MSOPE). In that way, on the specified range, according to the indicator of the maximum module of RA and DE residual, without rejecting the worst measurements, the new methods provide the results almost twice best than MSOPE.

The efficiency of the offered methods was shown during trial operation as parts of the CoLiTec program [18-20] of automatic asteroids and comets discovering on the set of the CCD frame.

As a result of CoLiTec exploitation in the H15, C15, D00 and A50 observatories more than 1500 small bodies of the solar system have been discovered.

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