# CHOICE AND EVALUATION METHODICS OF INVESTMENT PROJECTS

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## Metodiky výberu a hodnotenia investičných projektov

**Abstract:** The article provides building approaches of mathematical model and the evaluation of the investment projects, which is based on a duplex hierarchical structure.
**Key words**: investment projects; experts; criteria; evaluation; fuzzy information; membership function.

**Abstrakt:** Článok prezentuje vytváranie prístupov s využitím matematického modelu a hodotenia investičných projektov na základe duplexnej hierarchickej štruktúry.
**Kľúčové slová:** investičný project, experti, kritériá, hodnotenie, fuzzy informácia, funkcia členstva

## INTRODUCTION

Investment activity – is an important component of economical development. The lack of investments – is a painful question about the economical development in any country of the world. Along with the investment projects there are a lot of others such as logistical base updating, production capacity accretion, new activities development and etc. The realization of such projects need resources which today are in lack. That's why investors are trying to find the solution of one or other problem very carefully. In this case the problem of investment projects becomes very relevant.

The aim of this article is the construction of the mathematical model, which gives an opportunity to rank the investment project depending to the investors aim. This model must count the factors of uncertainty in decisions, it must base on hierarchical structure and consider the investors wishes on the final choice stage.

## FORMULATION OF PROBLEM

Let's describe the problem of investment project choice in the next way. Just imagine we have a set of investment projects $X = \{x_1, x_2, ..., x_n\}$ , which should be ranked. The set of projects is evaluated by several experts, so let's mark them $E^1, E^2, ..., E^k$ , and the investor(a person, which makes decisions) $E^0$.

Every expert and investor is using an own set of criteria for the investment project evaluation. Criteria can be qualitative and quantitative. Quantitative criteria are built on the base of known incoming marks about projects, and qualitative criteria are determined by the qualitative characteristics about the object.

Let's imagine this problem as a duplex hierarchic structure. On the top level is the investor, on the bottom level are the experts. Every expert is evaluating a set of investment projects. The investor counts the advantages of experts and has an opportunity to evaluate investment projects by his own set of criteria.

In this manner, on each level a problem of multicriterion choice of project ranking is solved, and on the top the choice of the best, or again project ranking. Now let's discuss the mathematic model of multicriterion choice.

### Mathematic model of the problem

Let's discuss the duplex hierarchic structure of decision. On each level of hierarchy a multicriterion choice problem is solved, the investor is solving his problem as well as each expert. A set of criteria, by which projects are evaluated, usually each has its own.

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Now let's formulate the multicriterion choice problem in the general form. Imagine a set of investment projects $X = \{x_j\}, j = \overline{1,n}$ and a set of criteria $K = \{K_i\}, i = \overline{1,m}$, by which projects are evaluated. Projects should be ranked relatively to the integrated function of utility $U$. The result of evaluation is submitted in the matrix of solution:

| | $x_1$ | $x_2$ | $\cdots$ | $x_n$ |
|---|---|---|---|---|
| $K_1$ | $O_{11}$ | $O_{12}$ | $\cdots$ | $O_{1n}$ |
| $K_2$ | $O_{21}$ | $O_{22}$ | $\cdots$ | $O_{2n}$ |
| . | | | | |
| . | | | | |
| . | | | | |
| $K_m$ | $O_{m1}$ | $O_{m2}$ | $\cdots$ | $O_{mn}$ |

Or in the matrix of marks $O = (O_{ij}), \quad i = \overline{1,m}; \quad j = \overline{1,n}$.

On the base of the matrix of marks the function of the projects utility relatively to which ranking is conducted, is made by each participant. The best value of the function of utility can receive maximal and minimal values and values from this interval.

The concept of the best value is a fuzzy concept. That's why we propose to build functions of utility by fuzzy sets. At the present stage of science development two types of fuzzy sets are distinguished:

- fuzzy sets, which are determined on a numerical scale, scilicet fuzzy numbers and fuzzy intervals;
- fuzzy sets, which are determined on a not numerical set.

The fuzzy set – is a set of objects, which is written in the next way: $A = \{x, \mu(x)\}$ where $x$ – is a set of objects, and $\mu(x)$ – is the function of membership of objects in this set. The most difficult problem is to build the function of membership.

The function of membership for relative criteria of this problem we will choice as one from the examples: linear s-type or z-type[3]. In general s-type and z-type functions of membership are given by relative analytical expressions:

$$\mu_s(x,a,b) = \left\{ \begin{array}{cc} 0, & x \le a \\ 2\left(\dfrac{x-a}{b-a}\right)^2, & a < x \le \dfrac{a+b}{2} \\ 1-2\left(\dfrac{b-x}{b-a}\right)^2, & \dfrac{a+b}{2} < x < b \\ 1, & x \ge b \end{array} \right\}; \qquad (1)$$

$$\mu_z(x,a,b) = \left\{ \begin{array}{cc} 1, & x \le a \\ 1-2\left(\dfrac{x-a}{b-a}\right)^2, & a < x \le \dfrac{a+b}{2} \\ 2\left(\dfrac{b-x}{b-a}\right)^2, & \dfrac{a+b}{2} < x < b \\ 0, & x \ge b \end{array} \right\}. \qquad (2)$$

Where $a, b$ are numerical parameters, which can accept criteria of evaluation and are ordered by correlation: $a < b$.

On the base of this matrix $O$, we can build the matrix by using a definite type of membership function:

$$Q = (Q_{ij}), \quad i = \overline{1,m}; \quad j = \overline{1,n}, \qquad (3)$$

[3] Маляр М.М. Нечітка модель оцінки фінансової кредитоспроможності підприємств/ Маляр М.М., Поліщук В.В.// Східно-Європейський журнал передових технологій. Сер. Математика і кібернетика – фундаментальні і прикладні аспекти. – Харків, 2012. - №3/4(57). – С.8-16. ISSN 1729-3774

where $Q_{ij}$ – is a value(mark) of the membership function *j* and alternative by *i* criteria.

Each expert and investor have to evaluate investment projects. These marks we will mark with a set $E = (E_1, E_2, ..., E_n)$. We can obtain these marks in the next way: we will build a membership function as a convolution of numerical marks. Imagine that the participant knows (he can set) the weighting coefficients to each criteria of efficiency $\{w_1, w_2, ..., w_m\}$ from the interval [0,a]. Then we can determine the normalized weighting coefficients for each criteria:

$$a_i = \frac{w_i}{\sum_{i=1}^{m} w_i},\ i = \overline{1,m};\ a_i \in [0,1],\ \sum_{i=1}^{m} a_i = 1. \tag{4}$$

Now let's build the function of utility for every participant, as one of the proposed convolutions, depending to psychosomatic mood:

1. $$U_E^1(x_j) = \frac{1}{\sum_{i=1}^{m} \frac{a_i}{Q_{ij}}} \text{ - pessimistic;} \tag{5}$$

2. $$U_E^2(x_j) = \prod_{i=1}^{m} \left(Q_{ij}\right)^{a_i} \text{ - careful;} \tag{6}$$

3. $$U_E^3(x_j) = \sum_{i=1}^{m} a_i \cdot Q_{ij} \text{ - average;} \tag{7}$$

4. $$U_E^4(x_j) = \sqrt{\sum_{i=1}^{m} a_i \cdot (Q_{ij})^2} \text{ - optimistic.} \tag{8}$$

After that marks from the set $E = (E_1, E_2, ..., E_n)$ will be determined by one of convolutions $E_j = U_E^r(x_j),\ (j = \overline{1,n};\ r = \overline{1,4})$.

Profiles of participants are built by analogical consideration, on the base of which the next matrix is built:

| | $x_1$ | $x_2$ | $\cdots$ | $x_n$ |
|---|---|---|---|---|
| $E^0$ | $E_1^0$ | $E_2^0$ | $\cdots$ | $E_n^0$ |
| $E^2$ | $E_1^1$ | $E_2^1$ | $\cdots$ | $E_n^1$ |
| $\vdots$ | | | | |
| $E^k$ | $E_1^k$ | $E_2^k$ | $\cdots$ | $E_n^k$ |

On the first stage we are building a fuzzy set on the numerical scale, and on the second not numerical. For this we should introduce a concept of the "point of pleasure" (point of limited rationality).

The point is called "point of pleasure", if all of its coordinates represent marks by themselves, which would satisfy the participant of making decisions. This concept will count the investors wishes[4].

On the second stage we should determine the next values:

$$z_{lj} = 1 - \frac{\left|t_l - E_j^l\right|}{\max\{t_l - \min_j E_j^l; \max_j E_j^l - t_l\}},\quad l = 0,1,\ldots,k;\ j = 1,\ldots,n. \tag{9}$$

Each of these values we can consider, as a value of the membership function of a fuzzy set "close to the point of pleasure". The matrix $Z = \{z_{lj}\}$, which is determined, characterizes relative marks of alternative $x_j$

[4] Маляр Н. Н., Поліщук В.В. Двухуровневая модель нечеткого рационального выбора// ITHEA International Journal "Problem of Computer Intellectualizacion", Kyiv-Sofia 2012. – P.242-248. ISBN: 978-966-02-6529-5

closeness by columns up to the "point of pleasure" $T$ by every correct mark of the expert and removes the question from several evaluating scales.

The Investor due to his consideration $E^0$ and each expert $E^1, E^2, ..., E^k$ sets weighting coefficients $\{p_0, p_1, ..., p_k\}$ from the interval [0, a]. The normalized weighting coefficients are determined by analogical way: $\beta_l = p_l \Big/ \sum_{l=0}^{k} p_l$ , $l = \overline{0,k}$; $\sum_{l=0}^{k} \beta_l = 1$.

For investment project ranking a making the final decision, we should use one the convolutions (5)-(8), but instead matrix elements $Q$ , we will use element of the created matrix $Z$ and normalized weight $\beta$ . For example the average convolution will look so:

$$U_E^3(x_j) = \sum_{l=0}^{k} \beta_l \cdot z_{lj}, \quad j = \overline{1,n}. \tag{10}$$

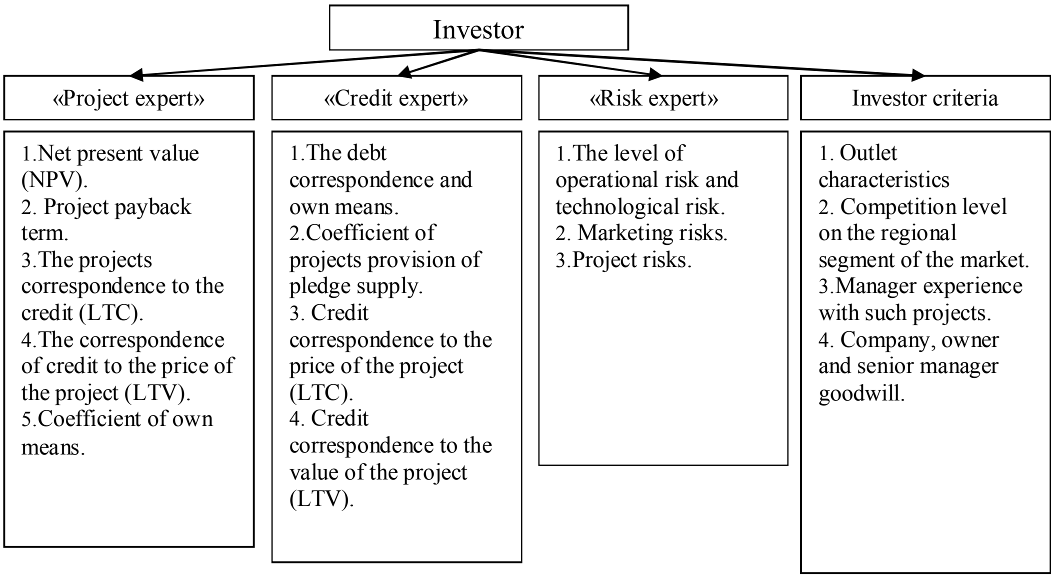
## SET OF XPERTS AND THE THEIR CRITERIA BUILDING

We have a set of investment projects, which must be ranked. Let's propose a set of experts, investors consideration, set of criteria by for ranking and for their membership function. We formalize criteria with fuzzy logic apparatus in a power set which equals to one.

For investment project evaluation we can use next experts:

1. Project expert – realizes analysis of investment project profitability;
2. Credit expert – does the debt analysis;
3. Risk expert – analyses the risk of the project.

Each expert gives his own mark, the investor also has his own considerations and mark criteria. Then we can observe the duplex hierarchic scheme and criteria of investor and experts:




Let's observe the expert and their criteria.

"**Project expert**" $E^1$ makes analysis due to the next criteria:

1. Net present value (NPV)[5].

[5] Чернов В.Г. Модели поддержки принятия решений в инвестиционной деятельности на основе апарата нечетких множеств. / В.Г. Чернов — М.: Горячая линия — Телеком, 2007. — 312 с. ISBN 978-5-93517-353-0

We will calculate the NPV due to the formula:

$$NPV = \sum_{k=1}^{n} \frac{CF_k}{(1+r)^k} - CF_0;$$

where $CF_0$ – is an initial investment, $CF_k$ - stream of payment on the stage, or period; $r$ – is a discount bet, which reflects the speed of money value changes in time, $n$ – is determined, as an average of a simple payback term for the observed investment projects.

As a result of the calculating process of this formula will be the price of the project. In this formula the initial investment is expressed, as own funds. In this case, we should consider even investment funds (also here we can add the percentage, which are accrued on own funds).

Imagine $K_1 = \frac{NVP}{IC}$, where $IC$- this the total cost (investment budget) of the project without percents, then we will build the membership function for this criteria, like a s-type due to the formula (1), in the next way:

$$\mu(K_1;\quad 0{,}2;\quad 1) = \begin{cases} 0, & if \quad K_1 \le 0{,}2; \\ \frac{(5K_1-1)^2}{8}, & if \quad 0{,}2 < K_1 \le 0{,}6; \\ 1-\frac{(5-5K_1)^2}{8}, & if \quad 0{,}6 < K_1 < 1; \\ 1, & if \quad K_1 \ge 1. \end{cases}$$

2. Simple payback term of the project (in years).

The time, which is needed for the payment on investments (without discount).
The function of membership for this criteria we will build as a z-type, formula (2), where $a = \min_j O_{ij}, b = \max_j O_{ij}$ . That's how the membership function will have the next content: the less is the payback term, the more the function of membership will go lead to one, in the opposite case to zero.

As a partial variant, the we can observe the payback term on the interval of years [1;5], then the function of membership would be:

$$\mu(K_2;\quad 1;\quad 5) = \begin{cases} 1, & if \quad K_2 \le 1; \\ 1-\frac{(K_2-1)^2}{8}, & if \quad 1 < K_2 \le 3; \\ \frac{(5-K_2)^2}{8}, & if \quad 3 < K_2 < 5; \\ 0, & if \quad K_2 \ge 5. \end{cases}$$

3. The projects correspondence to the credit (LTC).

We can determine this correspondence by this formula: $K_3 = \frac{Cs}{IC}$, where $Cs$ the loan amount (main debt). In that case, we understand the total expense of the project under its price. It's understood that the criteria is $K_5 \in (0;1]$ . Z-type function of membership will look so:

$$\mu(K_3;\quad 0{,}2;\quad 0{,}8)=\begin{cases} 1, & if \quad K_3 \le 0{,}2; \\ 1-\dfrac{2(5K_3-1)^2}{9}, & if \quad 0{,}2 < K_3 \le 0{,}5; \\ \dfrac{2(4-5K_3)^2}{9}, & if \quad 0{,}5 < K_3 < 0{,}8; \\ 0, & if \quad K_3 \ge 0{,}8. \end{cases}$$

4. The correspondence of credit to the price of the project (LTV).

The formula for calculating the criteria is: $K_4 = \dfrac{Cs}{VM}$, $VM$ - market price of assets, property, which was bought for the project. The total market price of the object is set when the project is finished. Under this criteria we understand the market price of the object, which is determined as the most probable price, for which it might be sold on the market in the case of competition. The z-type function of membership will look so:

$$\mu(K_4;\quad 0{,}3;\quad 0{,}9)=\begin{cases} 1, & if \quad K_4 \le 0{,}3; \\ 1-\dfrac{(10K_4-3)^2}{18}, & if \quad 0{,}3 < K_4 \le 0{,}6; \\ \dfrac{(9-10K_4)^2}{18}, & if \quad 0{,}6 < K_4 < 0{,}9; \\ 0, & if \quad K_4 \ge 0{,}9. \end{cases}$$

5. Coefficient of own means.

The formula for calculating the criteria is $K_5 = \dfrac{O}{IC}$, where $O$ is own means, - $IC$ is the total price (investment budget) of the project without percentage. The function of this criteria we should build as a s-type due to the formula (1), and here it is:

$$\mu(K_5;\quad 0{,}2;\quad 1)=\begin{cases} 0, & if \quad K_5 \le 0{,}2; \\ \dfrac{(5K_5-1)^2}{8}, & if \quad 0{,}2 < K_5 \le 0{,}6; \\ 1-\dfrac{(5-5K_5)^2}{8}, & if \quad 0{,}6 < K_5 < 1; \\ 1, & if \quad K_5 \ge 1. \end{cases}$$

"**Credit expert**" $E^2$ has the next criteria:

1. The debt correspondence and own means.

This correspondence we should determine by the formula: $K_6 = \dfrac{C_s}{O}$. The z-type function of membership is:

$$\mu(K_6;\quad 0{,}5;\quad 1)=\begin{cases} 1, & if \quad K_6 \le 0{,}5; \\ 1-\dfrac{(2K_6-1)^2}{2}, & if \quad 0{,}5 < K_6 \le 1; \\ \dfrac{(3-2K_6)^2}{2}, & if \quad 1 < K_6 < 1{,}5; \\ 0, & if \quad K_6 \ge 1{,}5. \end{cases}$$

In that case, only borrowed and own means are considered, which go on the project sponsorship.

2. Coefficient of projects provision of pledge supply.

This criteria we should calculate by the formula: $K_7 = \dfrac{Mv}{Cs}$ where $Mv$ – is the mortgage value of provision. We will build the function of membership as a s-type, and it's outlook is here:

$$\mu(K_7;\quad 0{,}8;\quad 1{,}4) = \begin{cases} 0, & if \quad K_7 \le 0{,}8; \\ \dfrac{(10K_7 - 8)^2}{18}, & if \quad 0{,}8 < K_7 \le 1{,}1; \\ 1 - \dfrac{(14 - 10K_7)^2}{18}, & if \quad 1{,}1 < K_7 < 1{,}4; \\ 1, & if \quad K_7 \ge 1{,}4. \end{cases}$$

In addition to the above said, the credit expert uses two criteria: LTC and LTV.

"**Risk expert"** $E^3$ evaluates by quality indicators. Let's observe the set of quality indicators and the scale of their evaluating.

1. The level of operational risks (stuff mistakes, IT system failures, supplier work failures, Force Majeures) and technological risk - $K_8$.

[0,9; 1] – absent - the operational and technological risks do not exist in the project;
[0,6; 0,9] –low level – management has predicted possible risks and had a plan to solve them;
[0,4; 0,6] – middle level – management has a list of possible risks, but the ways of their solution are not elaborated;
(0; 0,4] – high level – management has no idea about possible risks.

2. Marketing risks (are related with the sale of products/services) – $K_9$.

[0,9; 1] – absent – marketing risks are either absent or are almost eliminated, product sales is guaranteed by contracts or procurement.
[0,6; 0,9] – low risk – potential demand for the production is high, partially confined contracts for product realization;
[0,4; 0,6] – average risk – potential demand for the production is average , demand increasing is predicted, partially confined contracts for product realization;
(0; 0,4] – high risk - potential demand for the production is low, contracts for product realization are absent, demand increasing is not predicted.

3. Project risks - $K_{10}$.

[0,9; 1] – absent;
[0,6; 0,9] – low risk;
[0,4; 0,6] – average risk;
(0; 0,4] – high risk.

Now let's observe a set of criteria, which the investor $E^0$ considers to use them for investment project evaluating.

1. Outlet characteristics – $K_{11}$.

[0,7; 1] – today a significant increase of product outlet is observed;
[0,4 ; 0,7] - today the outlet capacity is stable;
(0; 0,4] – today the outlet is shortening.

2. Competition level on the regional segment of the market – $K_{12}$.

(0, 0;5] – high competition, aggressive competitor policy;
[0,5; 0,8] – relatively high competition, aggressive policy of market leaders;
[0,8; 1] – low competition, opportunity to increase the markets share.

3. Manager experience with such projects(except the definite project) – $K_{13}$.

[0,7; 1] – multiple completing of analogical projects;
[0,5; 0,7] – there is enough experience for such projects;
(0; 0,5] – managers don't have experience to realize the project from the beginning.

4. Company, owner and senior managers goodwill - $K_{14}$.

[0,7; 1] absence of information about possible sanctions, scandals, trials relative to the company or to company owners and senior managers;

[0,5; 7] – information existence about possible sanctions, scandals, trials relative to the company or to company owners and senior managers, the result of which are evaluated as minor and they can't influence the financial result.
(0; 0,5] - information existence about possible sanctions, scandals, trials relative to the company or to company owners and senior managers, the result of which can influence the financial result of the company.

**Example of model application**

Imagine that for investment projects $X = \{x_1, x_2, x_3, x_4\}$ were received, which must be evaluated, and determine the best of them. Let's observe the incoming values $A$ due to the next table:

| | Incoming parameters | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|---|
| $A_1$ | $CF_0$ - initial investment (thousands) | 300 | 1000 | 500 | 850 |
| $A_2$ | $r$ - discount bet | 0,1 | 0,1 | 0,1 | 0,1 |
| $A_3$ | Simple payback term of the project(years) | 2 | 3 | 4 | 3 |
| $A_4$ | $Cs$ - credit amount (main debt) (thousands) | 100 | 800 | 300 | 350 |
| $A_5$ | $IC$ - total price (investment budget)of the project without percentage (thousands) | 300 | 1000 | 500 | 850 |
| $A_6$ | $VM$ - market price of the assets, property, which was bought for the project(thousands) | 200 | 1500 | 700 | 900 |
| $A_7$ | $O$ - own means (thousands) | 200 | 200 | 200 | 500 |
| $A_8$ | $Mv$ - mortgage price of the project (thousands) | 150 | 900 | 600 | 400 |

The next step is built for expert evaluation $E^1, E^2, E^3$, and consideration of the investor $E^0$. First of all let's calculate the quantitative criteria for experts $E^1, E^2$, by data-in. we will wright the result into the table:

| $E^1$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $K_1$, $n=3$ | 0,66 | 0,12 | 0,24 | 0,32 |
| $K_2$ | 2 | 3 | 4 | 3 |
| $K_3$ | 0,33 | 0,8 | 0,6 | 0,41 |
| $K_4$ | 0,5 | 0,53 | 0,43 | 0,39 |
| $K_5$ | 0,67 | 0,2 | 0,4 | 0,59 |

On the next step we should determine the values of the projects by experts.

"Project expert" $E^1$ . This expert had evaluated the importance of his criteria with the next set {10, 10, 9, 8, 7}. Then due to the formula (4) we determine the normalized weighting coefficients - {0,23; 0,23; 0,20; 0,18; 0,16}. Now we calculate the functions of membership for expert $E^1$, and by the average convolution (7) we determine the values, and write the result in the matrix of solutions:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $\mu(K_1)$ | 0,64 | 0 | 0,01 | 0,05 |
| $\mu(K_2)$ | 0,88 | 0,5 | 0,13 | 0,5 |
| $\mu(K_3)$ | 0,9 | 0 | 0,22 | 0,76 |
| $\mu(K_4)$ | 0,78 | 0,7 | 0,93 | 0,96 |
| $\mu(K_5)$ | 0,66 | 0 | 0,13 | 0,48 |
| $E^1$ | 0,78 | 0,24 | 0,27 | 0,53 |

"Credit expert" $E^2$. This expert had evaluated the importance of his criteria with the next set {6, 7, 9, 8}, then due to the formula (4) we determine the normalized weighting coefficients {0,20; 0,23; 0,30; 0,27}. Now we calculate the functions of membership for expert $E^2$, and by the average convolution (7) we determine the values:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $\mu(K_6)$ | 1 | 0 | 0 | 0,92 |
| $\mu(K_7)$ | 1 | 0,6 | 1 | 0,63 |
| $\mu(K_3)$ | 0,9 | 0 | 0,22 | 0,76 |
| $\mu(K_4)$ | 0,78 | 0,7 | 0,93 | 0,96 |
| $E^2$ | 0,91 | 0,33 | 0,55 | 0,82 |

"Risk expert" $E^3$. This expert had evaluated the importance of his criteria {6, 8, 10} and relative normalized weighting coefficients {0,25; 0,33; 0,42}. The scale of criteria and values due to the (7) for this expert are here:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $\mu(K_8)$ | 0,45 | 0,5 | 0,6 | 0,9 |
| $\mu(K_9)$ | 0,7 | 0,5 | 0,8 | 0,8 |
| $\mu(K_{10})$ | 0,5 | 0,4 | 0,6 | 0,7 |
| $E^3$ | 0,55 | 0,46 | 0,67 | 0,78 |

The investor $E^0$ had evaluated the importance of his criteria {5, 6, 7, 8} and relative normalized weighting coefficients {0,19; 0,23; 0,27; 0,31}. The investor $E^0$ has determined his considerations and values by the average convolution here:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $\mu(K_{11})$ | 0,5 | 0,3 | 0,4 | 0,7 |
| $\mu(K_{12})$ | 0,8 | 1 | 0,5 | 0,6 |
| $\mu(K_{13})$ | 0,3 | 1 | 0,7 | 0,6 |
| $\mu(K_{14})$ | 0,6 | 0,4 | 0,7 | 0,9 |
| $E^0$ | 0,55 | 0,68 | 0,6 | 0,71 |

And now let's move to investors' decision stage.

The validity of experts and his considerations the investor had evaluated is number from the [0,10] interval relatively to: {9,8,9,7}. We search the normalized weighting coefficients $\beta_l, l = 1,2,3,4$ analogically {0,27; 0,24; 0,27; 0,21}. "the point of pleasure" the investor have set so: {0,79; 0,82; 0,83; 0,73}. Let's write it in the table, which consist of expert values $E^1, E^2, E^3$, investors considerations $E^0$ "point of pleasure" and normalized weighting coefficients:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $T$ | $\alpha_i$ |
|---|---|---|---|---|---|---|
| $E^1$ | 0,78 | 0,24 | 0,27 | 0,53 | 0,79 | 0,27 |
| $E^2$ | 0,91 | 0,33 | 0,55 | 0,82 | 0,82 | 0,24 |
| $E^3$ | 0,55 | 0,46 | 0,67 | 0,78 | 0,83 | 0,27 |
| $E^0$ | 0,55 | 0,68 | 0,60 | 0,71 | 0,73 | 0,21 |

Let's calculate the values by the formula (9) and let's write the result in a matrix Z:

$$Z = \begin{pmatrix} 0{,}98 & 0{,}00 & 0{,}05 & 0{,}53 \\ 0{,}82 & 0{,}00 & 0{,}45 & 1{,}00 \\ 0{,}24 & 0{,}00 & 0{,}57 & 0{,}86 \\ 0{,}00 & 0{,}72 & 0{,}28 & 0{,}89 \end{pmatrix}.$$

For making a decision, about the best investment project we can choose the average convolution, which we should calculate due to the formula (10).

| | $U_E^3(x_j)$ The average |
|---|---|
| $x_1$ | 0,5320 |
| $x_2$ | 0,1532 |
| $x_3$ | 0,3374 |
| $x_4$ | 0,8107 |

Then the investment projects are ranked in the next way $x_4, x_1, x_3, x_2$. From here we can say that the best and the least risky project is $x_4$.

**CONCLUSION**

The result of this scientific research is a model of investment project evaluation, which is based on cases, when the experts uncertainty exists in conclusions. And the validity in making decisions, due to the choice of investment project lets us to reduce the investment risks.

The simplicity, the clarity and transparency of calculations – are the qualities which investors want to see in mathematic models. This model can be used for several investment and financial institutions, which have an opportunity to form a group of own experts, criteria set, and complement it anytime, also it has to determine criteria importance and to set own levels (“points of pleasure”) for making decisions.

**LITERATURE**

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