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Levy–Baxter theorems for one class of non-Gaussian random fields

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Abstract. The Levy–Baxter limit theorems were obtained for a class of random fields with increments of class K.

Keywords. Levy–Baxter theorem, vector with property K, fields with increments of class K.

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1 Introduction

In 1971, S. M. Krasnitskii [10] had proved the limit theorem of Baxter type for the random fields with Gaussian m-order increments. The convergence of Baxter sums for Gaussian random fields was investigated also by T. V. Arak [1], T. Kawada [7], C. M. Deo and S. F. Wong [5], X. Guyon [6], C. Xiong and P. Xia [14]. O. O. Kurchenko [11] also investigated the convergence of Baxter sums for non–Gaussian random fields. V. V. Buldygin, V. M. Melnik, V. G. Shportjuk [3] obtained Levy–Baxter theorems for shot-noise fields. V. V. Buldygin and Y. V. Kozachenko [2] obtained the conditions of the convergence of Baxter sums to non-random constant for jointly strictly sub–Gaussian random processes and jointly pseudo-Gaussian random processes. In 2009, O. O. Kurchenko [12] proved the Baxter type theorem for jointly strictly sub-Gaussian random fields. The Levy–Baxter theorems were used for parametric estimation in statistics of random processes and fields in papers by Y. V. Kozachenko, O. O. Kurchenko [8], B. L. S. Prakasa Rao [13]. Y. V. Kozachenko and O. O. Kurchenko established in the paper [9] the Baxter type theorems for a certain class of random processes with K-increments.

In this paper we have obtained the limit Levy–Baxter theorems for one class of non-Gaussian random fields. The paper is organized as follows. In Section 2 we present the definition of random vectors of class K and their basic properties. Section 3 contains the definition of random fields with increments of class K and Baxter type theorems for these fields. We give an example of a non-Gaussian random field in Section 3 which satisfies the conditions of the Levy–Baxter theorems.

In the problem of the stochastic simulation of random field from some parametric family random fields it is necessary to estimate the unknown parameter by available observations. The Levy–Baxter theorem which had been obtained in this paper could be used for the estimation of this parameter on the certain conditions.

2 Random vectors with property K

Let (Ω, F, P) be a standard probability space.

Definition 2.1 ([9]). A random vector $(\xi, \eta) \in L_4(\Omega) \times L_4(\Omega)$ has property K if

(1) $E\xi = E\eta = 0$,

(2) $E(\xi \pm \eta)^4 \le 3(E(\xi \pm \eta)^2)^2$.

The class of all two-dimensional vectors with property K is denoted by K. Let us define the subclass K_1 of the class K as the set of all vectors of class K for which $E(\xi \pm \eta)^4 = 3(E(\xi \pm \eta)^2)^2$.

Example 2.2. Gaussian two-dimensional vectors with zero mean belong to the subclass K_1.

Example 2.3 ([9]). Let ξ_1, ξ_2 be independent and uniformly distributed on $[-a, a]$ $(a > 0)$ random variables. Then the random vector (ξ_1, ξ_2) belongs to the class K.

Lemma 2.4 ([9]). *Let $\xi_1, \xi_2 \in L_4(\Omega)$ be independent random variables for which $E\xi_1 = E\xi_2 = 0$, $E\xi_i^4 \le 3(E\xi_i^2)^2$, $i = 1, 2$. Then the random vector (ξ_1, ξ_2) belongs to the class K.*

Remark 2.5. Let the conditions of Lemma 2.4 hold and $E\xi_i^4 = 3(E\xi_i^2)^2$, $i = 1, 2$. Then the random vector (ξ_1, ξ_2) belongs to the class K_1.

Lemma 2.6 ([9]). *Let a random vector (ξ, η) belong to the class K. Then the following inequality holds:*

$$E(\xi^2\eta^2) \le 2(E\xi\eta)^2 + E\xi^2 E\eta^2 + \frac{1}{2}\left((E\xi^2)^2 - \frac{1}{3}E\xi^4\right) + \frac{1}{2}\left((E\eta^2)^2 - \frac{1}{3}E\eta^4\right). \tag{2.1}$$

Lemma 2.7. *Let ξ be a random variable, $E\xi = 0$, $E\xi^4 \le 3(E\xi^2)^2$. Then for all $\alpha, \beta \in \mathbb{R}$ the random vector $(\alpha\xi, \beta\xi)$ belongs to the class K.*

Proof. We have

$$E(\alpha\xi \pm \beta\xi)^4 = (\alpha \pm \beta)^4 E\xi^4 \le 3\big(E\xi^2(\alpha \pm \beta)^2\big)^2 = 3\big(E(\alpha\xi \pm \beta\xi)^2\big)^2.$$

The lemma is proved. □

Remark 2.8. Let ξ be a random variable, $E\xi = 0$, $E\xi^4 = 3(E\xi^2)^2$. Then for all $\alpha, \beta \in \mathbb{R}$ the random vector $(\alpha\xi, \beta\xi)$ belongs to the class K_1.

Lemma 2.9. *Let ξ_1, ξ_2 be independent random variables such that $E\xi_i = 0$ and $E\xi_i^4 \le 3(E\xi_i^2)^2$, $i = 1, 2$. Then for all $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \mathbb{R}$ the random vector*

$$(\alpha_1\xi_1, \beta_1\xi_1) + (\alpha_2\xi_2, \beta_2\xi_2) = (\alpha_1\xi_1 + \alpha_2\xi_2, \beta_1\xi_1 + \beta_2\xi_2)$$

belongs to the class K.

Proof. We have

$$\begin{aligned}
E\big((\alpha_1 + \beta_1)\xi_1 + (\alpha_2 + \beta_2)\xi_2\big)^4 \\
&= E(\alpha_1 + \beta_1)^4\xi_1^4 + 6E(\alpha_1 + \beta_1)^2\xi_1^2(\alpha_2 + \beta_2)^2\xi_2^2 + E(\alpha_2 + \beta_2)^4\xi_2^4 \\
&\le 3\Big(\big((\alpha_1 + \beta_1)^2 E\xi_1^2\big)^2 + 2E\big((\alpha_1 + \beta_1)^2\xi_1^2\big)E\big((\alpha_2 + \beta_2)^2\xi_2^2\big) \\
&\qquad + \big((\alpha_2 + \beta_2)^2 E\xi_2^2\big)^2\Big) \\
&= 3\big((\alpha_1 + \beta_1)^2 E\xi_1^2 + (\alpha_2 + \beta_2)^2 E\xi_2^2\big)^2 \\
&\le 3\Big(E\big((\alpha_1 + \beta_1)\xi_1 + (\alpha_2 + \beta_2)\xi_2\big)^2\Big)^2.
\end{aligned}$$

The lemma is proved. □

Remark 2.10. Let ξ_1, ξ_2 be independent random variables such that $E\xi_i = 0$ and $E\xi_i^4 = 3(E\xi_i^2)^2$, $i = 1, 2$. Then for all $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \mathbb{R}$ the random vector

$$(\alpha_1\xi_1, \beta_1\xi_1) + (\alpha_2\xi_2, \beta_2\xi_2) = (\alpha_1\xi_1 + \alpha_2\xi_2, \beta_1\xi_1 + \beta_2\xi_2)$$

belongs to the class K_1.

Example 2.11. Let ξ_1, ξ_2 be independent identically distributed random variables with density

$$f(x) = \begin{cases} 0, & |x| < 1, \\ \frac{s-1}{2|x|^s}, & |x| \ge 1 \end{cases}, \quad \text{where } s \ge 3 + \sqrt{6}.$$

The random variables ξ_1, ξ_2 have zero mean and satisfy the inequality

$$E\xi_i^4 \le 3\big(E\xi_i^2\big)^2, \quad i = 1, 2.$$

Indeed,

$$E\xi_i^4 = (s-1)\int_1^\infty \frac{x^4}{x^s}dx = \frac{s-1}{s-5}, \quad i = 1, 2,$$

and

$$E\xi_i^2 = (s-1)\int_1^\infty \frac{x^2}{x^s}dx = \frac{s-1}{s-3}, \quad i = 1, 2.$$

For $s \geq 3 + \sqrt{6}$ the inequality

$$E\xi_i^4 \leq 3\big(E\xi_i^2\big)^2, \quad i = 1, 2,$$

holds. So, the random vector (ξ_1, ξ_2) belongs to the class K. If $s = 3 + \sqrt{6}$, then (ξ_1, ξ_2) belongs to the class K_1.

3 Baxter theorems for random fields with increments of class *K*

Let $X(t)$, $t \in [0,1]^d$, be a random field. The increment of the random field $X(t), t \in [0,1]^d$, on $\Pi = [t_1, t_1 + h_1] \times \cdots \times [t_d, t_d + h_d]$, where $t = (t_1, \ldots, t_d)$, $h = (h_1, \ldots, h_d)$, $h_i > 0$, $1 \leq i \leq d$, is the following random variable:

$$X_\Pi = \sum_{i(1),\ldots,i(d)=0}^{1} (-1)^{i(1)+\cdots+i(d)} X(t_1 + i(1)h_1, \ldots, t_d + i(d)h_d).$$

Definition 3.1. The random field $X(t)$, $t \in [0,1]^d$, with zero mean is called random field with increments of class K (resp. K_1) if for arbitrary parallelepipeds $P, Q \subset [0,1]^d$ without inner common points the random vector (X_P, X_Q) belongs to the class K (resp. K_1).

Example 3.2. Let $\varphi_i : [0,1]^d \to [0,1]$, $i \geq 1$, be a sequence of Borel functions, and (ξ_i), $i \geq 1$, be a sequence of independent identically distributed random variables such that:

(1) $E\xi_i = 0, i \geq 1$,

(2) $E\xi_i^4 \leq 3(E\xi_i^2)^2, i \geq 1$.

Let us assume that for all $t \in [0,1]^d$ the series $\sum_{i=1}^\infty \xi_i \varphi_i(t)$ converges in $L_4(\Omega)$. Then the random field

$$X(t) = \sum_{i=1}^\infty \xi_i \varphi_i(t), \quad t \in [0,1]^d, \tag{3.1}$$

has the increments of the class K.

Indeed, for increments $\xi = X_P$, $\eta = X_Q$, where parallelepipeds $P, Q \subset [0,1]^d$ do not have common inner points, the random variable $\xi + \eta$ is the series

$$\sum_{i=1}^{\infty} \alpha_i \xi_i,$$

where $\alpha_i = (\varphi_i)_P + (\varphi_i)_Q$, $i \geq 1$, which converges in $L_4(\Omega)$. For all $n \in \mathbb{N}$ according to the conditions on the sequence of independent variables ξ_i, $i \geq 1$, and Lemma 2.9, we have

$$E\left(\sum_{i=1}^{n} \alpha_i \xi_i\right)^4 \leq 3\left(E\left(\sum_{i=1}^{n} \alpha_i \xi_i\right)^2\right)^2.$$

In previous inequality we pass to the limit as $n \to \infty$ and obtain

$$E\left(\sum_{i=1}^{\infty} \alpha_i \xi_i\right)^4 \leq 3\left(E\left(\sum_{i=1}^{\infty} \alpha_i \xi_i\right)^2\right)^2.$$

So the field (3.1) is a random field with increments of class K. If for random variables ξ_i, $i \geq 1$, the following conditions $E\xi_i^4 = 3(E\xi_i^2)^2$, $i \geq 1$, hold, then the field (3.1) is a random field with increments of class K_1.

Let $X(t)$, $t \in [0,1]^d$, be a random field with zero mean, $(a_n) \subset \mathbb{N}$ be an increasing sequence, $a_n \to \infty$. We denote by λ_n the regular partition of the d-dimensional parallelepiped $[0,1]^d$ on a_n^d congruent parallelepipeds:

$$\lambda_n = \{Q(k) \mid k = (k_1, \dots, k_d),\ 0 \leq k_i \leq a_n - 1,\ 0 \leq i \leq d\},$$

where $Q(k) = \left[\frac{k_1}{a_n}, \frac{k_1+1}{a_n}\right] \times \cdots \times \left[\frac{k_d}{a_n}, \frac{k_d+1}{a_n}\right]$, $n \geq 1$.

Let

$$S_n = \sum_{Q \in \lambda_n} X_Q^2, \quad n \geq 1,$$

be a sequence of Baxter sums.

Theorem 3.3. *Let $X(t)$, $t \in [0,1]^d$, be a random field with increments of the class K satisfying the following conditions:*

(i) $V_n^{(1)} = \sum_{P,Q \in \lambda_n} (EX_P X_Q)^2 \to 0$, $n \to \infty$,

(ii) $V_n^{(2)} = a_n^d \sum_{Q \in \lambda_n} ((EX_Q^2)^2 - \frac{1}{3} EX_Q^4)^2 \to 0$, $n \to \infty$.

Then $S_n - ES_n \to 0$ in the mean square as $n \to \infty$.

Proof. We have

$$\begin{aligned}\operatorname{Var} S_n &= E\Big(\sum_{Q\in\lambda_n}(X_Q^2 - EX_Q^2)\Big)^2 \\ &= E\Big(\Big(\sum_{P\in\lambda_n}(X_P^2 - EX_P^2)\Big)\Big(\sum_{Q\in\lambda_n}(X_Q^2 - EX_Q^2)\Big)\Big) \\ &= \sum_{P,Q\in\lambda_n}\big(E(X_P^2 X_Q^2) - EX_P^2 EX_Q^2\big).\end{aligned}$$

Then, with regard to Lemma 2.6, we obtain

$$\begin{aligned}\operatorname{Var} S_n &\le \sum_{P,Q\in\lambda_n}\Big(2(EX_P X_Q)^2 + \frac{1}{2}\Big((EX_P^2)^2 - \frac{1}{3}EX_P^4\Big) \\ &\qquad + \frac{1}{2}\Big((EX_Q^2)^2 - \frac{1}{3}EX_Q^4\Big)\Big) \\ &= 2\sum_{P,Q\in\lambda_n}(EX_P X_Q)^2 + a_n^d\sum_{Q\in\lambda_n}\Big((EX_Q^2)^2 - \frac{1}{3}EX_Q^4\Big) \\ &= 2V_n^{(1)} + V_n^{(2)} \to 0, \quad n\to\infty.\end{aligned}$$

It follows from conditions (i)–(ii) of this theorem that $S_n - ES_n \to 0$ in the mean square as $n\to\infty$. The theorem is proved. □

For the random field $X(t)$, $t\in[0,1]^d$, with increments of class K_1 we have

$$EX_Q^4 = 3(EX_Q^2)^2 \quad \text{and} \quad V_n^{(2)} = 0.$$

Then condition (ii) of Theorem 3.3 hold.

Corollary 3.4. *Let $X(t)$, $t\in[0,1]^d$, be a random field with increments of class K_1 and conditions* (i) *of Theorem* 3.3 *hold. Then $S_n - ES_n \to 0$ in the mean square as $n\to\infty$.*

Theorem 3.5. *Let $X(t)$, $t\in[0,1]^d$, be a random field with increments of class K (resp. K_1) satisfying the conditions of Theorem* 3.3 *and the series*

$$\sum_{n=1}^{\infty}(2V_n^{(1)} + V_n^{(2)})$$

is convergent. Then $S_n - ES_n \to 0$ with probability one as $n\to\infty$.

Proof. From the proof of Theorem 3.3 we have

$$\operatorname{Var} S_n \le 2V_n^{(1)} + V_n^{(2)}, \quad n\ge 1.$$

Taking into account the convergence of the series $\sum_{n=1}^{\infty}(2V_n^{(1)} + V_n^{(2)})$, the series $\sum_{n=1}^{\infty} \mathrm{Var} S_n$ converges too. So, we obtain $S_n - ES_n \to 0$ with probability one as $n \to \infty$. □

Theorem 3.6. *Let $X(t)$, $t \in [0,1]^d$, be a random field with increments of class K, $\alpha \in \mathbb{R}$ satisfying the following conditions:*

(a) $a_n^{\alpha-d} \sum_{Q\in\lambda_n} EX_Q^2 \to c$, $0 < c < +\infty$, $n \to \infty$,

(b) $\tilde{V}_n^{(1)} = a_n^{2(\alpha-d)} \sum_{P,Q\in\lambda_n} (EX_P X_Q)^2 \to 0$, $n \to \infty$,

(c) $\tilde{V}_n^{(2)} = a_n^{2\alpha-d} \sum_{Q\in\lambda_n} ((EX_Q^2)^2 - \frac{1}{3} EX_Q^4)^2 \to 0$, $n \to \infty$.

Then

$$a_n^{\alpha-d} \sum_{Q\in\lambda_n} X_Q^2 \to c$$

in mean square as $n \to \infty$. If moreover the series $\sum_{n=1}^{\infty}(\tilde{V}_n^{(1)} + \tilde{V}_n^{(2)})$ converges, then

$$a_n^{\alpha-d} \sum_{Q\in\lambda_n} X_Q^2 \to c$$

with probability one as $n \to \infty$.

Theorem 3.7. *Let $X(t)$, $t \in [0,1]^d$, be a random field with increments of class K_1 with zero mean and covariance function $r(t,s) \in C([0,1]^{2d})$, $t,s \in [0,1]^d$, satisfying the following conditions:*

(I) *there is a constant $\alpha > 0$ and a function $u : [0,1]^d \to (0,\infty)$ such that*

$$\frac{EX_Q^2}{h^\alpha} \to u(\cdot) \quad \text{uniformly on } [0,1]^d \text{ as } h \to 0+,$$

where $Q = [t_1, t_1 + h] \times \cdots \times [t_d, t_d + h]$,

(II) *there are constants $L \geq 0$ such that for all*

$$t,s \in \{(t,s) \mid t,s \in [0,1]^d,\ t_i \neq s_i,\ 1 \leq i \leq d\}$$

one has

$$\left| \frac{\partial^{2d} r(t,s)}{\partial t_1 \dots \partial t_d \partial s_1 \dots \partial s_d} \right| \leq \frac{L}{|t_1 - s_1|^{\theta_1} \cdots |t_d - s_d|^{\theta_d}},$$

where

$$0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_k < \frac{1}{2} = \theta_{k+1} = \cdots = \theta_{k+l} < \theta_{k+l+1} \leq \cdots \leq \theta_d,$$

(III) $2\alpha + k < 5d + 2(\theta_1 + \cdots + \theta_k)$, $\alpha > 0$, $d \geq 2$, $k \leq d$.

Then

$$\hat{S}_n = a_n^{\alpha-d} \sum_{Q\in\lambda_n} X_Q^2 \to \int_{[0,1]^d} u(t)dt$$

in mean square as $n \to \infty$.

Proof. According to uniform convergence in condition (I) and the continuity in mean square of the random field $X(t)$, $t \in [0,1]^d$, we have that $u \in C([0,1]^d)$ and

$$E\hat{S}_n \to \int_{[0,1]^d} u(t)dt, \quad n \to \infty.$$

It follows from Theorem 3.3 that

$$\operatorname{Var} \hat{S}_n \le 2a_n^{2(\alpha-d)} \sum_{P,Q\in\lambda_n} (EX_P X_Q)^2.$$

To prove that

$$a_n^{2(\alpha-d)} \sum_{P,Q\in\lambda_n} \big(E(X_P X_Q)\big)^2 \to 0, \quad n \to \infty,$$

we put

$$A(n) = \big\{P = P(k), Q = Q(j) \mid k = (k_1, \dots, k_d),\ j = (j_1, \dots, j_d),\\ 0 \le k_i, j_i \le a_n - 1,\ |k_i - j_i| > 3,\ 1 \le i \le d\big\}$$

and

$$B(n) = \lambda_n \setminus A(n).$$

Let us divide the sum $\sum_{P,Q\in\lambda_n} (E(X_P X_Q))^2$ into two sums

$$\sum_{P,Q\in\lambda_n} \big(E(X_P X_Q)\big)^2 = \sum_{P,Q\in A(n)} \big(E(X_P X_Q)\big)^2 + \sum_{P,Q\in B(n)} \big(E(X_P X_Q)\big)^2.$$

Using the Cauchy–Bunyakovsky inequality and condition (I), we obtain

$$\begin{aligned} a_n^{2(\alpha-d)} \sum_{P,Q\in B(n)} \big(E(X_P X_Q)\big)^2 &\le a_n^{2(\alpha-d)} \sum_{P,Q\in B(n)} EX_P^2 EX_Q^2 \\ &= O\left(a_n^{2(\alpha-d)} a_n^{2d-1} \frac{1}{a_n^{2\alpha}}\right) \\ &= O\left(\frac{1}{a_n}\right), \quad n \to \infty. \end{aligned}$$

For $P(k), Q(j) \in A(n)$ from condition (II) we have

$$\begin{aligned}
&\left(EX_{P(k)}X_{Q(j)}\right)^2 \\
&\quad = \left(\int_{P(k)\times Q(j)} \frac{\partial^{2d} r(t,s)}{\partial t_1 \dots \partial t_d \partial s_1 \dots \partial s_d} dt_1 \dots dt_d ds_1 \dots ds_d\right)^2 \\
&\quad \le \left(\int_{P(k)\times Q(j)} \frac{L}{|t_1 - s_1|^{\theta_1} \cdots |t_d - s_d|^{\theta_d}} dt_1 \dots dt_d ds_1 \dots ds_d\right)^2 \\
&\quad \le \left(\frac{L}{|k_1 - j_1 - 1|^{\theta_1} \cdots |k_d - j_d - 1|^{\theta_d}} \left(\frac{1}{a_n}\right)^{2d}\right)^2 \\
&\quad \le \frac{L^2}{|k_1 - j_1 - 1|^{2\theta_1} \cdots |k_d - j_d - 1|^{2\theta_d}} \cdot \frac{1}{a_n^{4d}}.
\end{aligned}$$

Then

$$\begin{aligned}
a_n^{2(\alpha-d)} \sum_{P,Q\in A(n)} \left(E(X_P X_Q)\right)^2 &\le a_n^{2(\alpha-d)} a_n^d \sum_{l=2}^{a_n} \frac{1}{l^{2\theta_1}} \cdots \sum_{l=2}^{a_n} \frac{1}{l^{2\theta_d}} \cdot \frac{1}{a_n^{4d}} \\
&= O\left(a_n^{2(\alpha-d)} a_n^d (\ln a_n)^l a_n^{k-2(\theta_1+\dots+\theta_k)} \frac{1}{a_n^{4d}}\right) \\
&= O\left(a_n^{2\alpha-5d+k-2(\theta_1+\dots+\theta_k)}\right),
\end{aligned}$$

because

$$\sum_{l=2}^{a_n} \frac{1}{l^{2\theta_i}} = \begin{cases} O(1), & \theta_i > \frac{1}{2},\ 1 \le i \le k, \\ O(\ln a_n), & \theta_i = \frac{1}{2},\ k+1 \le i \le k+l, \\ O(a_n^{1-2\theta_i}), & \theta_i < \frac{1}{2},\ k+l+1 \le i \le d, \end{cases} \quad n \to \infty.$$

Then from condition (III) it follows that $\operatorname{Var} \hat{S}_n \to 0$, $n \to \infty$. The theorem is proved. □

Remark 3.8. If for all $\alpha > 0$ the series $\sum_{n=1}^{\infty} \frac{1}{a_n^{\alpha}} < +\infty$, then

$$a_n^{\alpha-d} \sum_{Q\in\lambda_n} X_Q^2 \to \int_{[0,1]^d} u(t)dt$$

with probability one as $n \to \infty$.

Example 3.9. A Gaussian random field $W(t)$, $t \in [0,1]^d$, with zero mean and covariance function

$$EW(t)W(s) = \min(t_1, s_1) \cdots \min(t_d, s_d)$$

for all $t = (t_1, \dots, t_d), s = (s_1, \dots, s_d) \in [0,1]^d$ is called Chentsov random field, cf. [4].

Let us consider the case when $d = 2$. Let $W(t,s)$, $t, s \in [0,1]$, be a Chentsov random field such that

$$EW(t,s) = 0, \quad EW(t_1,t_2)W(s_1,s_2) = \min(t_1,s_1)\min(t_2,s_2),$$

where $t = (t_1,t_2), s = (s_1,s_2) \in [0,1]^2$. For the random field $W(t,s)$, $t, s \in [0,1]$, we have the following Karhunen–Loeve expansion:

$$W(t,s) = 2\sum_{k,j=1}^{\infty} \frac{\sin(k-\frac{1}{2})\pi t}{(k-\frac{1}{2})\pi} \cdot \frac{\sin(j-\frac{1}{2})\pi s}{(j-\frac{1}{2})\pi} \xi_{kj}, \quad t, s \in [0,1],$$

where ξ_{kj}, $k, j \geq 1$, are independent Gaussian random variables with

$$E\xi_{kj} = 0, \quad E\xi_{kj}^2 = 1.$$

Let η_{kj}, $k, j \geq 1$, be independent random variables such that

$$E\eta_{kj} = 0, \quad E\eta_{kj}^2 = 1, \quad E\eta_{kj}^4 = 3\big(E\eta_{kj}^2\big)^2.$$

We put

$$X(t,s) = 2\sum_{k,j=1}^{\infty} \frac{\sin(k-\frac{1}{2})\pi t}{(k-\frac{1}{2})\pi} \cdot \frac{\sin(j-\frac{1}{2})\pi s}{(j-\frac{1}{2})\pi} \eta_{kj}, \quad t, s \in [0,1]. \tag{3.2}$$

For all $t, s \in [0,1]$ the series (3.2) converges in $L_4(\Omega)$. The proof of the fact that the field $X(t,s)$ is a random field with increments of class K_1 is similar to that of Example 3.2. For the random field $X(t,s)$, $t, s \in [0,1]$, the conditions of Theorem 3.7 hold for $u(t) = 1$, $t = (t_1,t_2) \in [0,1]^2$ and $L = 1$, $d = 2$, $\alpha = 2$, $\theta_1 = \theta_2 = 0$, $k = 2$.

4 Conclusions

In this work we obtained the Levy–Baxter theorems for the random fields with increments of class K. Also there was considered an example of the random field with increments of class K_1 which satisfies the Baxter type theorems. Further we plane to use the obtained results for the estimation of the unknown parameters of the covariance function of non-Gaussian random fields.

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