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# Time of quantum information transfer from one atom qubit to another taking into account damping of quantum states

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## Abstract

The dynamics of the model quantum system consisting of two stationary identical two-level atoms, one of which is irradiated by a real photons field. The influence of states damping on the process of quantum information transfer between two atoms qubits at arbitrary distances is studied, taking into account the retarded dipole-dipole interaction of atoms. Under rather general assumptions, the system of non-stationary equations is obtained that describes the evolution of the amplitudes of possible states of the compound system “atom  $A(1)$  + atom  $A(2)$  + field” in symmetric and antisymmetric channels of interaction of an atom pair with the field of real photons. For the first time, the combined influence of retarded dipole-dipole interaction and state damping effects on the process of transmission of quantum information from one atom qubit to another at arbitrary interatomic distances is investigated.

## 1 Introduction

Because of the rapid development of quantum optics, many-particle problems describing systems of qubits controlled by external fields have been of increasing interest lately [1]. There are many different quantum systems modelling qubits – carriers of a unit of quantum information [1, 2]. In this capacity, one of the possible options is the use of two-level atoms. Usually, in the problems of quantum optics and quantum informatics, the connection of atoms is carried out via retarded interaction of atoms with each other, and the coherent control of the system is carried out via their interaction with the field of real photons [1, 3, 4].

Promising schemes for implementing two-qubit quantum logic operations with neutral atoms can be based on the effect of resonant transfer of quantum information between two distant qubits, taking into account the retarded dipole-dipole interaction of atoms in the field of real photons. The process of quantum information transfer from one atom to another due to the resonance interaction of atoms

was previously studied [5, 6] under the rather strict condition that the time  $\tau$  of the excitations exchange between atoms is significantly shorter than the life time  $\Gamma_n^{-1}$  of an atom in the excited state  $|n\rangle$ . However, this approach is acceptable only when the interatomic distance  $R$  is much smaller than the characteristic wavelength  $\lambda_0 = 2\pi c/\omega_0$  ( $\omega_0 = (E_n - E_0)/\hbar$ ) in the spectrum of interacting two-level atoms ( $R \ll \lambda_0$ ). The closer the atoms are to each other, the faster the transfer of excitation energy from one atom to another occurs. Therefore, in the works [5, 6], the damping of states in the process of quantum information transfer between qubits was not taken into account. However, the further the atoms from each other, the more the role of state damping effects. It is also significant that already at  $R \gtrsim \lambda_0$ , the effects of the interaction retardation begin to appear as well. Therefore, rather general questions about the role of damping of states and effects of interaction retardation in the process of quantum information transfer from one atom qubit to another at arbitrary interatomic distances require special study.

The problem considered in this paper on the time  $\tau$  of quantum information transfer from one atom to another has two stages. The first stage consists in the study of the influence of the retarded dipole-dipole interaction of atoms on the optical properties of the energy levels of a system of two identical two-level atoms, and the second one – in solving the dynamic problem – the system of time equations (18) for the probability amplitudes at resonant (non-resonant) absorption of a photon by one of the atoms of this system.

This paper is a logical continuation and development of research started by the authors at the first stage in [7]. On the basis of a detailed study of the pair interaction [8, 9, 10, 11, 12] of two hydrogen-like atoms located at an arbitrary distance from each other, general analytical expressions for the widths and shifts of the collective (symmetric and antisymmetric) states of the system of two dipole-interacting two-level atoms are below obtained.

## 2 The system of time equations for the amplitudes of states in the two-level approximation

In this section, we will study the interaction of two identical two-level atoms  $A(1)$  and  $A(2)$  with a free electromagnetic field in the electro-dipole approximation. Throughout below, the concept of a complex (“compound”) object [13] is used, namely “atom  $A(1)$  + atom  $A(2)$  + field  $F$ ”. In accordance with this concept, when analyzing the processes of quantum information transfer between qubits  $A(1)$  and  $A(2)$ , it is convenient to consider the field as a system with a defined number of quanta  $n_\omega$  and include it in the unperturbed Hamiltonian  $\hat{H}$ . We will consider the state of the diatomic quantum system “ $A(1) + A(2) + F$ ” in the coordinate representation, and the state of the photon field – in the occupation number representation, which is convenient when studying systems with a variable number of particles. In the future, we will assume that the atoms are at an arbitrary distance from each other. In this case, it is necessary to take into account the retarded dipole-dipole interaction of atoms. Without taking into account the interaction of atoms with the field, the Hamiltonian  $\hat{\mathcal{H}}_0$  of the complete compound

system “ $A(1) + A(2) + F$ ” consists of the Hamiltonian  $\hat{H}_F$  of free quanta and the Hamiltonian  $\hat{H}$  of the system of two interacting two-level atoms  $A(1)$  and  $A(2)$ :

$$\hat{\mathcal{H}}_0 = \hat{H} + \hat{H}_F = \hat{H}_1(\vec{r}_1) + \hat{H}_2(\vec{r}_2) + \hat{V}_{dip}^{(\pm)}(\vec{r}_1, \vec{r}_2; R) + \hat{H}_F. \quad (1)$$

Here  $\hat{H}_1, \hat{H}_2$  are Hamiltonians of isolated atoms  $A(1)$  and  $A(2)$ , respectively;  $\vec{R}$  is the distance vector between atomic nuclei;  $\hat{V}_{dip}^{(\pm)}(\vec{r}_1, \vec{r}_2; R)$  is the interaction operator of atoms  $A(1)$  and  $A(2)$  at an arbitrary distance from each other in the electric dipole approximation (see [7]); electron radius vectors  $\vec{r}_1$  and  $\vec{r}_2$  refer to the first  $A(1)$  and the second  $A(2)$  atom, respectively. In the representation of second quantization, the Hamiltonian  $\hat{H}_F$  and the energy  $E_\omega$  of the photon free field are defined by standard expressions

$$\hat{H}_F = \sum_{\omega} \hbar\omega \hat{a}_{\omega}^{\dagger} \hat{a}_{\omega}, \quad E_{\omega} = \sum_{\omega} \hbar\omega n_{\omega}, \quad (2)$$

where  $n_{\omega}$  is the number of photons of frequency  $\omega$  in the normalizing volume  $V_R$ ,  $\hat{a}_{\omega}^{\dagger}$  and  $\hat{a}_{\omega}$  are the operators of photons generation and annihilation in the representation of the occupation numbers  $n_{\omega}$  of the photon states  $|n_{\omega}\rangle$ . Below, transitions accompanied by a change in the number of photons of a specific field mode will be considered everywhere, so the indices characterizing the polarization  $\alpha$  ( $\alpha = 1, 2$ ) and the direction of the wave vector  $\vec{k}$  are omitted everywhere. The Hamiltonian of the field  $\hat{H}_F$  (2) is written in the standard form where zero energy is omitted using the appropriate choice of the origin.

As a basis for writing the time wave function of the compound system, we will choose the products of the eigenwave functions of the Hamiltonians  $\hat{H}$  and  $\hat{H}_F$ . Let the state of the compound system “ $A(1) + A(2) + F$ ” at the initial moment of time  $t_1 = 0$  (in the absence of interaction of atoms with the field) be described by a wave function

$$\tilde{\Psi}_m^{(0)} = |n_{\omega}\rangle \Psi_m^{(0)} \equiv |n_{\omega}\rangle \Psi_0(1) \Psi_0(2). \quad (3)$$

Here, the eigenfunction of the field  $|n_{\omega}\rangle$  corresponds to a certain number of photons, and the wave function  $\Psi_m^{(0)}$  of the initial state of a pair of identical atoms is given by the expression [7]

$$\begin{aligned} \Psi_0(1) \Psi_0(2) &= \tilde{\varphi}_0(1) \tilde{\varphi}_0(2) + \\ &+ \sum_{n_1 n_2} \frac{\langle \tilde{\varphi}_{n_1}(1) \tilde{\varphi}_{n_2}(2) | \hat{V}_{dip}^{(\pm)} | \tilde{\varphi}_0(1) \tilde{\varphi}_0(2) \rangle}{2E_0 - E_{n_1} - E_{n_2}} \tilde{\varphi}_{n_1}(1) \tilde{\varphi}_{n_2}(2). \end{aligned} \quad (4)$$

The initial state of the  $j$ th atom  $A(j)$  is determined here by the index 0 with the energy  $E_0$  and the wave function  $\tilde{\varphi}_0(j)$ . The summation in (4) is carried out over all possible intermediate states in the spectrum of interacting atoms, except for the initial state  $|00\rangle$ . The second term on the right-hand side of the expression (4) coincides with the first-order corrections of the non-relativistic perturbation theory to the wave functions of a system of two atoms with non-degenerate energy levels, where the interaction between atoms with the operator of retarded dipole-dipole interaction  $\hat{V}_{dip}^{(\pm)}$  from [7] is taken as a perturbation.

The process of photon absorption by one of the atoms of the  $A(1) + A(2)$  system can be formally considered as a quantum transition  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_n^{(0)}$  of the complete compound system “ $A(1) + A(2) + F$ ” into a new state

$$\tilde{\Psi}_n^{(0)} = |n_\omega - 1\rangle \Psi_n^{(0)} \equiv |n_\omega - 1\rangle \Psi_{s(a)} \equiv \tilde{\Psi}_{s(a)}^{(0)}, \quad (5)$$

where the function  $|n_\omega - 1\rangle$  determines the state of the photon field, and the function  $\Psi_{s(a)}$  describes the symmetric (antisymmetric) state of a pair of identical two-level atoms:

$$\Psi_s = \Phi_s \exp(-i\delta E_s t_1/\hbar), \quad \Psi_a = \Phi_a \exp(-i\delta E_a t_1/\hbar). \quad (6)$$

Here

$$\Phi_s(1, 2) = \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) + \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)], \quad (7)$$

$$\Phi_a(1, 2) = \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) - \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)], \quad (8)$$

are superpositions of different coherent states of atoms, one of which is in the ground state  $|0\rangle$ , and the other is in the excited state  $|n\rangle$ ;  $t_1$  is the local time for the pair of atoms associated with the location of the first atom  $A(1)$ .

We determine the transition energy  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_n^{(0)}$  by equations

$$E_m^{(0)} - E_n^{(0)} = E_0 - E_n \mp \delta E_s = -\hbar(\omega + \Delta), \quad (9)$$

where the upper minus sign corresponds to the symmetric state  $\Psi_s$  of the pair of atoms, and the lower plus sign corresponds to the antisymmetric state  $\Psi_a$ ;  $\omega$  is the frequency of a real photon;  $\Delta = \omega - \omega_0$  is the shift of the field frequency  $\omega$  from the atomic transition frequency  $\omega_0$ , and the shift of the energy levels of the symmetric (7) and antisymmetric (8) states of pair of atoms is determined by the expression (see [7])

$$\delta E_{s,a}(R) = \pm \text{Re} \Delta E_{AA}(R) = \pm e^2 |\langle n|\vec{r}|0\rangle|^2 F(1, 2; R). \quad (10)$$

Here  $\langle n|\vec{r}|0\rangle$  is the matrix element of the atomic transition  $|n\rangle \rightarrow |0\rangle$ , and

$$F(1, 2; R) = \left[ \frac{\Phi(1, 2)}{R^3} - \frac{\omega_0^2 \Phi'(1, 2)}{c^2 R} \right] \cos\left(\frac{\omega_0 R}{c}\right) + \frac{\omega_0 \Phi(1, 2)}{c R^2} \sin\left(\frac{\omega_0 R}{c}\right), \quad (11)$$

where

$$\Phi(1, 2) \equiv \cos\theta_1^x \cos\theta_2^x + \cos\theta_1^y \cos\theta_2^y - 2\cos\theta_1^z \cos\theta_2^z,$$

and

$$\Phi'(1, 2) \equiv \cos\theta_1^x \cos\theta_2^x + \cos\theta_1^y \cos\theta_2^y$$

are the geometric factors depending on the orientation of dipole transitions in both atoms. At the same time,  $\theta_1^x, \theta_1^y, \theta_1^z$  are the angles formed with the axes  $Ox, Oy, Oz$  by the direction of the transition dipole moment in the first atom  $A(1)$ ;  $\theta_2^x, \theta_2^y,$

$\theta_2^{\pm}$  are the corresponding angles for the second atom  $A(2)$ . The plus and minus signs in the expression (10) correspond to the symmetric  $\Phi_s$  and antisymmetric  $\Phi_a$  wave functions of the pair of identical atoms.

Let the quantum transition  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_n^{(0)}$  corresponds to the elementary act of the disappearance of one real photon at the location of the atom  $A(2)$ . In the one-mode approximation, the operator of the vector potential  $\hat{A}$  binding the atom  $A(2)$  and the field is defined in the form of a standard linear form respect to the operators of generation  $\hat{a}_\omega^+(t_1)$  and annihilation  $\hat{a}_\omega(t_1)$  of photons, i.e.

$$\hat{A} = \vec{A}_0 \left[ e^{i\vec{k}\vec{r}'_2} \hat{a}_\omega(t_1) + e^{-i\vec{k}\vec{r}'_2} \hat{a}_\omega^+(t_1) \right], \quad (12)$$

where  $\vec{r}'_2$  is the radius vector of the electron in the  $A(2)$  atom, and the photon operators in the Heisenberg representation are described by free evolution:  $\hat{a}_\omega^+(t_1) = \hat{a}_\omega^+(0) \exp(i\omega t_1)$ ,  $\hat{a}_\omega(t_1) = \hat{a}_\omega(0) \exp(-i\omega t_1)$ , so the operators  $\hat{a}_\omega(0)$  and  $\hat{a}_\omega^+(0)$  do not depend on coordinates and time. In the expression (12), the following notation is introduced for convenience

$$\vec{A}_0 = \sqrt{\frac{2\pi\hbar c^2}{\omega V_R}} \vec{e},$$

where  $\vec{e}$  is the unit polarization vector of this mode photon. Focusing on electric dipole transitions and limiting ourselves to one-photon processes, we represent the Hamiltonian of the interaction of a one-electron atom  $A(2)$  with the field of real photons in the following form:

$$\hat{H}_{int} = -\frac{e}{mc} \hat{p}_2 \vec{A}, \quad (13)$$

where  $\hat{p}_2$  is the electron momentum operator of the atom  $A(2)$ . Then the complete Hamiltonian  $\hat{\mathcal{H}}$  of the compound system “ $A(1) + A(2) + F$ ” is of the form:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{H}_{int} = \hat{H} + \hat{H}_F + \hat{H}_{int}. \quad (14)$$

We will obtain the equations describing the quantum transition  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_n^{(0)} = \tilde{\Psi}_{s(a)}^{(0)}$ . In this case, one can choose the set of atom-field functions of the form (3) and (5) as a basis for writing the time wave function of the compound system “atom  $A(1) +$  atom  $A(2) +$  field  $F$ ”. We denote by  $a_m(t_1)$  the amplitude of the probability of being a pair of atoms  $A(1) + A(2)$  in the ground state  $\Psi_m^{(0)}$  (4) and the field in state  $|n_\omega\rangle$ . Then  $a_n(t_1) \equiv a_{s(a)}(t_1)$  acquires meaning of the probability amplitude of finding the compound system in the symmetric (antisymmetric) state  $\Psi_{s(a)}$  (5) by atomic degrees of freedom and in the  $|n_\omega - 1\rangle$  state by field degrees of freedom. Let us further take into account that the interaction of the  $A(2)$  atom with the field of real photons is determined by the operator (13), which has a certain type of symmetry with respect to the permutations of atoms. Accordingly, the states with different symmetries,  $\tilde{\Psi}_s^{(0)}$  and  $\tilde{\Psi}_a^{(0)}$ , are not related to each other, since the total Hamiltonian  $\hat{H}$  (1) of the system of two identical atoms (and in

particular the retarded dipole-dipole interaction of atoms  $\hat{V}_{dip}^{(\pm)}$  is symmetric with respect to the permutation of nuclei. Thus, the dynamics of symmetric  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_s^{(0)}$  and antisymmetric  $\tilde{\Psi}_n^{(0)} \rightarrow \tilde{\Psi}_a^{(0)}$  channels of interaction of the pair of atoms with the field of real photons can be considered separately from each other. In the case of a symmetric channel  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_s^{(0)}$ , for the states amplitudes  $a_m(t)$  and  $a_n(t)$  (i.e. the coefficients for the basis functions  $\tilde{\Psi}_m^{(0)}$  (3) and  $\tilde{\Psi}_s^{(0)}$  (5)) from the non-stationary Schrödinger equation with the Hamiltonian (14) we obtain in the usual way [14] the following system of differential equations:

$$\left. \begin{aligned} i\hbar \frac{da_m}{dt_1} &= F_{mn} \exp[i(\omega_{mn} + \omega + i\gamma_n)t_1] a_n = F_{mn} \exp(i(\varepsilon_+ + i\gamma_n)t_1) a_n, \\ i\hbar \frac{da_n}{dt_1} &= F_{nm} \exp(-i(\varepsilon_+ + i\gamma_n)t_1) a_m, \end{aligned} \right\} \quad (15)$$

where  $\varepsilon_+ = (\delta E_s - \hbar\Delta)/\hbar$ ,  $\gamma_n = \Gamma_n/2$ , and the damping constants  $\Gamma_n = \Gamma_{s(a)} = \gamma_0 + \gamma_{s(a)}$  of the symmetric (antisymmetric) state of the pair of atoms are determined by formulas (20)–(24) in [7]. The specificity of the influence of the retarded interatomic interaction  $\hat{V}_{dip}^{(\pm)}$  on the optical properties of diatomic systems  $A(1) + A(2)$  is that in the shifts  $\Delta E_{s(a)} = \delta E_{s(a)} - i\hbar\gamma_{s(a)}/2$  of energy levels  $E_{s(a)}$  of collective states  $\Psi_{s(a)}$  caused by this interaction appear imaginary terms, which are known [7] to lead to an additional broadening  $\gamma_{s(a)}$  of levels  $E_{s(a)}$  compared to the radiative broadening  $\gamma_0$ . In the system of equations (15),  $F_{mn}$  is the matrix element of the transition  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_s^{(0)}$ , which by introducing the effective dipole moment of the atom  $A(2)$  can be written in the form

$$F_{mn} = -\frac{i}{c} \sqrt{\frac{n_\omega}{2}} \omega_0 \vec{A}_0 \vec{d}_{n0}^{eff}(R) \exp(i\vec{k}\vec{R}). \quad (16)$$

Here  $\vec{A}_0$  is the amplitude of the vector potential,  $\vec{k}$  is the wave vector of a real photon ( $k = \omega/c$ ), which is absorbed at the location of the atom  $A(2)$  with the radius vector  $\vec{R}$ ;  $\vec{d}_{n0}^{eff}(R)$  is the matrix element of the electric dipole moment operator of the atom  $A(2)$ , calculated taking into account all terms of the wave function (4) of an initial state:

$$\vec{d}_{n0}^{eff}(R) = \vec{d}_{n0} + \vec{d}_{n0}^{\prime}(R) = \vec{d}_{n0} + \sum_{n_2} \frac{(E_{n_2} - E_0)}{\hbar\omega_0} \frac{\vec{d}_{0n_2} \langle \tilde{\varphi}_n(1) \tilde{\varphi}_{n_2}(2) | \hat{V}_{dip}^{(\pm)} | \tilde{\varphi}_0(1) \tilde{\varphi}_0(2) \rangle}{E_n + E_{n_2} - 2E_0}. \quad (17)$$

The first term  $\vec{d}_{n0}$  in (17) denotes the matrix element of the electric dipole moment operator of a single atom between the  $|n\rangle$  and  $|0\rangle$  states. The specific form of the dependence  $\vec{d}_{n0}^{\prime}(R)$  is related to the type of interaction between atoms  $A(1)$  and  $A(2)$ . In the case of the dipole interaction of atoms considered here,  $\vec{d}_{n0}^{\prime}(R)$  decreases exponentially with the increase of the interatomic distance  $R$ . Thus, the second term  $\vec{d}_{n0}^{\prime}(R)$  in the r. h. s. of (17) takes into account the influence of the interaction of atoms  $A(1)$  and  $A(2)$  on the absorption process of a real photon via the field of virtual photons. This interaction is shown in [15, 16, 17] to lead to



the inducing the so-called electronic polarizing field with the corresponding vector potential at the location of the observer atom  $A(1)$ .

With the substitution  $\tilde{a}_n(t_1) = a_n(t_1)e^{-\gamma_n t_1}$ , the initial system of differential equations (15) for the probability amplitudes  $\tilde{a}_n$  and  $a_m$  takes the well-known form (see, for example, [13, 18]):

$$\left. \begin{aligned} i\hbar \frac{da_m}{dt_1} &= F_{mn} \exp[i(\omega_{mn} + \omega)t_1] \tilde{a}_n = F_{mn} \exp(i\varepsilon_+ t_1) \tilde{a}_n, \\ i\hbar \frac{d\tilde{a}_n}{dt_1} &= -i\gamma_n \hbar \tilde{a}_n + F_{nm} \exp(-i\varepsilon_+ t_1) a_m. \end{aligned} \right\} \quad (18)$$

The system (18) should be solved under the initial condition  $a_m(0) = 1$ ,  $\tilde{a}_n(0) = 0$ . The form of the equations (18) is the same for both channels of interaction of the pair of atoms with the field of real photons; in the system (18), for the antisymmetric channel  $\tilde{\Psi}_m^{(0)} \rightarrow \tilde{\Psi}_a^{(0)}$  it is necessary to replace  $\Gamma_s$  with  $\Gamma_a$  and  $\varepsilon_+$  with  $\varepsilon_-$ , where  $\varepsilon_- = -(\delta E_s + \hbar\Delta)/\hbar$ . Based on the system of equations (18), it is possible to describe the mutual influence of atoms  $A(1)$  and  $A(2)$  in the field of real photons quite completely. At the same time, the single terms in (18) have the following physical meaning: 1) the term  $-i\hbar\gamma_n\tilde{a}_n$  describes the damping of symmetric (antisymmetric) states  $\tilde{\Psi}_{s(a)}^{(0)}$  of the compound system; 2) other terms in (18), proportional to the matrix elements  $F_{mn}$  and  $F_{nm}$ , describe the quanta absorption by one of the system atoms.

The system of equations (18) is adequate to the two-level approximation in the system of atoms [14], when in the Schrödinger wave equation the most significant effect arises from the resonant ( $\omega = \omega_0$ ) terms  $\sim \exp[\pm i(\omega_0 - \omega)t_1]$ , in which the dependence on time is determined by a small difference in frequencies  $\omega_0 - \omega$ . A similar system of equations without taking into account the damping of states ( $\gamma_n = 0$ ) was obtained in [6] (see also the book [14]).

A certain disadvantage of the constructed system of equations (18) (as well as the original system (15)) is its non-Hermiticity leading to non-conservation of the normalization of the amplitudes  $a_m(t_1)$ ,  $\tilde{a}_n(t_1)$  of collective states  $\tilde{\Psi}_m^{(0)}$  (3) and  $\tilde{\Psi}_n^{(0)}$  (5) as a result of their decay into the continuum. Physically, this is explained by the fact that a variable external perturbation (13) can ionize bound atomic states. These processes can be taken into account [13, 18] via introducing additional damping into  $\Gamma_{s(a)}$ :  $\Gamma_{s(a)} \rightarrow \tilde{\Gamma}_{s(a)} = \Gamma_{s(a)} + \gamma'_{s(a)}$ , where  $\gamma'_{s(a)}$  is the effective width taking into account the specified processes, and  $\tilde{\Gamma}_{s(a)}$  is the total width composed of contributions from all possible ways of “decay” of this quasi-stationary state. The result is the same system of equations (18), in which only the additional width  $\gamma'_{s(a)}$  is present from the states of the continuous spectrum. Thus, the difficulties with non-conserving the normalization of the amplitudes of discrete states can be circumvented in the same way as it is done in [13, 18]. However, below we will assume that  $\gamma'_{s(a)} t_1 \ll 1$  and therefore omit  $\gamma'_{s(a)}$ .

Let us consider the general solution of the system (18). It is clear that obtaining an exact solution of this system for an arbitrary form of  $F_{mn}(t_1)$  and at the same time for  $\gamma_n \neq 0$  is an obviously analytically unsolvable problem. In the absence of damping ( $\gamma_n = 0$ ), a time-periodic perturbation with frequency  $\omega$  would

directly lead to periodic oscillations of the amplitudes of both states  $\tilde{\Psi}_m^{(0)}$  (3),  $\tilde{\Psi}_n^{(0)}$  (5) with frequencies  $\omega^{(1,2)}$  determined by the roots of the corresponding secular equation [14]. Taking into account the damping ( $\gamma_n \neq 0$ ) leads to the appearance of imaginary parts in the frequencies  $\omega^{(1,2)}$  and, thereby, to the damping of the amplitudes of the collective states  $\tilde{\Psi}_m^{(0)}$  and  $\tilde{\Psi}_n^{(0)}$ . Omitting intermediate calculations, we will immediately give the final result for the amplitudes  $a_m$  and  $\tilde{a}_n$  in the case of a time-periodic perturbation (of the form (12), (13)):

$$a_m = \exp\left(-\frac{\gamma_n - i\varepsilon_+}{2}t_1\right) \left\{ \cos((\Omega_+ + i\beta_+)t_1) - \frac{i(\varepsilon_+ + i\gamma_n)}{2(\Omega_+ + i\beta_+)} \sin((\Omega_+ + i\beta_+)t_1) \right\},$$

$$\tilde{a}_n = a_n e^{-\gamma_n t_1} = -\frac{iF_{nm}}{(\Omega_+ + i\beta_+)\hbar} \exp\left(-\frac{\gamma_n + i\varepsilon_+}{2}t_1\right) \sin((\Omega_+ + i\beta_+)t_1). \quad (19)$$

Here  $\Omega_+$  and  $\beta_+$  are the real numbers determined by the expression:

$$\Omega_+ + i\beta_+ = \left\{ \frac{|F_{nm}|^2}{\hbar^2} + \frac{(\varepsilon_+ + i\gamma_n)^2}{4} \right\}^{1/2}.$$

The solution (19) for a symmetric interaction channel (and a similar solution for an antisymmetric one) depends complicatedly on three parameters:  $\varepsilon_+$ ,  $F_{nm}$ ,  $\gamma_n = \Gamma_s/2$  and time  $t_1$ . However, it can be verified that the general character of the dependence of  $|a_m(t_1)|^2$  and  $|\tilde{a}_n(t_1)|^2$  on time is of the form

$$\exp(-\gamma_n t_1) (A \exp(-\beta_+ t_1) + B \exp(-i\Omega_+ t_1) + C \exp(\beta_+ t_1)),$$

where the constants  $A$ ,  $B$ ,  $C$  are determined by the parameters  $\varepsilon_+$ ,  $F_{nm}$ ,  $\Gamma_s$ . Depending on the ratio between the three parameters  $\Gamma_s$ ,  $\beta_+$  and  $\Omega_+$ , the information system of two resonant atoms in the field of real photons is characterized by one or another mode of operation.

First of all, for any ratios between  $\Gamma_s$ ,  $\beta_+$  and  $\Omega_+$ , there is an initial stage of the process when  $t_1 \ll \Gamma_s^{-1}$ ,  $\beta_+^{-1}$ ,  $\Omega_+^{-1}$ . In this case, only terms containing  $F_{nm}$  and  $F_{mn}$  can be saved in the system (18). At the same time, when neglecting damping ( $\Gamma_s = \beta_+ = 0$ ), the solutions (19) turn into the well-known formulas of the book [14]. If  $\Gamma_s \gg \Gamma_s - \beta_+$ ,  $\Gamma_s^{-1} \ll t_1 \ll (\Gamma_s - \beta_+)^{-1}$ , then the quasi-stationary regime is realized. In a real situation, this regime corresponds to sufficiently long times ( $\Gamma_s t_1 \gg 1$ ) of observing the compound system. At the same time, the distance  $R$  between atoms can be significantly larger or comparable to the wavelength of external radiation  $\lambda = 2\pi c/\omega$ . In this case, the damping of the states has a very non-trivial influence on the time dependence of the probabilities amplitudes  $a_m(t_1)$  and  $\tilde{a}_n(t_1)$  (see (19)); the damping effect is stronger, the larger the damping constant  $\Gamma_s$  and the smaller the ratio  $(\Gamma_s - \beta_+)/\Gamma_s$ .

Other type of regime – oscillatory – is possible when  $\Omega_+ \gg \Gamma_s$ ,  $\Omega_+^{-1} \ll t_1 \ll \Gamma_s^{-1}$ . This mode of time evolution of the “ $A(1) + A(2) + F$ ” system is characterized by the formulas (19), where  $\Omega_+ = \Omega_0 = \sqrt{\varepsilon_+^2/4 + |F_{nm}|^2/\hbar^2}$  and  $\beta_+ = \beta_0 = \varepsilon_+ \Gamma_s / 8\Omega_0$ . In the case of  $\Gamma_s = 0$ , the oscillatory mode was considered in detail in [6].

All modes of temporal evolution of the compound system mentioned here also occur for the antisymmetric channel of interaction. By changing the parameters characterizing the compound system “ $A(1) + A(2) + F$ ”, one can trace a continuous transition from one mode to another.

A feature of the problem being solved is the need to take into account the exchange of excitations between resonantly interacting atoms. The study of this problem carried out in [5] shows that when taking into account the dipole-dipole interaction of atoms without retardation the exchange of excitations between atoms is determined by the characteristic exchange time  $\hbar/\delta E'_{s,a}$ , where  $\delta E'_{s,a}$  is given by the expression

$$\delta E'_{s,a} = \pm \frac{e^2}{R^3} |\langle n|\vec{r}|0\rangle|^2 \Phi(1, 2). \quad (20)$$

If at some point in time the system consisting of two identical atoms is in a state in which only one atom is excited, then as shown in [5] due to resonance interaction this excitation is transferred to another atom in the time  $\tau' = \hbar/\delta E'_s$ . The closer the atoms are to each other, the faster the transfer of excitation energy from one atom to another occurs. At the same time, the time  $\tau'$  of excitation transfer is significantly shorter than the lifetime  $\Gamma_n^{-1}$  of an atom in an excited state. Therefore, in [5], the damping of states in the process of atom-to-atom excitation transfer was not taken into account.

As can be seen from the solution (19) of the system (18), taking into account the damping of one of the levels leads to “transferring” this damping to another level. So, for example, if at the initial point of time ( $t_1 = 0$ ) the state of  $\tilde{\Psi}_m^{(0)}$  was stable, then at following points of time ( $t_1 > 0$ ) it may decay with a constant equal to half the damping constant of the state  $\tilde{\Psi}_n^{(0)}$ . This effect has an analogy in the problem of the destruction of the  $2S_{1/2}$  metastable level of the hydrogen atom by an external field [18]. The determining parameter for this effect is the quantity  $\Gamma_{s(a)}\tau_{12}$ , where  $\tau_{12}$  is the effective time of excitation energy transfer from atom  $A(2)$  to atom  $A(1)$ . The revealed effect of the loss of quantum states coherence interferes with quantum computing and therefore must be minimized to the utmost. In this regard, the use of ultracold electroneutral atoms in highly excited states with the principal quantum number  $n \gg 1$  as qubits of a quantum computer seems to be the most promising. This implementation of a quantum computer becomes possible due to the rapid development in recent years of the technique of laser cooling and capture of neutral atoms in optical lattices or dipole traps [2, 4, 19, 20]. The highly excited states of the atom, called Rydberg states, have a number of unique properties, including long lifetimes and the ability to sense each other over a significant interatomic distance [4]. These properties make it possible to carry out quantum calculations provided there is no spontaneous relaxation of the levels.

In accordance with the set initial conditions  $a_m(0) = 1$ ,  $\tilde{a}_n(0) = 0$  at the point of time  $t_1 = 0$ , the considered compound system is in the state  $\tilde{\Psi}_m^{(0)}$ . However, already at following points of time  $t_1$  under the influence of the perturbation (13),

it evolves into a superpositional state:

$$\begin{aligned}
\tilde{\Psi}_s(t_1) &= \frac{1}{\sqrt{2}} e^{(-\frac{\Gamma_s}{4}t_1 + i\frac{\varepsilon_+}{2}t_1)} \times \\
&\times \left[ \cos((\Omega_+ + i\beta_+)t_1) - \frac{i(\varepsilon_+ + i\Gamma_s/2)}{2(\Omega_+ + i\beta_+)} \sin((\Omega_+ + i\beta_+)t_1) \right] \tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle - \\
&- \frac{iF_{nm}}{\hbar(\Omega_+ + i\beta_+)} e^{(-\frac{\Gamma_s}{4}t_1 - i\frac{\varepsilon_+}{2}t_1)} \sin((\Omega_+ + i\beta_+)t_1) \times \\
&\times \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) + \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)] e^{(-i\delta E_s t_1/\hbar)} |n_\omega - 1\rangle. \tag{21}
\end{aligned}$$

This wave function corresponds to the symmetric channel of interaction of the atoms pair with the field of real photons. In the case of the antisymmetric interaction channel, an explicit expression for the temporal wave function  $\tilde{\Psi}_a(t_1)$  of the considered compound system can be obtained from (21) using the substitutions  $\Omega_+ \rightarrow \Omega_-$ ,  $\beta_+ \rightarrow \beta_-$ ,  $\Gamma_s \rightarrow \Gamma_a$ , and  $\varepsilon_+ \rightarrow \varepsilon_-$ :

$$\begin{aligned}
\tilde{\Psi}_a(t_1) &= \frac{1}{\sqrt{2}} e^{(-\frac{\Gamma_a}{4}t_1 + i\frac{\varepsilon_-}{2}t_1)} \times \\
&\times \left[ \cos((\Omega_- + i\beta_-)t_1) - \frac{i(\varepsilon_- + i\Gamma_a/2)}{2(\Omega_- + i\beta_-)} \sin((\Omega_- + i\beta_-)t_1) \right] \tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle - \\
&- \frac{iF_{nm}}{\hbar(\Omega_- + i\beta_-)} e^{(-\frac{\Gamma_a}{4}t_1 - i\frac{\varepsilon_-}{2}t_1)} \sin((\Omega_- + i\beta_-)t_1) \times \\
&\times \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) - \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)] e^{(+i\delta E_s t_1/\hbar)} |n_\omega - 1\rangle.
\end{aligned}$$

Here the following notations are used in order to simplify the writing of formulas:

$$\varepsilon_\pm = (\pm\delta E_s - \hbar\Delta)/\hbar, \quad \Omega_\pm + i\beta_\pm = \left\{ \frac{|F_{nm}|^2}{\hbar^2} + \frac{(\varepsilon_\pm + i\Gamma_{s(a)}/2)^2}{4} \right\}^{1/2}. \tag{22}$$

Thus, the final state of the compound system “ $A(1) + A(2) + F$ ” is a superposition (linear combination) of states  $\tilde{\Psi}_s(t_1)$  and  $\tilde{\Psi}_a(t_1)$ :

$$\tilde{\Psi}(t_1) = \frac{1}{\sqrt{2}} [\tilde{\Psi}_s(t_1) + \tilde{\Psi}_a(t_1)]. \tag{23}$$

The normalization factor in (23) is chosen to ensure equal probabilities of excitation of both  $\tilde{\Psi}_s$  and  $\tilde{\Psi}_a$  states in the superposition state described by the wave function  $\tilde{\Psi}(t_1)$ . After the point  $t_1 = 0$ , the compound system develops independently along symmetric and antisymmetric interaction channels, each of which, generally speaking, decays with its own decay constant. These two communication channels can be particularly useful for performing two-qubit logic operations and in a number of other applications [1].

Gathering together the formulas obtained for  $\tilde{\Psi}_s(t_1)$  and  $\tilde{\Psi}_a(t_1)$ , we represent the wave function  $\tilde{\Psi}(t_1)$  in the form of a linear combination:

$$\tilde{\Psi}(t_1) = A_1\tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle + A_2\tilde{\varphi}_n(1)\tilde{\varphi}_0(2)|n_\omega - 1\rangle + A_3\tilde{\varphi}_0(1)\tilde{\varphi}_n(2)|n_\omega - 1\rangle, \tag{24}$$

where the probability amplitudes of the possible states of the compound system are given by the expressions

$$A_1 = \frac{1}{2} \left\{ e^{(-\frac{\Gamma_s}{4} t_1 + i\frac{\varepsilon_+}{2} t_1)} \left[ \cos((\Omega_+ + i\beta_+)t_1) - \frac{i(\varepsilon_+ + i\Gamma_s/2)}{2(\Omega_+ + i\beta_+)} \sin((\Omega_+ + i\beta_+)t_1) \right] + e^{(-\frac{\Gamma_a}{4} t_1 + i\frac{\varepsilon_-}{2} t_1)} \left[ \cos((\Omega_- + i\beta_-)t_1) - \frac{i(\varepsilon_- + i\Gamma_a/2)}{2(\Omega_- + i\beta_-)} \sin((\Omega_- + i\beta_-)t_1) \right] \right\}, \quad (25)$$

$$A_2 = -\frac{iF_{nm}}{2\hbar} \left[ \frac{\sin((\Omega_+ + i\beta_+)t_1)}{\Omega_+ + i\beta_+} e^{(-\frac{\Gamma_s}{4} t_1 - i\frac{\varepsilon_+}{2} t_1)} e^{(-i\delta E_s t_1/\hbar)} + \frac{\sin((\Omega_- + i\beta_-)t_1)}{\Omega_- + i\beta_-} e^{(-\frac{\Gamma_a}{4} t_1 - i\frac{\varepsilon_-}{2} t_1)} e^{(i\delta E_s t_1/\hbar)} \right], \quad (26)$$

$$A_3 = \frac{iF_{nm}}{2\hbar} \left[ -\frac{\sin((\Omega_+ + i\beta_+)t_1)}{\Omega_+ + i\beta_+} e^{(-\frac{\Gamma_s}{4} t_1 - i\frac{\varepsilon_+}{2} t_1)} e^{(-i\delta E_s t_1/\hbar)} + \frac{\sin((\Omega_- + i\beta_-)t_1)}{\Omega_- + i\beta_-} e^{(-\frac{\Gamma_a}{4} t_1 - i\frac{\varepsilon_-}{2} t_1)} e^{(i\delta E_s t_1/\hbar)} \right]. \quad (27)$$

For times  $\Gamma_{s(a)} t_1 \ll 1$ , the probability amplitudes (25)–(27) are normalized by the condition:  $|A_1(t_1)|^2 + |A_2(t_1)|^2 + |A_3(t_1)|^2 = 1$ . Hence and from the formulas (25)–(27) it follows that the normalization of the total wave function  $\tilde{\Psi}(t_1)$  is conserved at  $\Gamma_{s(a)} t_1 \rightarrow 0$ . Resonance transfer of quantum information from atom  $A(2)$  to atom  $A(1)$  is characterized by a change in amplitudes  $A_1(t_1)$ ,  $A_2(t_1)$  and  $A_3(t_1)$  upon selective excitation of atom  $A(2)$  by the field of real photons.

Taking into account vanishing the matrix element of the operator (13) relating the states  $\tilde{\Psi}_s^{(0)}$  and  $\tilde{\Psi}_a^{(0)}$ , the amplitudes of the probabilities of finding a compound system in one or another state are represented as the sum of two groups of terms. The terms in (25)–(27) with the factor  $\exp(-\Gamma_s t_1/4 + i\varepsilon_+ t_1/2)$  correspond to the system decay along the symmetric channel of interaction, and the terms with the factor  $\exp(-\Gamma_a t_1/4 + i\varepsilon_- t_1/2)$  – along antisymmetric one. When separating atoms over long distances ( $R \gg \lambda_0$ ), the contribution to  $\Gamma_{s(a)}$  tends to zero due to the dipole-dipole interaction of atoms. In this case, the damping constants  $\Gamma_{s(a)}$  of states  $\Psi_{s(a)}$  (6) with a sufficient degree of accuracy can be considered equal to their asymptotic values (at  $R \rightarrow \infty$ ):  $\Gamma_s = \Gamma_a = \gamma_0$ , where  $\gamma_0$  is the radiation decay constant of a single atom. Coincidence of the damping constants  $\Gamma_s$  and  $\Gamma_a$  leads to quantum oscillations between states  $|0n\rangle$  and  $|n0\rangle$  in which only one of atom  $A(1)$  and  $A(2)$  is excited.

## Summary

The theory of resonance interaction of two identical two-level atoms via the field of virtual photons in the field of real photons is developed within the effects of the 2nd and 3rd orders of quantum electrodynamics [8, 15, 9, 10, 11, 12, 16]. On the basis of the developed formalism, the temporal dynamics of the amplitudes of the probabilities (see (25)–(27)) to detect the system of two dipole-interacting atoms

qubits in one or another state (24) at absorption of a resonant photon by one of the atoms of the system.

Regarding the formulas (25)–(27), we note the following. First of all, let us pay attention to the fact that these formulas make it possible to detect and precisely control all the effects associated with the influence of retarded interaction of atoms and damping of states on the process of quantum information transfer between two atoms qubits. From the exchange-resonance nature of the transfer of excitation energy from one atom to another, it follows that the efficiency of quantum information transfer must strongly depend on the interatomic distance and the mutual orientation of the dipole moments of the transition of atoms. At the same time, the interaction of two-level atoms with each other, which is described by the operator  $\hat{V}_{dip}^{(\pm)}$ , and the interaction of one of them with the field of real photons constitute a set of simple tools for control of internal quantum states  $\tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle$ ,  $\tilde{\varphi}_n(1)\tilde{\varphi}_0(2)|n_\omega - 1\rangle$  and  $\tilde{\varphi}_0(1)\tilde{\varphi}_n(2)|n_\omega - 1\rangle$  of the compound system “ $A(1) + A(2) + F$ ”, which is of undeniable interest from the point of view of their practical application to the implementation of quantum logic operations NOT and CNOT.

In the next article, we will trace the temporal evolution of the amplitudes (25)–(27) at following points of time.

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