



First Announcement

Dear colleagues, we would like to inform you that

Department of Mathematics, University of Žilina

in cooperation with

Faculty of Electrical Engineering and Communication,

Brno University of Technology, Czech Republic

Institute of Mathematics, University of Bialystok, Poland

Kyiv National Economic University, Ukraine

will organize traditional

Conference on Differential and Difference Equations and Applications 2014 (CDDEA 2014)

in Hotel Družba, Jasná, Slovak Republic

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REDUCTION OF INTEGRAL BOUNDARY VALUE PROBLEMS TO A CERTAIN MODEL TYPE

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We consider the following non-linear integral boundary value problem

$$\frac{dx}{dt} = f(t, x), t \in [a, b], \int_a^b g(s, x(s)) ds = d.$$

Here we suppose that $f : [a, b] \times D \rightarrow \mathbb{R}^n$, $g : [a, b] \times D \rightarrow \mathbb{R}^n$ are continuous and f is locally Lipschitzian with respect to the second variable.

Let Ω_a and Ω_b be a convex subsets, in which one looks for initial value $x(a)$ and the value $x(b)$ respectively. The problem is to find a continuously differentiable solution $x : [a, b] \rightarrow D$ with initial value $x(a) = z \in \Omega_a$. We note, that the domain D will be defined by using convex linear combinations of subsets Ω_a and Ω_b .

At first we simplify the integral boundary conditions and reduce it to some two-point separated linear model-type conditions. Namely, we introduce the vectors of parameters

$$z := \text{col}(z_1, z_2, \dots, z_n) = x(a), \quad \eta := \text{col}(\eta_1, \eta_2, \dots, \eta_n) = x(b).$$

Instead of the given integral boundary value problem we will study the following "model-type" two-point BVP with separated parametrized conditions :

$$\frac{dx}{dt} = f(t, x), t \in [a, b], \quad x(a) = z, \quad x(b) = \eta.$$

We connect the introduced model type problem with the special parametrized sequence of functions $\{x_m(t, z, \eta)\}_{m=0}^{\infty}$ satisfying the boundary conditions $x(a) = z$, $x(b) = \eta$ for all $z, \eta \in \mathbb{R}^n$.

We prove the uniform convergence of the above sequence of functions to a certain limit function $x_{\infty}(t, z, \eta) = \lim_{m \rightarrow \infty} x_m(t, z, \eta)$ on the interval $t \in [a, b]$. The limit function $x_{\infty}(t, z^*, \eta^*)$ will be a solution of the original integral boundary value problem if and only if the pair of parameters (z^*, η^*) satisfies the following system of $2n$ algebraic equations:

$$[\eta - z] - \int_a^b f(s, x_{\infty}(s, z, \eta)) ds = 0, \quad \int_a^b g(s, x_{\infty}(s, z, \eta)) ds - d = 0.$$