INTRODUCTION

Formulation of the problem. Innovative methods of teaching natural and physical-mathematical disciplines in higher education institutions involving computer modeling, as well as their use by teachers, are the object of special attention in scientific pedagogical literature (Hodson, 2014; Kafyulilo et al., 2015; Mar'yan et al., 2020). However, special training of future teachers in digital information processing and computer modeling of natural phenomena is not widespread enough and needs additional attention. This is evidenced...
both by the analysis of the bibliography of scientific and pedagogical research, and by the analysis of the curricula of educational programs for the training of natural science teachers. For example, the curriculum for training future physics teachers in Slovak universities does not include this subject (Yurkovych et al., 2017). Students - future teachers can familiarize themselves with the content of computer modeling of natural phenomena, in particular, in the process of studying special disciplines: "Digital technologies in teaching physics", "Computer information technologies in physics", "Computing technology in physics". Unfortunately, a similar situation with mastering the content of these disciplines is observed in other universities in Slovakia and Ukraine (Sladek et al., 2011; Mar'yan et al., 2020).

The analysis of current research. In the process of teaching natural sciences, attention is mostly focused on the large amount of material and its unstructured nature. Insufficiently developed are: interconnection and correlation with other disciplines; practical application of modern information technologies. It is worth emphasizing the essential role of intuitive thinking in the implementation of the fractal approach to teaching natural sciences. Intuitive perception is directly related to the use of visualization tools, modern advances in programming and artificial intelligence, namely object-oriented programming (Sugden, 2009; Yurkovych et al., 2017; Mar'yan et al., 2020). We note that the implementation of the fundamental principles of object-oriented programming (encapsulation, inheritance, polymorphism) ensures the efficiency of complex assimilation of information (Yurkovych et al., 2017). They define the concept of the information architecture of the perception of physical-mathematical/natural sciences educational disciplines. Therefore, their use in the field of physical and mathematical/science education is extremely relevant.

The purpose of the article. The purpose of the study is to substantiate the possibility of applying the fractal approach in science education using computer modeling. The subject of the study is the introduction of an innovative fractal approach, which enables future teachers to independently and creatively conduct computer modeling of natural phenomena (Luft, 2001; Yurkovych et al., 2017).

METHODS OF THE RESEARCH

Significant volumes of information and their unstructuredness are not positive aspects of modern physical and mathematical education. There is an internal natural interrelationship and structuredness between various fields of education, science, information technologies, which is manifested and can be demonstrated using the fractal approach (Yurkovych et al., 2017). Features of the fractal approach in teaching natural sciences reproduce the unique functioning of fractals in the natural environment. Namely, it is the implementation of a "simple" algorithm, self-similarity on spatial and temporal scales, self-sufficiency due to the reproduction of the minimum of energy dissipation (Frame & Mandelbrot, 2002; Mar'yan et al., 2020). At the same time, this algorithm implements extremely complex structures. With regard to physical and mathematical education, this means that when teaching one of the sections (subsections) of the discipline, a conjugate algorithm can be defined, which is developed and implemented in subsequent sections. And, in this way, a complex structure is formed while preserving the integrity of the perception of the material. In this case, the use of computer modeling is essential (Yurkovych et al., 2017). It is important to note that during the formation of this and subsequent iterations, the "algorithm" of the fractal structure, laid down at the level of the previous iteration, is preserved. This ensures self-sufficiency and localization of perception. Computer modeling and features of modern object-oriented programming languages, developed on the principles of artificial intelligence (encapsulation, inheritance and polymorphism), determine the thoroughness and spontaneity of teaching the material. In this regard, each subsequent iteration uses the properties of the previous one, and at the same time, it should contain new information (property, method) (Mar'yan et al., 2020) and realize the spontaneity of the educational process.

Fractal, in the context of the application of physical and mathematical education, is an information branched structure, the spatial dimension of which is described by a fractional number (Frame & Mandelbrot, 2002). It is this aspect of fractality that ensures the integrity and completeness of information flows when forming complex structures. However, the complexity is achieved through minimum energy dissipation and energy-informational self-similarity (Mar'yan et al., 2020). This testifies to the expediency and adequacy of their application in the teaching of science pedagogical disciplines, which at the current stage are insufficiently developed and argued (Windschitl, 2004; Kuo et al., 2013; Kafyulillo et al., 2015). Aspects of geometric, algebraic and stochastic fractals (Fig. 1) can appear as attributes of the functioning of the educational process.

![Fractal Structures](image)

Fig.1. The example of fractal structures (geometrical fractal — Koch's snowflake, algebraic fractal — Mandelbrot's set, stochastic fractal — flow of bubbles) (Frame & Mandelbrot, 2002).

To implement the approach, the visual programming environment Delphi and the algorithmic programming language Object Pascal, which is built on the principles of object-oriented programming and the latest modern information technologies, are used (Lotter et al., 2007; Mar'yan et al., 2020). Along with using Object Pascal, other object-oriented languages such as C++, Java, and Python can be recommended. Students of two universities participated in the educational experiment: Uzhhorod National University (Uzhhorod, Ukraine) and University of Preshov in Preshov (Preshov, Slovakia). Control groups consisted of 10-18 students. After completion of certain
stages of students’ training, a test control was conducted, which was supposed to reveal the effectiveness of the proposed approach. Part of the control questions is aimed specifically at identifying students’ creative interest in further study and application of computer modeling of natural phenomena.

RESULTS OF RESEARCH

Iterations of the fractal structure were developed on the example of studying the sections of physics "Geometric Optics" and "Wave Optics" with the complex use of features of geometric, algebraic and stochastic fractals. The development and principles of the branches of the fractal structure are shown in Fig. 2-4. It should be noted that the concepts used in computer modeling are also based on the principles of object-oriented programming: encapsulation, inheritance and polymorphism. That is, it is object-oriented modeling.

Fig. 2. An example of a branch fractal structure: computer modeling of the Snell’s laws.

An example of the laws of reflection and refraction of light, the Snell’s law is shown in Fig. 2. The branch contains numerical calculations and computing of optical parameters (Fig. 2, a), 2th and 3th steps of computer modeling with the visual interface in the environment Delphi (Fig. 2, b-c), testing of perception of information (Fig. 2, d). The algorithm includes the features of computer modeling of the light propagation process at the interface of two environments, and is further used for the following phenomena (steps), forming a more complex but internally self-sufficient fractal structure. During of the lecture students consider the basic laws of geometrical optics and types of light reflection: mirror (parallel light rays remain parallel after reflection (smooth even surfaces) and diffuse (parallel rays after reflection are scattered in all directions (rough uneven surfaces) with the immediate and spontaneous transition to computer modeling. After consideration of these laws in the environment of visual programming Delphi, the students create the own interface (Fig. 2): students have the opportunity to directly modify the parameters of the optical system (the factors of reflectivity and diffuseness, angle of incidence, refractive index), means of visualization of the rays in Delphi environment (types of lines, colors of the incident and reflected rays) and become active self-sufficient participants in conducting computer experiment. It is important to develop the algorithm of information perception by students on the intuitive level that will be used and developed further in later iterations (lectures).

Similarly, the following branches of the physics section "Geometric optics" and "Wave Optics" are considered:

- The total internal light reflection (Fig. 3).
• Determination of light transmittance of a thin film taking into account multiple reflections and using the methods of approximation and computing (Figs. 3, 4).
• Wave Optics.
• Organic use of the Internet, development of web pages on the topics “Geometric optics” and “Wave Optics”, interactive access to computer modeling and mobility of students from different educational institutions.
• Organic use and combination of distance learning, means and innovative communication technologies of Google Meet and Zoom.
• Other subsequent branches (sections of physics).

![Image of fractal structure](image)

Fig. 4. The branch of a fractal structure: visual interface of the modeling optical power and focal distance of a thick lens (a is numerical calculations of optical parameters, b-c are the modeling of optical parameters, d is testing of perception of information).

Each iteration (section of physics) is characterized by a synergy - adding new iteration provides a qualitative perception of the information (without a mechanical and artificial outside introduction of the division according to themes), the formation of the whole in which the student becomes an active participant. This synergy creates a unique fractal structure, capable of development and functioning. The iterations discussed above can be complemented and developed, in particular by involvement of learning using the testing tools, the exchange of information using the Internet. That is, the process generates and allows an infinite spontaneous number of steps, which is essential for fractal structures (Sherin, 2006; Mar'yan et al., 2020).

Based on the presented approach of the joint and synchronized synergetic usage of lectures on physics and computer modeling, the fractal structure is formed on an intuitive level (Table 1). Functioning of this structure is manifested in the transition to the presence of students’ self-sufficiency, involving the use of creative approach, increased the hyper sensibility to the radically new, and the desire to apply the obtained information in cardinally new areas. For example, the phenomenon of total internal reflection along with the classical perception of physics as a process of dissemination of light are also transistor and diode effects of dissemination information, cellular communication, cloud technologies (Informational and Technological Sciences), preparing a hamburger, croissant, cupcake (Technological and Culinary Sciences), the Belousov-Zhabotinsky reaction with the formation of dancing rhythms Latin Dance, Greek Syrtaki, Slovakian Folk Chardash, German Folk Dances (Humanities Sciences), and so similar. An example of the formation of a integrated fractal structure in the teaching of sections of physics “Geometric optics”, “Wave optics” is given in Table 1. It should be noted that this approach can be combined and synchronized with other subject areas, for example, mathematics, economics, geography, society, etc.

**Table 1.**

<table>
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<tr>
<th>STEPS</th>
<th>LEVELS OF INFORMATION PERCEPTION</th>
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<tr>
<td>THE FIRST STEP</td>
<td>Physics (“Geometric optics”, “Wave optics”)</td>
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<tr>
<td>THE SECOND STEP</td>
<td>Physics and Computer Modeling using object-oriented programming</td>
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<td>THE THIRD STEP</td>
<td>The intuitive level perception of information by students and the formation of the fractal structure</td>
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<tr>
<td>THE FOURTH STEP</td>
<td>The hyper sensibility and spontaneous distribution of fractal structure in the integrated environment</td>
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<td>THE FIVE STEP</td>
<td>The formation of an integrated fractal structure</td>
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**DISCUSSION**
Thus, a fractal structure in teaching one of the sections of physics, “Geometric optics,” is formed (it can be easily and spontaneous spread to other branches of physics). The advantages of the proposed approach are obvious: the corresponding physics section is perceived as a single unit without the mechanical separation into its component parts; and the possibility of forming branched structures according to a single algorithm that can be extended to other branches of physics, while maintaining the integrity (fractality) at the level of several sections. It should be noted that unlike the classical approach, which is based on the assimilation and learning of a certain amount and volume of material, the fractal connections reflect the internal structure of the sections that are assigned spontaneously (Windschitl, 2004; Sherin, 2006; Yurkovych et al., 2017).

The offered fractal approach was tested in Uzhhorod National University (Ukraine) for students at the Faculty of Physics and the University of Preshov (Slovakia) for students at the Faculty of Humanities and Natural Sciences at the Department of Physics, Mathematics and Technology. The study carried out the evaluation of the quality of training the discipline “Programming and mathematical modeling” for students according to the specializations “physics teacher”, “physics – applied physics and robotics” (Figs. 5, 6). The resulting dependencies are presented in Figs. 5, 6. They are not accidental; this naturally manifests itself thanks to the fractal approach to physics learning processes.

![Fig. 5. The diagram of the dependence of assessment ratings of the students in 2018/2019 academic year.](chart.png)

![Fig. 6. The diagram of academic performance in the study discipline “Programming and mathematical modeling” for 2018/2019 - 2021/2022 academic years.](chart2.png)

The research in University of Preshov was conducted in 2018/2019, 2019/2020 and 2021/2022 academic years. The experiment involved 10-15 students in the study subject “Methods of teaching physics” including computer modeling of physics sections “Geometrical optics”, “Wave optics”. The conducted rating control indicates the students' interest in computer simulation used in teaching physics in 88-93% of future physics teachers and the increase in their level of mastering material by 9-11%. During these classes, the activation of students, and in-depth perception of the material have been noted (Lotter et al., 2007; Özcan, 2015; Sladek et al., 2011; Mar'yan et al., 2020).

In Fig. 7 shown one of the elements of the implementation for the fractal approach when studying the physics section "Wave Optics", which qualitatively demonstrated the synergy effect. The analysis of the conducted research and the obtained results confirmed the innovativeness of the technologies of the fractal approach (Yurkovych et al., 2017).
CONCLUSIONS AND PERSPECTIVES FOR A FURTHER RESEARCH

The implementation of the fractal approach to the teaching of physical and mathematical disciplines using computer modeling in the object-oriented Delphi programming environment is substantiated. The formation of a fractal structure is established and iterations are defined that reflect the integrity and spontaneity of information presentation. The involvement of students of related specialties from two universities - Uzhhorod and Preshov - demonstrates the feasibility of using the fractal approach in training future teachers of natural sciences at higher educational institutions. Scientific studies of the behavior and adaptation of self-organized fractal structures in the context of physical and mathematical education are effective and will be continued in the future. The possibility of introducing tried-and-tested aspects of the fractal approach to science teaching in higher education institutions of Central Europe will also be developed and promoted.

REFERENCES