УДК 512.56
DOI https://doi.org/10.24144/2616-7700.2024.44(1).7-14

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CLASSIFICATION OF THE POSETS OF MM-TYPE BEING THE SYMMETRIC OVERSUPERCRITICAL POSET OF ORDER 9

Representations of posets were introduced by L. A. Nazarova and A. V. Roiter in 1972, and the first author was one of those who took an active part in the development of the relevant theory. The first criterion in it was the criterion on finiteness of the representation types obtained by M. M. Kleiner. In 1992 he proved that a posets S is of finite representation type if and only if it does not contain full subposets of the form $K_1 = (1,1,1,1), K_2 = (2,2,2), K_3 = (1,3,3), K_4 = (1,2,5)$ and $K_5 = (\mathrm{N},4)$. These posets are called critical posets (relative to the finiteness of type) in the sense that they are minimal posets with an infinite number, up to equivalence, of indecomposable representations. Now they are called the Kleiner's posets. In 1974, Yu. A. Drozd proved that a poset S has finite representation type if and only if its Tits quadratic form

$$q_S(z) := z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j,i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i$$

is weakly positive (i.e., positive on the set of non-negative vectors). Consequently, the Kleiner's posets are also critical relative to weak positivity of the Tits quadratic form. In 2005, the authors proved that a poset is critical relative to the positivity of the Tits quadratic form if and only if it is minimax isomorphic to a Kleiner's poset.

A similar situation takes place for posets of tame representation type. In 1975, L. A. Nazarova proved that a poset S is tame if and only if it does not contain full subsets of the form $N_1 = (1,1,1,1,1)$, $N_2 = (1,1,1,2)$, $N_3 = (2,2,3)$, $N_4 = (1,3,4)$, $N_5 = (1,2,6)$ and $(\mathrm{N},5)$. So these posets are critical relative to the tameness and she called them supercritical. They are also critical relative to weak non-negativity of the Tits quadratic form. In 2009, the authors proved that a poset is critical relative to non-negativity of the Tits quadratic form if and only if it is minimax isomorphic to a supercritical poset.

The first author suggested to introduce posets (called oversupercritical) which differ from the supercritical posets to the same extent as the supercritical posets differ from the critical ones.

In previous papers, the authors described all posets that are minimax isomorphic to any oversupercritical poset except $(1,4,4)$ and studied some of their combinatorial properties. The case of the poset $(1,4,4)$ is considered in this paper.

Keywords: representation, critical and supercritical poset, oversupercritical poset, Tits quadratic form, finite and tame representation type, positivity and weak positivity, non-negativity and weak non-negativity.

1. Introduction. Representations of posets were introduced by L. A. Nazarova and A. V. Roiter [1], and Kyiv algebraists actively participated in development of the relevant theory (see, e.g., [2] – [11]). The first criterion in it was the criterion on finiteness of the representation types obtained by M. M. Kleiner [2]. He proved that a posets S is of finite representation type if and only if it does not contain full subposets of the form $K_1 = (1,1,1,1), K_2 = (2,2,2), K_3 = (1,3,3), K_4 = (1,2,5)$ and $K_5 = (\mathrm{N},4)$ (see below Remark 1). These posets are called critical posets (relative to the finiteness of type) in the sense that they are minimal posets with an infinite number, up to equivalence, of indecomposable representations. Now they are called the Kleiner's posets. On the other hand, Yu. A. Drozd [3] proved that a poset S has finite representation type if and only if its Tits quadratic form

$$q_S(z) := z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i$$

is weakly positive (i.e., positive on the set of nonnegative vectors). Consequently, the critical posets are also critical relatively to the weak positivity of the Tits quadratic form. In [12] the authors proved that a poset is critical relative to the positivity of the Tits quadratic form if and only if it is minimax isomorphic to a Kleiner's poset (such isomorphism was introduced by the first author in [13]); in this paper all such posets are fully described (they are named by the authors as P-critical).

A similar situation takes place for posets of tame representation type. L. A. Nazarova [4] proved that a poset S is tame if and only if it does not contain full subsets of the form $N_1 = (1,1,1,1,1)$, $N_2 = (1,1,1,2)$, $N_3 = (2,2,3)$, $N_4 = (1,3,4)$, $N_5 = (1,2,6)$ and $(\mathrm{N},5)$ (see below Remark 1). So these posets are critical relative to the tameness and she called them supercritical. They are also critical relative to weak non-negativity of the Tits quadratic form. In [14] the authors proved that a poset is critical relative to non-negativity of the Tits quadratic form if and only if it is minimax isomorphic to a supercritical poset. In [15] all such posets are fully described (they are named by the authors as NP-critical).

The importance of studying minimax isomorphic posets is determined by the fact that their Tits quadratic forms are $\mathbb{Z}$-equivalent, and minimax isomorphism itself is a fairly general constructively defined $\mathbb{Z}$-equivalence for posets.

In [16] were introduced *1-oversupercritical posets* which differ from supercritical sets to the same extent as the letter differ from critical ones; often, including in this paper, they are simply called *oversupercritical*. Such posets are exhausted (up to isomorphism) by the following:

1) $(1,1,1,1,1,1)$, 2) $(1,1,1,1,2)$, 3) $(1,1,2,2)$, 4) $(1,1,1,3)$,

5) $(2,3,3)$, 6) $(2,2,4)$, 7) $(1,4,4)$, 8) $(1,3,5)$, 9) $(1,2,7)$, 10) $(\mathrm{N},6)$.

Remark 1. For posets X, Y, Z=(X,Y) is denoted their direct sum, i.e., $Z = X \cup Y$ and any elements $x \in X$ and $y \in Y$ are incomparable; (m) denotes the linearly ordered set $1 \prec 2 \prec \cdots \prec m$ and N the poset $1 \prec 2, 3 \prec 4, 1 \prec 4$. These notations are used as a rule when the posets are specified by their Hasse diagrams.

In previous papers [16] – [19], the authors described all posets that are minimax isomorphic to a oversupercritical poset, except the single asymmetric one of order greater than 8 (i.e., $(1,4,4)$) and studied some of their combinatorial properties. The case of the poset $(1,4,4)$ is considered in this paper.

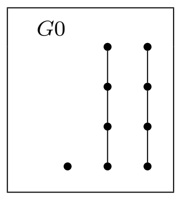
2. The main result. We consider only finite posets and identify them with their Hasse diagrams.

For a poset S and its minimal (respectively maximal) element a, we denote by $T = S_a^{\uparrow}$ (respectively $T = S_a^{\downarrow}$) the following poset: $T = S$ as usual sets, $T \setminus a = S \setminus a$ as posets, the element a is maximal (respectively minimal) in T, and a is comparable with x in T if and only if they are incomparable in S. Two posets S and T are called (min, max)-*equivalent* if there are posets $S_1, \ldots, S_p$ ($p \geq 0$) such that, if we put $S = S_0$ and $T = S_{p+1}$, then, for every $i = 0, 1, \ldots, p$, either $S_{i+1} = (S_i)_{x_i}^{\uparrow}$ or $S_{i+1} = (S_i)_{y_i}^{\downarrow}$ [13]. Obviously, any poset is (min, max)-equivalent to itself (if one put $p = 0$). Since some time we also use the term *minimax equivalence*.

The notion of minimax equivalence can be naturally continued to the notion of *minimax isomorphism*: posets S and S' are minimax isomorphic if there exists a poset T which is minimax equivalent to S and isomorphic to S'.

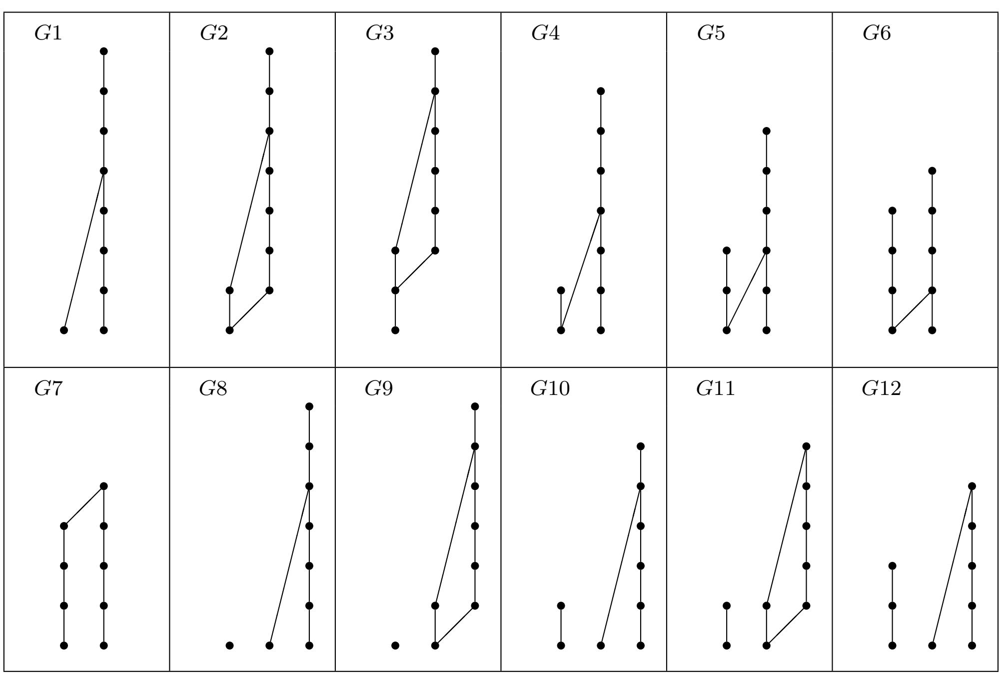
Let P be a fix poset. A poset S is called of *MM-type P* if S is minimax isomorphic to P [20]. In the case when the poset P is an oversupercritical one we say that S is of *oversupercritical MM-type*.

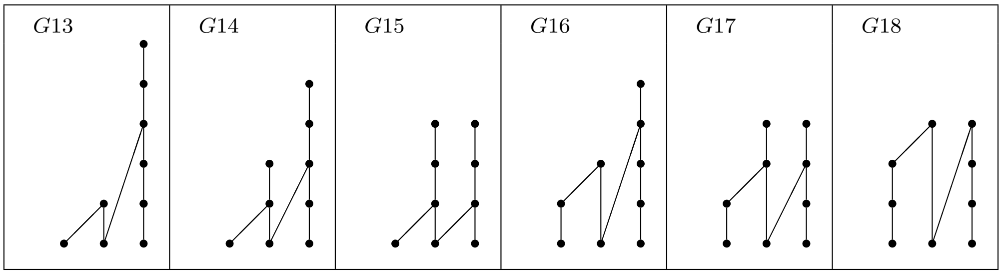
The main result of this paper describes all posets of oversupercritical MM-type P with P to be the oversupercritical poset of order 9, i.e., P is equal to $G_0 = (1, 4, 4)$:



Recall that a poset T is called *dual* to a poset S and is denoted by S^{op} if $T = S$ as usual sets and $x < y$ in T if and only if $x > y$ in S.

Theorem 1. *Up to isomorphism and duality, the complete set of posets minimax isomorphic to* $G0$ *consists of, in addition to* $G0$ *itself, the posets indicated in the following table:*





3. Proof of Theorem 1. The definition of posets of the form $T = S^{\uparrow}_{a}$ can be extended to posets of the form $T = S^{\uparrow}_{A}$, where A is a lower subposet of S, i.e., $x \in A$ whenever $x < y$ and $y \in A$. Namely, $T = S^{\uparrow}_{A}$ is defined as follows: $T = S$ as usual sets, partial orders on A and $S \setminus A$ are the same as before, but comparability and incomparability between elements of $x \in A$ and $y \in S \setminus A$ are interchanged and the new comparability can only be of the form $x > y$. In the special case, when $A = \{a\}$ is a one-element subposet, we identify A with a. Instead of $(S^{\uparrow}_{A})^{\uparrow}_{B}$ write write $S^{\uparrow\uparrow}_{AB}$.

For subposets X, Y of S of a poset S, $X < Y$ means that $x < y$ for any $x \in X, y \in Y$. Subposets X and X' of S are called *strongly isomorphic* if there exists an automorphism $\varphi : S \to S$ such that $\varphi(X) = X'$ (as equality of subposets). Similarly, pairs (Y, X) and (Y', X') of subposets of S are called *strongly isomorphic* if there exists an automorphism $\varphi : S \to S$ such that $\varphi(Y) = Y'$ and $\varphi(X) = X'$.

In [12], the authors propose the following algorithm for finding (up to isomorphism) all posets that are minimax isomorphic to a given one.

I. Describe, up to strongly isomorphic, all lower subposets of $P \neq S$ in S, and, for every of them, build the poset $S^{\uparrow}_{P}$ ($P = \varnothing$ is not excluded).

II. Describe. up to strongly isomorphic, all pairs (Q, P) consisting of a proper lower subposet Q in S and a nonempty lower subposet P in Q such that $P < S \setminus Q$; for every such pair, build the poset $S^{\uparrow\uparrow}_{QP}$.

III. Among the posets obtained in I and II, choose one from each class of isomorphic posets.

For the poset $G0$, we denote the partial order by $\prec$ and number the points with numbers $1, 2, 3, \ldots$ in such a way that $i < j$ whenever $i \prec j$ or i is (in the picture) to the left of j. Then the poset $G0$ consists of the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ and we have $2 \prec 3 \prec 4 \prec 5, 6 \prec 7 \prec 8 \prec 9$.

Now we apply our algorithm to the proof of the theorem.

Step I. Describe (up to strongly isomorphic) all lower subposets. They are:

for $G0$ — $X_0 = \varnothing$, $X_1 = \{1\}$, $X_2 = \{2\}$, $X_3 = \{1, 2\}$, $X_4 = \{2, 3\}$, $X_5 = \{2, 6\}$, $X_6 = \{1, 2, 3\}$, $X_7 = \{1, 2, 6\}$, $X_8 = \{2, 3, 4\}$, $X_9 = \{2, 3, 6\}$, $X_{10} = \{1, 2, 3, 4\}$, $X_{11} = \{1, 2, 3, 6\}$, $X_{12} = \{2, 3, 4, 5\}$, $X_{13} = \{2, 3, 4, 6\}$, $X_{14} = \{2, 3, 6, 7\}$, $X_{15} = \{1, 2, 3, 4, 5\}$, $X_{16} = \{1, 2, 3, 4, 6\}$, $X_{17} = \{1, 2, 3, 6, 7\}$, $X_{18} = \{2, 3, 4, 5, 6\}$, $X_{19} = \{2, 3, 4, 6, 7\}$, $X_{20} = \{1, 2, 3, 4, 5, 6\}$, $X_{21} = \{1, 2, 3, 4, 6, 7\}$, $X_{22} = \{2, 3, 4, 5, 6, 7\}$, $X_{23} = \{2, 3, 4, 6, 7, 8\}$, $X_{24} = \{1, 2, 3, 4, 5, 6, 7\}$, $X_{25} = \{1, 2, 3, 4, 6, 7, 8\}$, $X_{26} = \{2, 3, 4, 5, 6, 7, 8\}$, $X_{27} = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $X_{28} = \{2, 3, 4, 5, 6, 7, 8, 9\}$.

Denote by K_i the poset $G0^{\uparrow}_{X_i}$. Then it is easy to see that

$K_0 \cong G0$, $K_1 \cong G7$, $K_2 \cong G12$, $K_3 \cong G6^{\rm op}$, $K_4 \cong G10$, $K_5 \cong G18$, $K_6 \cong G5^{\rm op}$, $K_7 \cong G15^{\rm op}$, $K_8 \cong G8$, $K_9 \cong G16$, $K_{10} \cong G4^{\rm op}$, $K_{11} \cong G14^{\rm op}$, $K_{12} \cong G1$, $K_{13} \cong$

$G13$, $K_{14} \cong G17$, $K_{15} \cong G1^{\mathrm{op}}$, $K_{16} \cong G13^{\mathrm{op}}$, $K_{17} \cong G17^{\mathrm{op}}$, $K_{18} \cong G4$, $K_{19} \cong G14$, $K_{20} \cong G8^{\mathrm{op}}$, $K_{21} \cong G16^{\mathrm{op}}$, $K_{22} \cong G5$, $K_{23} \cong G15$, $K_{24} \cong G10^{\mathrm{op}}$, $K_{25} \cong G18^{\mathrm{op}}$, $K_{26} \cong G6$, $K_{27} \cong G12^{\mathrm{op}}$, $K_{28} \cong G7^{\mathrm{op}}$.

Step II. Describe (up to strongly isomorphic) all pairs of lower subposets (see the algorithm). They are:

for $G0$ — $X'_1 = (X_{20}, \{6\})$, $X'_2 = (X_{24}, \{6\})$, $X'_3 = (X_{24}, \{6,7\})$, $X'_4 = (X_{27}, \{6\})$, $X'_5 = (X_{27}, \{6,7\})$, $X'_6 = (X_{27}, \{6,7,8\})$.

Denote by K'_i the poset $(G0^{\uparrow}_{V})^{\uparrow}_{W}$ and $(V, W) = X'_j$. Then it is easy to see that $K'_1 \cong G2^{\mathrm{op}}$, $K'_2 \cong G9^{\mathrm{op}}$, $K'_3 \cong G3$, $K'_4 \cong G11$, $K'_5 \cong G9$, $K'_6 \cong F2$.

Step III. It is easy to verify that in I and II each of the posets Gi, indicated in the condition of the theorem, and dual to them (in the non-dual cases) occurs only once. And hence the theorem is proved.

4. Coefficients of transitivity. Let S be a (finite) poset and $S^2_< := \{(x,y) \,|\, x, y \in S, x < y\}$. If $(x,y) \in S^2_<$ and there is no z satisfying $x < z < y$, then we say that x and y are *neighboring*. We put $n_w = n_w(S) := |S^2_<|$ and denote by $n_e = n_e(S)$ the number of pairs of neighboring elements. The ratio $k_t = k_t(S)$ of the numbers $n_w - n_e$ and n_w are called the *coefficient of transitiveness of* S; if $n_w = 0$ (then $n_e = 0$), we assume $k_t = 0$ (see [20]).

In this part of the paper we calculate k_t for the posets of MM-type to be $G0$.

Theorem 2. *The following holds for posets Gi:*

N	n_e	n_w	k_t
$G0$	6	12	0,5

N	n_e	n_w	k_t	N	n_e	n_w	k_t	N	n_e	n_w	k_t
$G1$	8	32	0,75	$G7$	8	20	0,6	$G13$	8	20	0,6
$G2$	9	32	0,71875	$G8$	7	24	0,70833	$G14$	8	18	0,55556
$G3$	9	32	0,71875	$G9$	8	24	0,66667	$G15$	8	18	0,55556
$G4$	8	26	0,69231	$G10$	7	18	0,61111	$G16$	8	16	0,5
$G5$	8	22	0,63636	$G11$	8	18	0,55556	$G17$	8	16	0,5
$G6$	8	20	0,6	$G12$	7	14	0,5	$G18$	8	14	0,42857

The transitivity coefficients are written out with an accuracy of five decimal places. The value is exact if and only if the number of decimal places is less than five, and two values equal to exactly five digits are equal at all.

The proof is carried out by direct calculations using [21, Lemmas 1 -5].

Recall that the greatest length among the lengths of all linear ordered subsets of a poset S is called its *height*. An element of a poset is called *nodal*, if it is comparable with all the others elements. A subposet X of T is said to be *dense* if there is not $x_1, x_2 \in X$, $y \in T \setminus X$ such that $x_1 < y < x_2$.

Note that a poset of MM-type $G0$ can have at most four nodal elements.

Corollary 1. *The coefficient $k_t(S)$ of a poset S is the largest among all the posets of MM-type $G0$ if and only if S contains a dense subposet with four nodal elements.*

5. Conclusions. In this paper we describe the finite posets that are minimax isomorphic to the oversupercritical poset $(1, 4, 4)$ which is a single symmetric one of the order 9. We also study combinatorial properties of these posets, namely calculate their transitivity coefficients.

Analogous results for the rest of the oversupercritical posets were obtained by the authors earlier.

The importance of studying minimax isomorphic posets is determined by the fact that their Tits quadratic forms are $\mathbb{Z}$-equivalent.

The obtained results (together with the corresponding research methods) can be used in the study of other classes of posets.

References

1. Nazarova, L. A., & Roiter, A. V. (1972). Representations of partially ordered sets. *Zap. Nauchn. Sem. LOMI,* 28, 5–31 [in Russian].
2. Kleiner, M. M. (1972). Partially ordered sets of finite type. *Zap. Nauchn. Sem. LOMI,* 28, 32–41 [in Russian].
3. Drozd, Yu. A. (1974). Coxeter transformations and representations of partially ordered sets. *Funkts. Anal. Prilozh.,* 8(3), 34–42 [in Russian].
4. Nazarova, L. A. (1975). Partially ordered sets of infinite type. *Izv. Akad. Nauk SSSR Ser. Mat.,* 39(5), 963–991 [in Russian].
5. Bondarenko, V. M., Zavadskij, A. G., & Nazarova, L. A. (1979). On representations of tame partially ordered sets. *Representations and Quadratic Forms. Inst. Math. Acad. Sci. Ukrain. SSR,* 75–105 [in Russian].
6. Bondarenko, V. M. (1983). Exact partially ordered sets of infinite growth. *Linear Algebra and Representation Theory. Inst. Math. Acad. Sci. Ukrain. SSR,* 68–85 [in Russian].
7. Bondarenko, V. M., Nazarova, L. A„ & Roiter, A. V. (1986). Representations of partially ordered sets with involution. *Inst. Math. Acad. Sci. Ukrain. SSR, Preprint 86.80*, 24p. [in Russian].
8. Bondarenko, V. M. (1988). Bundles of semi-chains and their representations. *Inst. Math. Acad. Sci. Ukrain. SSR, Preprint 88.50*, 32p. [in Russian]
9. Nazarova, L. A., Bondarenko, V. M., & Roiter, A. V. (1991) Tame partially ordered sets with involution. *Proc. Steklov Inst. Math.,* 183, 149–159 [in Russian].
10. Bondarenko, V. M., & Zavadskij, A. G. (1991) Posets with an equivalence relation of tame type and of finite growth. *CSM Conf. Proc.,* 11, 67–88.
11. Zavadskij, A. G. (1991) Differentiation algorithm and classification of representations. *Izv. Akad. Nauk SSSR Ser. Mat.,* 55(5), 1007–1048 [in Russian].
12. Bondarenko, V. M., & Styopochkina, M. V. (2005). (Min, max)-equivalence of partially ordered sets and the Tits quadratic form. *Problems of Analysis and Algebra: Zb. Pr. Inst. Mat. NAN Ukr.,* 2(3), 18–58 [in Russian].
13. Bondarenko, V. M. (2005). On (min, max)-equivalence of posets and applications to the Tits forms. *Bull. of Taras Shevchenko University of Kyiv. (series: Physics & Mathematics),* (1), 24–25.
14. Bondarenko, V. M., & Styopochkina, M. V. (2008). (Min, max)-equivalence of posets and nonnegative Tits forms. *Ukr. Math. J.,* 60(9), 1349–1359.
15. Bondarenko, V. M., & Styopochkina, M. V. (2009). Description of posets critical with respect to the nonnegativity of the quadratic Tits form. *Ukr. Math. J.,* 61(5), 734–746.
16. Bondarenko, V. V., Bondarenko, V. M., Styopochkina, M. V., & Chervyakov, I. V. (2011). 1-oversupercritical partially ordered sets with trivial group of automorphisms and min-equivalence. *Scien. Bull. of Uzhhorod Univ. Series of Math. and Inform.,* 22(2), 17–25 [in Russian].
17. Bondarenko, V. V., & Styopochkina, M. V. (2013). Non-primitive 1-oversupercritical partially ordered set and min-equivalence. *Scien. J. of NPU named after Dragomanov. Series 1. Phys.-Math. sciences,* 14, 55–61 [in Russian].
18. Bondarenko, V. M., & Styopochkina, M. V. (2021). On posets of sixth order having over-

supercritical MM-type. *Scien. Bull. of Uzhhorod Univ. Series of Math. and Inform.,* 38(1), 7–15.

19. Bondarenko, V. M., & Styopochkina, M. V. (2023). Classification of the posets of minmax type which are symmetric oversupercritical posets of the eighth order. *Math. methods and phys.-mech. fields,* 66(1-2), 1–11.
20. Bondarenko, V. M., & Styopochkina, M. V. (2018). On properties of posets of MM-type (1,3,5). *Scien. Bull. of Uzhhorod Univ. Series of Math. and Inform.,* 32(1), 50–53.
21. Bondarenko, V. M., & Styopochkina, M. V. (2023). Classification of the posets of minmax type which are symmetric oversupercritical posets of the eighth order. *Math. Sci. (N.Y.),* 274(5), 583–593.

Бондаренко В. М., Стьопочкіна М. В. Класифікація частково впорядкованих множин, MM-тип яких дорівнює симетричній надсуперкритичній множині порядку 9.

Зображення ч. в. множин (частково впорядкованих множин) над полем ввели Л. А. Назарова і А. В. Ройтер в 1972 р., і перший автор був одним із тих, хто брав активну участь у розвитку відповідної теорії. Першим критерієм у ній був отриманий М. М. Клейнером критерій скінченності зображувального типу. У 1992 р. він довів, що ч. в. множина S має скінченний зображувальний тип тоді і лише тоді, коли вони не містить повних ч. в. підмножин вигляду $K_1 = (1,1,1,1), K_2 = (2,2,2), K_3 = (1,3,3), K_4 = (1,2,5)$ і $K_5 = (\mathrm{N},4)$. Ці ч. в. множин називаються критичними ч. в. множинами (щодо скінченності типу) в тому сенсі, що це мінімальні ч. в. множин з нескінченною кількістю нерозкладних зображень, з точністю до еквівалентності). Тепер їх також називають ч. в. множинами Клейнера. У 1974 р. Ю. А. Дрозд довів, що ч. в. множина S має скінченний зображувальний тип тоді і лише тоді, коли її квадратична форма Тітса

$$q_S(z) := z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j, i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i$$

є слабко додатною (тобто додатною на множині невід'ємних векторів). Отже, ч. в. множини Клейнера є також критичними щодо слабкої додатності квадратичної форми Тітса. У 2005 р. автори довели що ч. в. множин є критичною щодо додатності квадратичної форми Тітса тоді і лише тоді, коли вона є мінімаксно ізоморфна деякій ч. в. множині Клейнера.

Подібну ситуацію маємо з ч. в. множинами ручного зображувального типу. У 1975 р. Л. А. Назарова довела, що ч. в. множина S є ручною тоді і лише тоді, коли вона не містить ч. в. підмножин вигляду $N_1 = (1,1,1,1,1)$, $N_2 = (1,1,1,2)$, $N_3 = (2,2,3)$, $N_4 = (1,3,4)$, $N_5 = (1,2,6)$ і $(\mathrm{N},5)$. Отже, ці ч. в. множини є критичними щодо ручного зображувального типу і вона назвала їх суперкритичними; вони є також критичними щодо слабкої невід'ємності квадратичної форми Тітса. У 2009 році автори довели, що ч. в. множина є критичною щодо невід'ємності квадратичної форми Тітса тоді і лише тоді, коли вона мінімаксно ізоморфна деякій суперкритичній ч. в. множині.

Перший автор запропонував ввести ч. в. множини (названі надсуперкритичними), які відрізняються від суперкритичних ч. в. множин в тій же мірі, що суперкритичні відрізняються від критичних.

У попередніх статтях автори описали (з точністю до ізоморфізму) всі ч. в. множини, мінімаксно ізоморфні довільній надсуперкритичній множині, окрім (1,4,4), і вивчили деякі їхні комбінаторні властивості. У цій статті розглядається випадок ч. в. множини $(1,4,4)$.

Ключові слова: зображення, критична та суперкритична ч. в. множина, надсуперкритична ч. в. множина, квадратична форма Тітса, скінченний і ручний зображувальний тип, додатність і слабка додатність, негативність і слабка негативність.

Список використаної літератури

1. Назарова Л. А., Ройтер А. В. Представления частично упорядоченных множеств. *Зап. науч. сем. ЛОМИ.* 1972. Т. 28. С. 5–31.
2. Клейнер М. М. Частично упорядоченные множества конечного типа. *Зап. науч. семинаров ЛОМИ.* 1972. Т. 28. С. 32–41.
3. Дрозд Ю. А. Преобразования Кокстера и представления частично упорядоченных множеств. *Функц. анализ и его прил.* 1974. Т. 8, Вып. 3. С. 34–42.
4. Назарова Л. А. Частично упорядоченные множества бесконечного типа. *Изв. АН СССР. Изв. АН СССР. Сер. матем.* 1975. Т. 39, Вып. 5. С. 963–991.
5. Бондаренко В. М., Назарова Л. А., Завадский А. Г. О представлениях ручных частично упорядоченных множеств. *Представления и квадратичные формы. – Киев: Ин-т математики АН УССР.* 1979. С. 75–105.
6. Бондаренко В. М. Точные частично упорядоченные множества бесконечного роста. *Линейная алгебра и теория представлений. – Киев: Ин-т математики АН УССР.* 1983. С. 68–85.
7. Бондаренко В. М., Назарова Л .А., Ройтер А. В. Представления частично упорядоченных множеств с инволюцией. *Препр. Ин-т математики АН УССР.* 1986. 86.80. 24 с.
8. Бондаренко В. М. Связки полуцепных множеств и их представления. *Препр. Ин-т математики АН УССР.* 1988. 88.60. 32 с.
9. Назарова Л. А., Бондаренко В. М., Ройтер А. В. Ручные частично упорядоченные множества с инволюцией. *Труды матем. ин-та АН СССР им. В. А. Стеклова.* 1990. Т. 183. С. 149–159.
10. Bondarenko V. M., Zavadskij A. G. Posets with an equivalence relation of tame type and of finite growth. *CSM Conf. Proc.* 1991. Vol. 11. P. 67–88.
11. Завадский А. Г. Алгоритм дифференцирования и классификация представлений. *Изв. АН СССР. Сер. матем.* 1991. Т. 55, вып. 5. С. 1007–1048.
12. Бондаренко В. М., Степочкина М. В. (Min, max)-эквивалентность частично упорядоченных множеств и квадратичная форма Титса. *Проблеми аналізу і алгебри: Зб. праць Ін-ту математики НАН України.* 2005. Т. 2, вып. 3. С. 18–58.
13. Bondarenko V. M. On (min, max)-equivalence of posets and applications to the Tits forms. *Bull. of Taras Shevchenko Kyiv Nation. Univer. Math. Mechanics.* 2005. No. 1. P. 24–25.
14. Бондаренко В. М., Степочкина М. В. (Min, max)-эквивалентность частично упорядоченных множеств и неотрицательные формы Титса. *Укр. мат. журнал.* 2008. Т. 60, № 9. С. 1157–1167.
15. Бондаренко В. М., Степочкина М. В. Описание частично упорядоченных множеств, критических относительно неотрицательности квадратичной формы Титса. *Укр. мат. журнал.* 2009. Т. 61, № 5. С. 611–624.
16. Бондаренко В. В., Бондаренко В. М., Степочкина М. В., Червяков И. В. 1-надсуперкритические частично упорядоченные множества с тривиальной группой автоморфизмов и min- эквивалентность. *Наук. вісн. Ужгород. ун-ту. Сер. матем. і інформ.* 2011. Вип. 22, № 2. С. 17–25.
17. Бондаренко В. В., Степочкина М. В. Непримитивное 1-надсуперкритическое частично упорядоченное множество и min-эквивалентность. *Науковий часопис НПУ ім. Драгоманова. Серія 1. фіз.-мат. науки.* 2013. № 14. С. 55–61.
18. Bondarenko V. M., Styopochkina M. V. On posets of sixth order having oversupercritical MM-type. *Scien. Bull. of Uzhhorod University. Series of Math. and Inform.* 2021. Vol. 38, No. 1. P. 7–15.
19. Bondarenko V. M., Styopochkina M. V. Classification of the posets of minmax type which are symmetric oversupercritical posets of the eighth order. *Math. methods and phys.-mech. fields.* 2023. Vol. 66, No. 1-2. P. 1–11.
20. Bondarenko V. M., Styopochkina M. V. On properties of posets of MM-type (1,3,5). *Scien. Bull. of Uzhhorod University. Series of Math. and Inform.* 2018. Vol. 32, No. 1. P. 50–63.
21. Bondarenko V. M., Styopochkina M. V. On transitivity coefficients for minimal posets with non-positive quadratic Tits form. *Math. methods and phys.-mech. fields.* 2021. Vol. 64, No. 1. P. 5–14.

Одержано 16.04.2024