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V. M. Bondarenko, I. V. Chervyakov (Institute of Mathematics of NAN of Ukraine, Kyiv)
M. V. Styopochkina (Zhytomyr National University of Agriculture and Ecology, Zhytomyr)

ON PROPERTIES OF THE HASSE DIAGRAM OF P-CRITICAL POSETS

We consider the posets critical with respect to positivity of the Tits quadratic form and study their combinatoric properties.

Ми розглядаємо частково впорядковані множини, критичні відносно додатності квадратичної форми Тітса і вивчаємо їх комбінаторні властивості.

1. Introduction. The Hasse diagram is a type of diagram that represents a finite poset in the plane. Namely, for a poset S one represents each element of S as a vertex and each pair of elements x, y of S, such that y covers x (i. e. $x < y$ and there is no z satisfying $x < z < y$), as an edge (a line segment or curve) that goes upward from x to y. We denote such diagram by $H(S)$. For a class of finite posets $\mathcal{X}$ we denote by VA($\mathcal{X}$) the set of pairs (s, k) of non-negative integer numbers, such that s and k are respectively the number of vertices and edges of $H(X)$ for an $X \in \mathcal{X}$.

In this paper we study properties of the Hasse diagram of some class of posets connected with the Tits quadratic form. All posets are assumed to be finite.

Let S be a poset without an element denoted by 0. The Tits quadratic form of S is by definition the form $q_S : \mathbb{Z}^{S\cup 0} \to \mathbb{Z}$ defined by the equality

$$q_S(z) = z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j, i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i.$$

A poset S is called critical with respect to positivity of the Tits quadratic form or, briefly, P-critical if the Tits form of any its proper subset is positive but the Tits form of S is not positive [1]. The set of all P-critical posets will be denoted by $\mathcal{P}_c$.

The aim of this paper is to prove the following theorem.

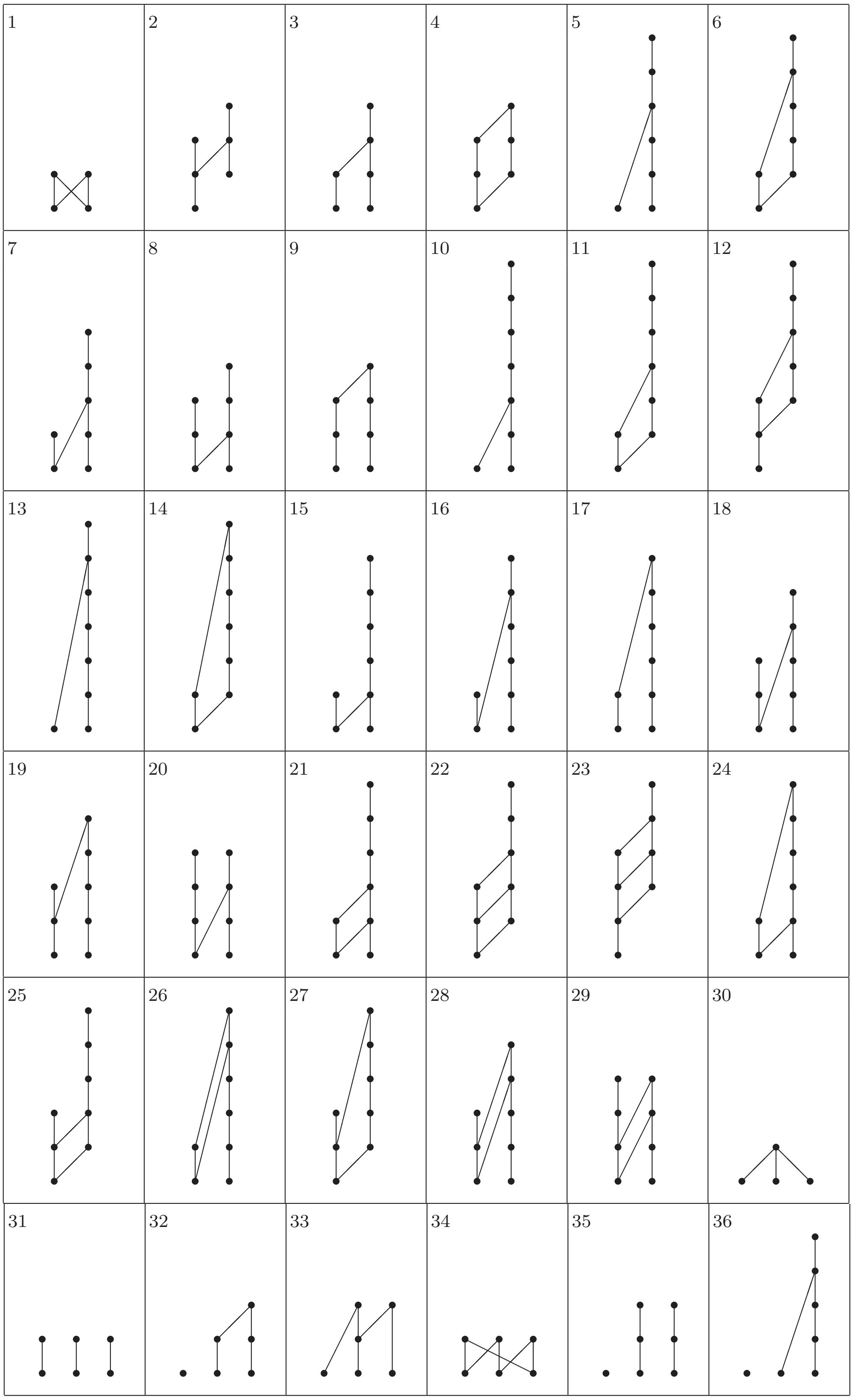
Theorem 1. *VA*$(\mathcal{P}_c)$ *consists of the following pairs:*
$(4,0)$, $(4,3)$, $(4,4)$,
$(6,3)$, $(6,4)$, $(6,5)$, $(6,6)$,
$(7,4)$, $(7,5)$, $(7,6)$, $(7,7)$,
$(8,5)$, $(8,6)$, $(8,7)$, $(8,8)$, $(8,9)$.

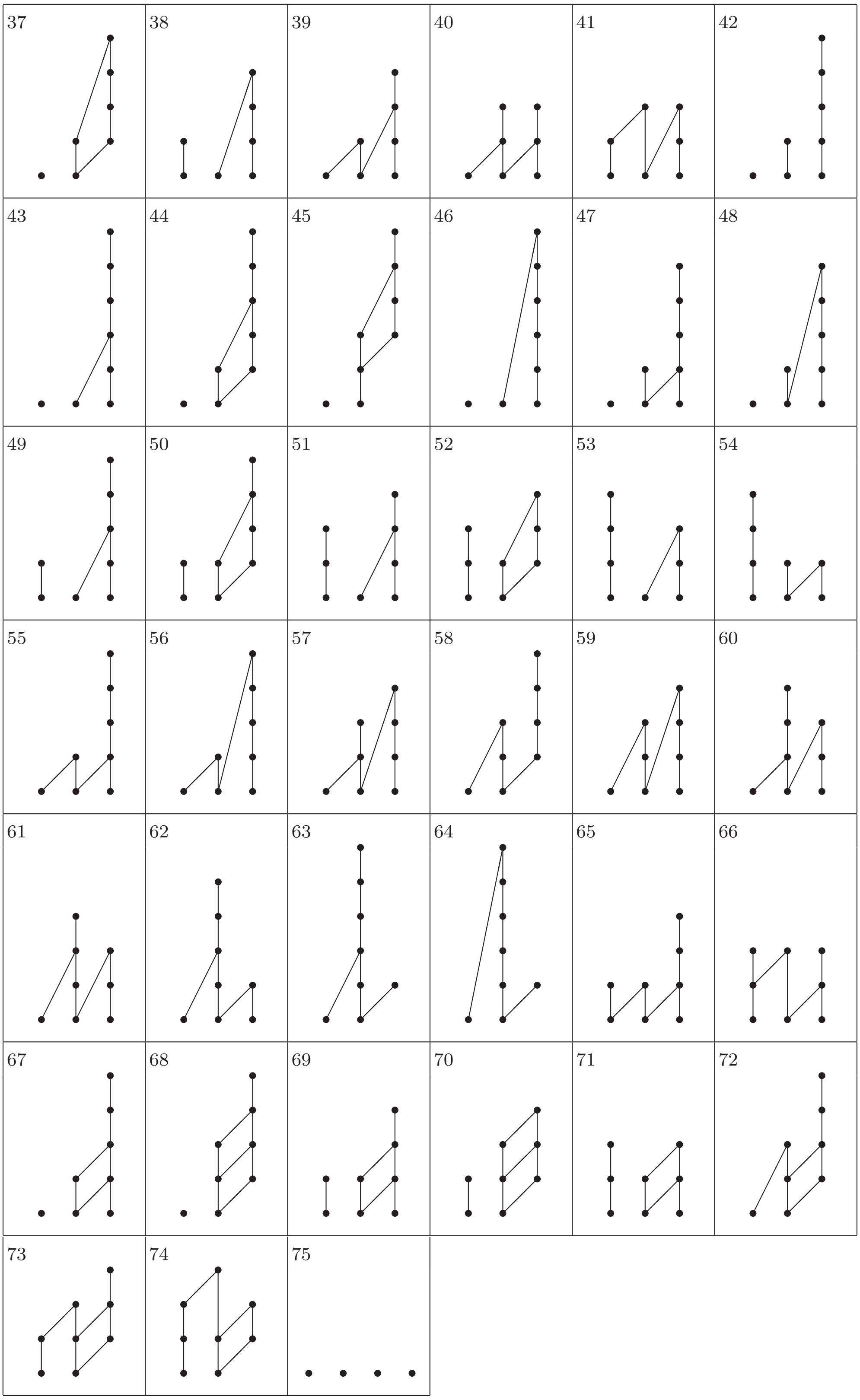
From this theorem we have the following corollaries.

Corollary 1. *Let* $(s,i),(s,j) \in VA(\mathcal{P}_c)$ *and* $i < k < j$. *If* s *is not equal to* 4 (*the smallest first coordinate for the pairs of* $VA(\mathcal{P}_c)$), *then* $(s,k) \in VA(\mathcal{P}_c)$.

Corollary 2. *Let* $(s,k) \in VA(\mathcal{P}_c)$. *If* s *is not equal to* 9 (*the biggest second coordinate for the pairs of* $VA(\mathcal{P}_c)$), *then* $s \geq k$.

2. ***P*-critical posets.** The P-critical posets were classified in [1]. They are given (up to isomorphism and anti-isomorphism) by the following table.





3. Proof of Theorem 1. The theorem follows from the following table in which one indicates the P-critical posets numbers, their numbers of vertices and edges.

№	vertices	edges	№	vertices	edges
1	4	4	39	7	6
2	6	5	40	7	6
3	6	5	41	7	6
4	6	6	42	8	5
5	7	6	43	8	6
6	7	7	44	8	7
7	7	6	45	8	7
8	7	6	46	8	6
9	7	6	47	8	6
10	8	7	48	8	6
11	8	8	49	8	6
12	8	8	50	8	7
13	8	7	51	8	6
14	8	8	52	8	7
15	8	7	53	8	6
16	8	7	54	8	6
17	8	7	55	8	7
18	8	7	56	8	7
19	8	7	57	8	7
20	8	7	58	8	7
21	8	8	59	8	7
22	8	9	60	8	7
23	8	9	61	8	7
24	8	8	62	8	7
25	8	8	63	8	7
26	8	8	64	8	7
27	8	8	65	8	7
28	8	8	66	8	7
29	8	8	67	8	7
30	4	3	68	8	8
31	6	3	69	8	7
32	6	4	70	8	8
33	6	5	71	8	7
34	6	6	72	8	8
35	7	4	73	8	8
36	7	5	74	8	8
37	7	6	75	4	0
38	7	5			

References

1. Bondarenko V. M., Stepochkina M. V. (Min, max)-equivalence of partially ordered sets and the Tits quadratic form // Problems of Analysis and Algebra: Institute of Mathematics of Ukrainian NAS, Kiev. – 2005. – **2**, N3. – P. 18-58 (in Russian).

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