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# The accuracy of modeling of Gaussian stochastic processes in some Orlicz spaces

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**Abstract**
The main purpose of this study is construction of a model of a Gaussian stochastic process with given reliability and accuracy in the Orlicz space. In the paper, the suitable model is presented, conditions for the model parameters are derived, and some examples of their calculations are given.

**Keywords** Sub-Gaussian process, stochastic processes, model of a process, Orlicz space, the accuracy and reliability.

**AMS 2010 subject classifications** 65C05, 60G07.

## 1. Introduction

One of the actual problems of the theory of stochastic processes is the construction of mathematical models of stochastic processes and studying properties of these models. During the twentieth century a number of simulation methods were developed, among them the method of minimal transformation, canonical representations, autoregression, double randomization and the method of partition and randomization of the spectrum. Among active creators of these new simulation methods we should mention G. A. Mikhailov and his students.

For example, the method of double randomization was analysed in the papers by A. V. Vojtishek [18], B. A. Kargin and S. M. Prigarin [6]. Other simulation methods were investigated in the papers by V. V. Bykov [3], N. L. Dergalin and V. V. Romantsev [4], S. M. Ermakov and G. A. Mikhailov [5], T. M. Tovstik [16], A. S. Shalygin and Y. I. Palagin [15], V. A. Ogorodnikov [14].

Among all papers, dedicated to the problem simulation of stochastic processes and random fields, the papers [1], [7] – [11] by Yu. V. Kozachenko and his followers should be distinguished. The essential difference in these works is that the authors not only construct the models of stochastic processes and fields, but also investigate the accuracy and reliability of these models.

In this paper, we continue the research begun in the papers by R. G. Antonini, Yu. V. Kozachenko, A. M. Tegza [1] and by N. V. Troshki [17]. In the mentioned papers, the reliability and accuracy of the constructed models in the space $L_U(\mathbb{T})$ were investigated. In the current paper, we construct a model of a Gaussian stochastic process using a modified method of partition and randomization of the spectrum. The modified version of this method was

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proposed in the paper N. V. Troshki [17]. In addition, we investigate the reliability and accuracy of the model in some Orlicz spaces, which are wider than $L_U(\mathbb{T})$.

The main result of this paper is a theorem about the accuracy and reliability of the approximation model of a Gaussian stochastic process in the Orlicz space. As examples, we consider the problem in some special cases. In particular, we studied the problem in the case where the covariance function of a process is represented with the help of the Bessel functions of the first kind. In addition, we managed to find the smallest $M$ such that the constructed model approximated the Gaussian stochastic process in the Orlicz space of exponential type with a given accuracy and reliability. Also, the following two cases were considered: in the first case, there were imposed certain restrictions on the distribution function, and in the second one, this function was not subject to any restrictions.

The paper consists of the introduction and two sections. The first section provides preliminaries that substantiate and facilitate understanding of the results obtained in this paper. The main results of the paper are concentrated in the second section. In addition to the theorem about the approximation of a Gaussian stochastic process in the Orlicz space, we presented some examples and constructed a model of a Gaussian stochastic process.

## 2. Construction of the model of Gaussian stochastic process

Let $\{\Omega, \mathcal{B}, P\}$ be a standard probability space.

**Definition 2.1** ([2])**.** A random variable $\xi$ is called sub-Gaussian, if there exists a number $a \geq 0$ such that the inequality

$$E\exp\{\lambda\xi\} \leq \exp\left\{\frac{a^2\lambda^2}{2}\right\}.$$

holds true for all $\lambda \in R$.

The class of all sub-Gaussian random variables defined on a standart probability space $\{\Omega, \mathcal{B}, P\}$ is denoted by $Sub(\Omega)$.

Consider the following numerical characteristic of the sub-Gaussian random variable $\xi$:

$$\tau(\xi) = \inf\left\{a \geq 0 : \ E\exp\{\lambda\xi\} \leq \exp\left\{\frac{a^2\lambda^2}{2}\right\}, \lambda \in R\right\}. \tag{1}$$

We shall call $\tau(\xi)$ the sub-Gaussian standard of a random variable $\xi$. By definition, $\xi \in Sub(\Omega)$ if and only if $\tau(\xi) < \infty$. In the book by Buldygin and Kozachenko ([2], 2000) it is shown that the space $Sub(\Omega)$ is a Banach space with the norm $\tau(\xi)$.

**Definition 2.2** ([2])**.** A stochastic process $X = \{X(t),\ t \in \mathbb{T}\}$, is called sub-Gaussian if for any $t \in \mathbb{T}$, $X(t) \in Sub(\Omega)$ and $\sup_{t\in\mathbb{T}} \tau(X(t)) < \infty$.

**Definition 2.3** ([2])**.** A continuous even convex function $U(\cdot)$ is called $C$- function if it is monotone increasing, $U(0) = 0$, $U(x) > 0$, as $x \neq 0$.
For example, $U(x) = \exp\{|x|^\alpha\} - 1$, $\alpha \geq 1$, is a $C$-function.

**Definition 2.4** ([2])**.** Let $U$ be a C-function. A family of random variables $L_U(\mathbb{T})$ such that for each $\xi \in L_U(\mathbb{T})$ there exists a constant $r_\xi > 0$ satisfying condition

$$EU\left(\frac{\xi}{r_\xi}\right) < \infty$$

is called Orlicz space of random variables.

The Orlicz space, generated by the function $U(x)$, is defined as the family of functions $\{f(t), t \in \mathbb{T}\}$, where for each function $f(t)$ there exists a constant $r$ such that

$$\int_{\mathbb{T}} U\left(\frac{f(t)}{r}\right) d\mu(t) < \infty.$$

The space $L_U(\mathbb{T})$ is a Banach space with respect to the norm

$$\|f\|_{L_U} = \inf\left\{r > 0\colon \int_{\mathbb{T}} U\left(\frac{f(t)}{r}\right) d\mu(t) \le 1\right\}. \tag{2}$$

The norm $\|f\|_{L_U}$ is called the Luxemburg norm.Properties of the random processes in the Orlicz space are described in the paper [12].

Let $\mathbb{T}$ be a parametric set and let $X = \{X(t), t \in \mathbb{T}\}$ be a real-valued Gaussian centered second-order stochastic process with the covariance function

$$R(t,s) = \int_0^\infty g(t,\lambda)g(s,\lambda)dF(\lambda) \ ,$$

where $F(\lambda)$ is a continuous distribution function.

*Theorem 2.1* ([7])
Let $U = \{U(x), x \in R\}$ be a $C$-function such that the function $G_U(t) = \exp\left\{\left(U^{(-1)}(t-1)\right)^2\right\}$ is convex for $t \ge 1$. Then with probability one $X \in L_U(\mathbb{T})$ and for all $\varepsilon$ such that

$$\varepsilon \ge \max(\mu(\mathbb{T}), 1) \cdot \tau \left(2 + \left(U^{(-1)}(1)\right)^{-2}\right)^{\frac{1}{2}}$$

we have

$$P\left\{\|X\|_{L_U} > \varepsilon\right\} \le \sqrt{e}\frac{\varepsilon U^{(-1)}(1)}{\hat{\mu}(\mathbb{T}) \cdot \tau} \cdot \exp\left\{-\frac{\varepsilon^2 (U^{(-1)}(1))^2}{2(\hat{\mu}(\mathbb{T}))^2 \cdot \tau^2}\right\}, \tag{3}$$

where $\hat{\mu}(\mathbb{T}) = max(\mu(\mathbb{T}), 1)$.

If $\{X(t), t \in \mathbb{T}\}$ is a centered Gaussian stochastic process with given covariance function, then, according to the Karhunen theorem, the process $X(t)$ can be represented in the following form

$$X(t) = \int_0^\infty g(t,\lambda)d\eta(\lambda), \ t \in \mathbb{T}, \tag{4}$$

where $\{\eta(\lambda),\ \lambda \ge 0\}$ is a Gaussian process with independent increments such that $E(\eta(b) - \eta(c))^2 = F(b) - F(c)$, $b > c$ and $E\eta(\lambda) = 0$.

Let $L > 0$ be a given number. Consider the partition $\Lambda = \{\lambda_0, \ldots, \lambda_M\}$ of the set $[0,\ \infty]$ such that $\lambda_0 = 0$, $\lambda_k < \lambda_{k+1}$, $\lambda_{M-1} = L$, $\lambda_M = \infty$. Then

$$X(t) = \sum_{k=0}^{M-1} \int_{\lambda_k}^{\lambda_{k+1}} g(t,\ \lambda)d\eta(\lambda), \ t \in \mathbb{T}. \tag{5}$$

Consider the process

$$X_M(t) = \sum_{k=0}^{M-1} \eta_k g(t,\ \zeta_k), \ t \in \mathbb{T}, \tag{6}$$

where $\eta_k$, $\zeta_k$ are independent random variables, $\eta_k$ are Gaussian random variables such that $E\eta_k = 0$,

$$E\eta_k^2 = F(\lambda_{k+1}) - F(\lambda_k) = b_k^2,$$

and $\zeta_k$ are random variables that take values on the segments $[\lambda_k, \lambda_{k+1}]$ such that

$$P\{\zeta_k < \lambda\} = F_k(\lambda) = \frac{F(\lambda) - F(\lambda_k)}{F(\lambda_{k+1}) - F(\lambda_k)}$$

if $b_k^2 > 0$, and $\eta_k = 0$, $\zeta_k = 0$ with probability 1 if $b_k^2 = 0$.

The process $X_M = \{X_M(t), t \in \mathbb{T}\}$ is called a model of the Gaussian random process $X = \{X(t), t \in \mathbb{T}\}$.

The covariance function of the process $X_M$ coincides with the covariance function of the stochastic process $X$. It is clear that different selection of the number $M$ provides different accuracy and reliability of the computer-simulated models of the process.

Consider

$$\eta_\Lambda(t) = X(t) - X_M(t) = \sum_{k=0}^{M-1} \int_{\lambda_k}^{\lambda_{k+1}} (g(t,\ \lambda) - g(t, \zeta_k)) d\eta(\lambda),\ t \in \mathbb{T}. \tag{7}$$

Let the following condition hold for the function $g(t, \lambda)$:

$$|g(t,\ \lambda) - g(t, u)| \le S(|u - \lambda|) \cdot Z(t),\ t \in \mathbb{T}, \lambda \ge 0, u \ge 0,$$

where $\{Z(t),\ t \in \mathbb{T}\}$ is a continuous function, and the function $S(\lambda)$ $\lambda \in \mathbb{R}$, monotonically increases, $S(\lambda) \to 0$ as $\lambda \to 0$.

*Theorem 2.2* ([17])
The stochastic process $\{\eta_\Lambda(t), t \in \mathbb{T}\}$ is sub-Gaussian and the following inequality holds:

$$\tau(\eta_\Lambda(t)) \le Z(t) \left[ \sum_{k=0}^{M-1} b_k^2 \sup_{m \ge 1} \left( ES^{2m}(|\zeta_k - \zeta_k^*|) \right)^{\frac{1}{m}} \right]^{\frac{1}{2}},$$

where $\zeta_k^*$ are random variables independent of $\zeta_k$ but with the same distribution as $\zeta_k$.

*Corollary 2.1* ([17])
If for all $\lambda, u \in \mathbb{R}_+$ there exists an absolute constant $C > 0$ such that

$$\sup_{t \in \mathbb{T}} |g(t,\ \lambda) - g(t, u)| \le C,$$

then we have

$$\tau(\eta_\Lambda(t)) \le \left[ \sum_{k=0}^{M-2} b_k^2 \sup_{m \ge 1} \left( ES^{2m}(|\zeta_k - \zeta_k^*|) \right)^{\frac{1}{m}} + C^2(1 - F(L)) \right]^{\frac{1}{2}},\ t \in \mathbb{T},$$

where $b_k^2$ and $\zeta_k^*$ are the same as in the previous theorem 2.2.

## 3. The accuracy and reliability of simulation of Gaussian random processes in the Orlicz space.

**Definition 3.1.** A model $X_M = \{X_M(t), t \in \mathbb{T}\}$ approximates the process $X = \{X(t), t \in \mathbb{T}\}$ with reliability $1 - \beta$, $0 < \beta < 1$, and accuracy $\delta > 0$ in the Orlicz space $L_U(\mathbb{T})$, if there exists a partition $\Lambda$ (described above) such that the inequality

$$P\left\{ \|X(t) - X_M(t)\|_{L_U} > \delta \right\} \le \beta$$

is fulfilled.

The above statements are applied to investigate the selection of such a partition of the set $[0,\infty]$ that a Gaussian process $X$ would be approximated by the model $X_M$ with a given accuracy and reliability in Orlicz space.

Consider the random process (4) and its model (6). For their difference $\eta_\Lambda(t) = X(t) - X_M(t)$, $t \in \mathbb{T}$, in the Theorem 2.2, an estimate of the sub-Gaussian standard $\tau(\eta_\Lambda(t))$ is obtained.

Let's denote

$$\tau(L,\ T) = \sup_{t\in\mathbb{T}} Z(t)\left[\sum_{k=0}^{M-1} b_k^2 \sup_{m\geq 1}\left(ES^{2m}(|\zeta_k - \zeta_k^*|)\right)^{\frac{1}{m}}\right]^{\frac{1}{2}},\ L>0, T>0.$$

Given the above assessment of sub-Gaussian standard and a similar theorem from the paper [1], we get a statement.

*Theorem 3.1*
Suppose that in the model $X_M$ the partition $\Lambda$ is such that the next inequalities hold

$$\begin{aligned} \tau(L,T) &\leq \frac{\delta}{\hat{T}\cdot\left(2+\left(U^{(-1)}(1)\right)^{-2}\right)^{\frac{1}{2}}}, \\ \tau(L,T) &\leq \frac{\delta U^{(-1)}(1)}{\hat{T}x(\beta)}\ , \end{aligned} \tag{8}$$

where $x(\beta) > 1$ is a root of the equation $\sqrt{e}x \cdot \exp\left\{-\frac{x^2}{2}\right\} = \beta$ and $\hat{T} = \max(T,1)$.

Then the model $X_M$ approximates the Gaussian process $X$ with reliability $\beta$, $0<\beta<1$, and accuracy $\delta>0$ in the Orlicz space $L_U([0,T])$, where $C$-function $U$ satisfies the conditions of Theorem 2.1 ($\mu(\cdot)$ is the Lebesgue measure).

*Proof*
The statement of this theorem follows from Theorem 2.1. In Theorem 2.1 we take $\mathbb{T}=[0,T]$, $\tau=\tau(\Lambda,T)$, $\varepsilon=\delta$, $X=\eta_\Lambda$. Since the next inequalities hold

$$U^{(-1)}(1) > \left(2+\left(U^{(-1)}(1)\right)^{-2}\right)^{-\frac{1}{2}},$$

$$\frac{\delta U^{(-1)}(1)}{\hat{T}\cdot\tau(\Lambda,T)} > \frac{\delta}{\hat{T}\tau(\Lambda,T)\left(2+\left(U^{(-1)}(1)\right)^{-2}\right)^{\frac{1}{2}}} > 1,$$

then we get $\delta > \hat{T}\cdot\tau(\Lambda,T)\cdot\left(2+\left(U^{(-1)}(1)\right)^{-2}\right)^{\frac{1}{2}}$. This implies that for $\varepsilon=\delta$ condition (3) holds.

The function $f(x)=\sqrt{e}x\exp\left\{-\frac{x^2}{2}\right\}$ decreases as $x>1$, $f(1)=1$. So, from (3) follows that

$$P\left\{\|\eta_\Lambda(t)\|_{L_U} > \delta\right\} \leq \sqrt{e}\frac{\delta U^{(-1)}(1)}{\hat{T}\cdot\tau(\Lambda,T)}\cdot\exp\left\{-\frac{\delta^2(U^{(-1)}(1))^2}{2\hat{T}^2\cdot\tau^2(\Lambda,T)}\right\} = \beta.$$

The inequality above holds true if $\frac{\delta U^{(-1)}(1)}{\hat{T}\cdot\tau(\Lambda,T)} \geq x(\beta)$, that is, when conditions in (8) are satisfied. Theorem is proved. □

Consider special cases.

*Example 3.1.* Let the covariance function have the representation:

$$R(t,s) = \int_0^\infty \cos\lambda t\cos\lambda s dF(\lambda),\ t,s\in\mathbb{T}.$$

Then the corresponding Gaussian process has the form

$$X(t) = \int_0^\infty \cos \lambda t d\eta(\lambda),\ t \in \mathbb{T}.$$

Let $\mathbb{T} = [0,\ T]$, and suppose that for the function $\{F(x), x \geq 0\}$ the following condition holds: $F(+\infty) - F(L) \leq \frac{1}{L^\alpha}$, where $0 \leq \alpha \leq 1$, $L > 1$. Then, using results of Example 2.1 from [17], we have that

$$\tau^2\left(\eta_\Lambda(t)\right) \leq 4^{1-\alpha} t^{2\alpha} \left( \frac{L^{2\alpha}}{(M-1)^{2\alpha}} + \frac{4}{L^\alpha} \right).$$

The right part of the resulting inequality reaches its minimum value at $L = 2^{\frac{1}{3\alpha}}(M-1)^{\frac{2}{3}}$. Substituting this value in the inequality above, we get the following estimate:

$$\tau\left(\eta_\Lambda(t)\right) \leq \frac{\sqrt{6} T^\alpha}{2^{\alpha - \frac{1}{3}}(M-1)^{\frac{\alpha}{3}}} = \tau(\Lambda, T),\ t \in [0,\ T]. \tag{9}$$

Let $T = 1$, $\alpha = 0.5$, accuracy $\delta = 0.01$, reliability $1 - \beta = 0.99$. Then in Theorem 3.1 the root of the corresponding equation is equal to $x_\beta = 3.58$. We take the following $C$-function: $U(x) = e^{x^2} - 1$. Then the inverse function is $U^{(-1)}(1) = \sqrt{\ln 2}$, and the system of inequalities (8) is of the form

$$\begin{cases} \frac{\sqrt{6}}{\sqrt[6]{2(M-1)}} \leq \frac{0.01}{(2+(\ln 2)^{-1})^{\frac{1}{2}}}, \\ \frac{\sqrt{6}}{\sqrt[6]{2(M-1)}} \leq \frac{0.01\sqrt{\ln 2}}{3.58}. \end{cases}$$

Solving it, we get

$$M \geq \frac{6^3}{2 \cdot 0.0023^6} + 1 \approx 7.296 \cdot 10^{17}.$$

If the accuracy and reliability are weakened, for example $\delta = 0.1$, $1 - \beta = 0.9$, then $x_\beta = 2.78$ and we get $M \geq 1.48 \cdot 10^{11}$.

*Example 3.2.* Let the covariance function have the same representation as in Example 1, but we have no restrictions on the function $\{F(x), x \geq 0\}$. Then

$$\tau^2(\eta_\Lambda(t)) \leq 4^{1-\alpha} t^{2\alpha} \left( F(L) \left( \frac{L}{M-1} \right)^{2\alpha} + 4(1 - F(L)) \right),\ t \in [0,\ T].$$

Consider special cases:

1. $F(L) = 1 - e^{-L}$, $T = 1$, $\alpha = 0.5$, accuracy $\delta = 0.1$, reliability $1 - \beta = 0.99$, $x_\beta = 3.58$, $C$-function $U(x) = e^{x^2} - 1$, $U^{(-1)}(1) = \sqrt{\ln 2}$. Then

$$\tau(\eta_\Lambda(t)) \leq \tau(L, 1) = \sqrt{2\frac{(1-e^{-L})L}{M-1} + 8e^{-L}},$$

and the system of inequalities (8) is of the form:

$$\begin{cases} \sqrt{2\frac{(1-e^{-L})L}{M-1} + 8e^{-L}} \leq \frac{0.1}{(2+(\ln 2)^{-1})^{\frac{1}{2}}} \approx 0.054, \\ \sqrt{2\frac{(1-e^{-L})L}{M-1} + 8e^{-L}} \leq \frac{0.1\sqrt{\ln 2}}{3.58} \approx 0.023. \end{cases}$$

Solving it, we get

$$M \geq \frac{2(1-e^{-L})L}{0.023^2 - 8e^{-L}} + 1.$$

The right-hand side of this inequality acquires the smallest value at $L = 12.2$ and is approximately equal to 49924. Consequently, we can assert that for $M \geq 49924$ the model $X_M$ approaches the Gaussian process $X$ with the reliability 0.99 and accuracy 0.1 in the Orlicz space $L_U([0,1])$.

2. $F(L) = 1 - \frac{1}{(1+L)^3}$, $T = 1$, $\alpha = 0.5$, accuracy $\delta = 0.1$, reliability $1 - \beta = 0.9$, $U^{(-1)}(1) = \sqrt{\ln 2}$. Then

$$\tau(L,1) = \sqrt{\left(1 - \frac{1}{(1+L)^3}\right)\frac{2L}{M-1} + \frac{8}{(1+L)^3}}.$$

Substituting this expression in the system of inequalities (8) and solving the system, we get

$$\sqrt{\left(1 - \frac{1}{(1+L)^3}\right)\frac{2L}{M-1} + \frac{8}{(1+L)^3}} \leq 0.03.$$

Hence, $M$ should be as follows:

$$M \geq \frac{2L(1+L)^3 - 2L}{0.0009(1+L)^3 - 8} + 1.$$

The right-hand side of the inequality above acquires the smallest value at $L = 31.6$ and is approximately equal to 94454.

*Example 3.3.* Let a covariance function have the following representation

$$R(t,s) = \int_0^\infty J_l(t\lambda)J_l(s\lambda)dF(\lambda), \quad t,s \in \mathbb{T},$$

where $J_l(u) = \frac{1}{\pi}\int_0^\pi \cos(l\phi - u\sin\phi)d\phi$ is the Bessel function of the first kind.

Then the corresponding Gaussian process has the form $X(t) = \int_0^\infty J_l(\lambda t)d\eta(\lambda)$. Since $|J_l(\lambda t) - J_l(tu)| \leq \frac{t|u-\lambda|}{\sqrt{\pi}}$ (see work [17]), $Z(t) = \frac{t}{\sqrt{\pi}}$, $S(\lambda) = |\lambda|$, $C = 2$, then, using the result of Corollary 2.1, we derive that

$$\tau^2(\eta_\Lambda(t)) \leq \frac{t^2}{\pi}\left(\frac{L^2}{(M-1)^2}F(L) + 4(1 - F(L))\right).$$

Consider partial cases:

1. $F(L) = 1 - e^{-L}$, $T = 1$, accuracy $\delta = 0.01$, reliability $1 - \beta = 0.99$, $x_\beta = 3.58$, $C$-function $U(x) = e^{x^2} - 1$, $U^{(-1)}(1) = \sqrt{\ln 2}$. In this case

$$\tau(\eta_\Lambda(t)) \leq \tau(L,1) = \frac{1}{\sqrt{\pi}}\sqrt{\frac{(1-e^{-L})L^2}{(M-1)^2} + 4e^{-L}},$$

and substituting this estimate to the system of inequalities (8), we get the common part

$$\frac{1}{\sqrt{\pi}}\sqrt{\frac{(1-e^{-L})L^2}{(M-1)^2} + 4e^{-L}} \leq \frac{0.01\sqrt{\ln 2}}{3.58} \approx 0.0023$$

Solving this inequality, we derive the condition for $M$:

$$M \geq \sqrt{\frac{(1-e^{-L})L^2}{\pi 0.0023^2 - 4e^{-L}}} + 1.$$

The right-hand side of the inequality above acquires the smallest value at $L = 14.8$ and is approximately equal to 4451. Substituting the number $M$ into the model (6) and computing the random variables $\eta_k$, $\zeta_k$ with $k = \overline{0, M-1}$, we can obtain a graphical representation of the model of Gaussian process.

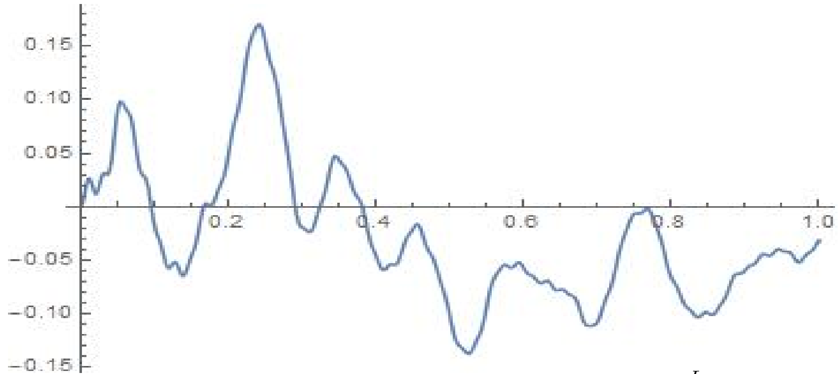


Figure 1. Illustration of Gaussian random process, where $F(L) = 1 - e^{-L}$ and $J_2(u) = \frac{1}{\pi} \int_0^{\pi} \cos(2\phi - u\sin\phi)d\phi$.

2. $F(L) = 1 - \frac{1}{1+L^5}$, $T = 1$, accuracy $\delta = 0.1$, reliability $1 - \beta = 0.99$, $x_\beta = 3.58$, $C$-function $U(x) = e^{x^2} - 1$, $U^{(-1)}(1) = \sqrt{\ln 2}$. Then

$$\tau(\eta_\Lambda(t)) \leq \tau(L, 1) = \frac{1}{\sqrt{\pi}} \sqrt{\frac{(1 - \frac{1}{1+L^5})L^2}{(M-1)^2} + \frac{4}{1+L^5}},$$

and, substituting this estimate in the system of inequalities (8), we get the common part:

$$\frac{1}{\sqrt{\pi}} \sqrt{\frac{(1 - \frac{1}{1+L^5})L^2}{(M-1)^2} + \frac{4}{1+L^5}} \leq \frac{0.1\sqrt{\ln 2}}{3.58} \approx 0.023.$$

Solving this inequality, we obtain

$$M \geq \sqrt{\frac{L^7}{\pi 0.023^2(1+L^5) - 4}} + 1.$$

The right-hand side of the last inequality acquires (in the domain of its definition) the smallest value at $L = 4.46$ and equals approximately 217.

## 4. Conclusions

In this paper we presented results of the investigations of the accuracy and reliability of the approximation model of a Gaussian stochastic process in the Orlicz space. We presented some examples of calculations for the model parameters. In one of the considered cases, the corresponding model of a Gaussian stochastic process is constructed and the graphical illustration is given.

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