TWO-STAGED MODEL OF MULTI-CRITERIA SELECTION

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Dvojúrovňový model multikriteriálneho výberu

ABSTRACT : The paper proposes a new conceptual approach to the problem of multi-criteria alternatives, based on the use of dynamic criteria of effectiveness, considering their tendency.
Keywords: multi-criteria selection, dynamic criteria, company lending.

Introduction

The main task for any organized activity is to make a decision to overcome problematic situations. Whether it's a simple solution, or a difficult organized multistage process, the decision is an act of choice on the set of variants (alternatives). In such cases a person (or a group of people) is exposed to the need to choose one or several alternative solutions (actions, plans, behavior). The necessity of selection is caused by the appearance of a problematic situation, which consists of real and desirable, and there is more than one variant to achieve the desired result. In this situation exists some kind of "freedom of choice", i.e. there is a finite (infinite) number of alternative solutions, the choice of which depends on the people who make decisions. In this paper we will examine a case, when from a built ranked row of alternatives advantages can't be determined clearly. Then we insert the consideration of dynamic criteria on the base of which we will build a new ranked row, with the use of predicted values.

Mathematic model

Let's consider the problem of selection, which we are going to describe by the following mathematical model. The set of alternatives we denote by X, and suppose, that it's finite, that acceptable alternatives can be counted $X = \{x_1, x_2, ..., x_n\}$. Let's denote $K = \{K_1, K_2, ..., K_m\}$ as a criteria set of effectiveness, by which each alternative from X set is evaluated. The problem of selection can be formed as follows: select the best alternative from the X set, if ranks of criteria on this set are known. The model of the problem can be presented in the table:

	x_1	x_2	$\cdots$	x_n
K_1	O_{11}	O_{12}	$\cdots$	O_{1n}
K_2	O_{21}	O_{22}	$\cdots$	O_{2n}
$\vdots$				
K_m	O_{m1}	O_{m2}	$\cdots$	O_{mn}

Or in the matrix of decision:

$$O = (O_{ij}),\ i = 1, \dots, m;\ j = 1, \dots, n; \qquad (1)$$

Where O_{ij} – is a rating of j-th alternative by i-th criterion. Each column of the matrix – a vector of ratings, which characterizes the alternative, and every line is a criteria.

Let using methods which are described in papers [1-3] constructed a ranked line of alternative marks $Z = (Z_1, Z_2, ..., Z_n)$ of descending values. By its' usage we order the vector of alternatives. Without loss of generality, let's consider that it's a vector of alternatives $\{x_1, x_2, ..., x_n\}$. Considering the case, when a number of built ranking line of alternatives $Z = (Z_1, Z_2, ..., Z_n)$ are close to the value to each other at some value

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$d = Z_{j+1} - Z_j$, on the first p place. Then exists the ambiguity and uncertainty about the choice of the person makes a decision about the best alternative. The decision maker doesn't know, how will behave the first, second…. *p-th* alternative decision in the future. Perhaps at such ranks the *p*-th alternative will be better than the first one. Thereby, a problem appears: which of alternatives of the ranking line $\{x_1, x_2, ..., x_n\}$, one must choose for making a decision. Let us build a new ranking line from chosen alternatives $\{x_1, x_2, ..., x_p\}$ using dynamic criteria, which will help to predict the behavior of alternative decisions in the future.

Let us consider a set of criteria $K_1, K_2, ..., K_h$ by which dynamic tracing of l period is possible. Let us present the criteria values of all period in the table and separately by each alternative ($i = 1,2,..., p$):

x_i	ε_1	ε_2	$\cdots$	ε_l
K_1	Q_{11}^i	Q_{12}^i	$\cdots$	Q_{1l}^i
K_2	Q_{21}^i	Q_{22}^i	$\cdots$	Q_{2l}^i
$\vdots$				
K_h	Q_{h1}^i	Q_{h2}^i	$\cdots$	Q_{hl}^i

Let us predict the Q_{hl}^i ranks by all criteria for the $l+1$ period on the base of pair linear regression [4]:

$$Y = a + bX, \tag{2}$$

where the values of a, b multiplier we will calculate by the least squares method.

In our case it is necessary to construct $h \cdot p$ equations, i.e. for each alternative by every criteria. Then we can build a matrix of solution on the base of these equations, for example, for the $l+1$ period, and to construct a ranking like on the base of this matrix.

In this case the equation of linear regression will be rewritten as:

$$Y_g^i(X) = a_g^i + b_g^i X, \quad i = \overline{1, p}, \quad g = \overline{1, h}. \tag{3}$$

b_g^i, a_g^i multipliers are calculated according to formula [5]:

$$b_g^i = \frac{l \cdot \sum_{k=1}^{l} \varepsilon_k \cdot Q_{gk}^i - \sum_{k=1}^{l} \varepsilon_k \cdot \sum_{k=1}^{l} Q_{gk}^i}{l \cdot \sum_{k=1}^{l} {\varepsilon_k}^2 - \left(\sum_{k=1}^{l} \varepsilon_k \right)^2}, \tag{4}$$

$$a_g^i = \overline{y_g^i} - b_g^i \cdot \bar{x}, \tag{5}$$

where $\overline{y_g^i} = \frac{1}{l} \sum_{k=1}^{l} Q_{gk}^i, \quad \bar{x} = \frac{1}{l} \sum_{k=1}^{l} \varepsilon_k, \quad i = \overline{1, p}, \quad g = \overline{1, h}.$

After the withdrawal of the regressive equations, we write the vectors of formed ranks of alternatives by criteria of effectiveness for the $X = l+1$ period in tabular form:

	x_1	x_2	$\cdots$	x_p
K_1	Y_1^1	Y_1^2	$\cdots$	Y_1^p
K_2	Y_2^1	Y_2^2	$\cdots$	Y_2^p
$\vdots$				
K_h	Y_h^1	Y_h^2	$\cdots$	Y_h^p

Or in a matrix of decision form:

$$L = \left(Y_g^i\right), i = \overline{1,p}; g = \overline{1,h}. \tag{6}$$

This matrix will characterize aggregated ranks of alternatives by dynamic criteria, from predicted on the next period. We can build a ranking line of matrix alternative (6) by analogical approaches as for the matrix (1) let us consider one of the cases.

We introduce into consideration a "point of satisfaction" [6] $T = \left(t_1, t_2, ..., t_h\right)$, i.e. an imaginary alternative, in which ranks by all criteria could satisfy the decision maker.

Whereas we know the matrix of decision [6] and "the point of satisfaction" T is set, we define the set of values as:

$$z_{gi} = 1 - \frac{\left|t_g - Y_g^i\right|}{\max\{t_g - \min_i Y_g^i; \max_i Y_g^i - t_g\}}, \quad g = \overline{1,h}; \quad i = \overline{1,p}. \tag{7}$$

Each values is a relative rank of matrix element proximity (6) for the corresponding element of the "point of satisfaction". So the defined matrix $Z = \{ z_{gi} \}$ characterizes by columns relative ranks of alternative proximity to the "point of satisfaction" by each concrete criterion, and removes the issue of different scales of ranking.

Then we need to select the best alternative, for this we build a function of membership, as a convolution of numerical ranks.

Let's assume that the decision maker knows and can set weight coefficients for each criterion of effectiveness $\{p_1, p_2, ..., p_h\}$ from the [0,a] interval. Then we can define the normalized weight coefficients for each criterion:

$$\alpha_g = \frac{p_g}{\sum_{g=1}^{h} p_g}, \ g = \overline{1,h}; \ \alpha_g \in [0,1]; \tag{8}$$

which correspond the condition $\sum_{g=1}^{h} \alpha_g = 1$

Then, we take one convolution for aggregated rank construction [7]. For example, we take an average weighted convolution:

$$A(x_i) = \sum_{g=1}^{h} \alpha_g z_{gi}, i = \overline{1,p}. \tag{9}$$

On the base of $A(x_i)$, we build a new ranking line:

$$A = (A_1, A_2, ..., A_p). \tag{10}$$

So, we resulted the technique, by which we can build a ranking line of alternatives on the base of dynamic criteria of effectiveness.

An example of the method

We consider the mathematic model of the problem on the example of company selection for credit issuance. Let's assume that six applications from companies enter the bank. We will consider companies as alternatives, between which the decision maker must select one for credit issuance.

x_1	x_2	x_3	x_4	x_5	x_6
0,79	0,32	0,75	0,57	0,82	0,81

Then, alternatives will be ranked as follows: $X = \{x_5, x_6, x_1, x_3, x_4, x_2\}$. The x_5, x_6, x_1, x_3 alternatives are side by side. Let's enter the $d = 0{,}03$ distance, then the x_3, x_4, x_2 alternatives we separate, so x_1, x_5, x_6 will remain. In such case there is uncertainty about the decision maker in deciding.

We evaluate the x_1, x_5, x_6 companies and rank them on the base of next dynamic criteria of effectiveness between 2010-2013 years:

Criterion	Criterion name	Weight (*p*)	Point of satisfaction (T)
K_1	Current liquidity coefficient	8	0,9
K_2	Coefficient of general liquidity	9	1,5
K_3	Coefficient of financial independence	7	2
K_4	Profitability of production	10	0,09

Firstly we calculate criteria of effectiveness on the base of financial reporting, we will write the result in the table separately for companies:

x_1	2010	2011	2012	2013
K_1	0,4	1,1	0,6	0,7
K_2	0,8	1	1,2	1,3
K_3	0,9	2,9	4,1	1,8
K_4	0,06	0,09	0,12	0,07

x_5	2010	2011	2012	2013
K_1	0,3	1,2	1,3	0,9
K_2	0,8	0,9	1,2	1
K_3	0,5	0,9	2,1	1,1
K_4	0,06	0,07	0,06	0,05

x_5	2010	2011	2012	2013
K_1	0,3	1,2	1,3	0,9
K_2	0,8	0,9	1,2	1
K_3	0,5	0,9	2,1	1,1
K_4	0,06	0,07	0,06	0,05

In the second stage we will build regressive equations on the base of data for each alternative by each criterion.

Firstly we construct a regressive equation for x_1 alternative by K_1 criterion.

We calculate the b_1^1 , a_1^1 coefficients by using formula (4) and (5):

$$b_1^1 = \frac{4 \cdot 5632{,}4 - 8046 \cdot 2{,}8}{4 \cdot 16184534 - 8046 \cdot 8046} = 0{,}04, \quad a_1^1 = \frac{1}{4} \cdot 2{,}8 - 0{,}04 \cdot \frac{1}{4} \cdot 8046 = -79{,}76.$$

Then the equation of linear regression will be:

$$Y_1^1(X) = -79{,}76 + 0{,}04 \cdot X.$$

The predicted value of current liquidity criterion K_1 for the 2014 year by the first alternative is: $Y_1^1(2014) = -79{,}76 + 0{,}04 \cdot 2014 = 0{,}8.$

The rest 11 regressive equations we construct analogically and calculate the predicted value. The result of the predicted value we will write in the table:

	x_1	x_5	x_6
K_1	0,8	1,4	0,75
K_2	1,5	1,2	1,4
K_3	3,4	1,9	2,55
K_4	0,1	0,05	0,035

On the third stage, we calculate the value of the matrix $Z = \{ z_{gi} \}$ by the formula:

$$Z = \begin{pmatrix} 0{,}80 & 0{,}00 & 0{,}70 \\ 1{,}00 & 0{,}00 & 0{,}67 \\ 0{,}00 & 0{,}93 & 0{,}61 \\ 0{,}82 & 0{,}27 & 0{,}00 \end{pmatrix}$$

We calculate the $\alpha = (0{,}24; \quad 0{,}26; \quad 0{,}21; \quad 0{,}29)$ weight coefficients on the base of formula (8).

On the last stage we calculate aggregated ranks by formula (9): $A = (0{,}69; \quad 0{,}27; \quad 0{,}47).$ we order the alternatives by descending: x_1, x_6, x_5 . We conclude, what is the best company for 2010-2013 years.

Conclusion

The paper presents an approach to the problem of multi criteria selection of alternatives the peculiarity is that this mathematical model uses dynamic criteria of effectiveness. The mathematical model can be used as a basic instrument of the multi criteria selection or as an auxiliary to enhance the degree validity of decision-making in a built ranking line of alternatives. As an example of the model application on economic problem of choosing the company to provide a bank loan.

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