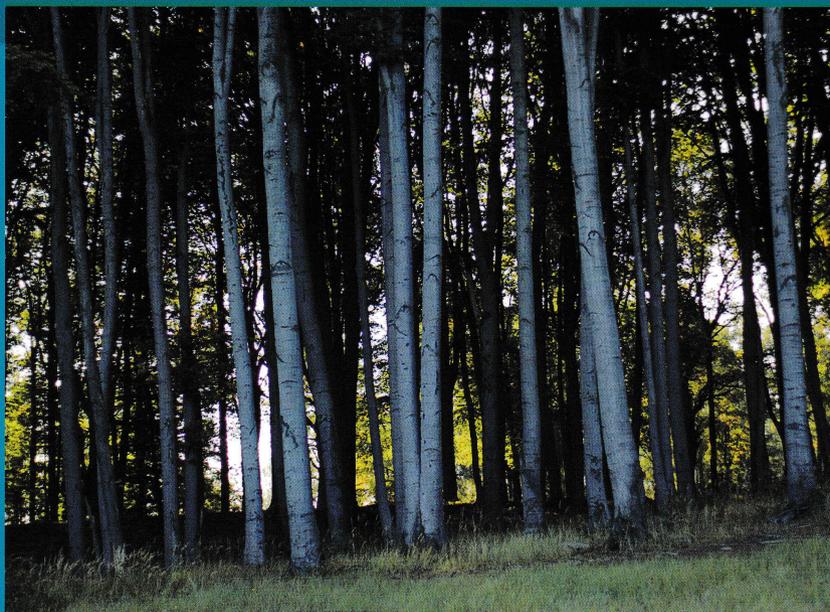


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**THE 16th SMALL TRIANGLE MEETING
on theoretical physics**

October 5–8, 2014 | Ptíče

ACADEMY OF SCIENCES



Quantum Teleportation and Information Transfer in the System of Two Resonant Atoms

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Abstract

The resonance transfer of quantum information from one two-level atom to another that is at an arbitrary distance from the former one is investigated. The symmetric Ψ_s and antisymmetric Ψ_a states of the two-atom system are considered taking into account dipole-dipole interaction between atoms of this system. The closed-form analytical expressions for shifts and widths of the considered collective states $\Psi_{s(a)}$ of two identical two-level atoms located at an arbitrary distance from each other. In the quite general considerations the system of nonstationary equations describing separately evolution of states amplitudes of the compound-system “ $A(1)$ atom+ $A(2)$ atom+field” in symmetric and antisymmetric channels at absorption of a photon by one of the compound-system atoms. Three cases of the obtained solutions for amplitudes of possible states of the compound-system at resonance and nonresonance absorption of a photon are considered. It has been shown that when atoms are at an infinite distance from one another, so that there is no dipole-dipole interaction of atoms, quantum information can be transferred from one atom to another with a characteristic time considerably shorter than the time it takes for a photon to cover the interatomic distance. This effect is referred to as the effect of quantum teleportation in a system of resonance atoms.

Introduction

Among basic physical problems related to realization of idea of quantum computers and quantum calculations it is necessary to mark the search of the concrete processes performed logic operations [1]. As it is shown in [1] at physical realization of the logical NOT operation it is need to influence on the carrier of quantum information (a qubit) by external pulse inverting qubit from ground state into excited one and vice versa, from excited state into ground one. At the present time many experimental groups (see for instance [2] and references therein) occupying ions in traps are able to prepare easy a single ion in the own state and then in the specified superposition of states with the coherence time larger than 1 sec. Two-qubit logic CNOT operation (Controlled NOT) is performed by means of influence

of exciting pulse on two qubits interacting with each other: herewith through interaction (switched on during realization of operation or existing constantly) one qubit controls a quantum evolution of second one.

In papers [3, 4, 5] the principle of operating the quantum computer on the basis of electric dipole transition in spectrum of two-level atom-qubits interacting selectively with short intensive optical pulses was proposed. Herewith the logic CNOT operation is performed by means of quantum information transfer, depending on time, at "far" (of order of 10^4 nm) distances from one two-level atom (qubit) to another because of retarded interaction of qubits in the field of optical radiation.

The present paper is devoted to study the proposed previously model [3] for description of resonance transfer of quantum information at an arbitrary (including arbitrary large) interatomic distances including the quantum teleportation effect. Realization of quantum teleportation of states in the considered system of two resonance atoms opens new opportunities in solving the problem of transfer of breakable superpositional states at large distances without losses of their coherence. This problem is a stumbling block for creation of quantum computers.

The elegant proof of principle possibility of transfer of the quantum information at far distances from one two-level atom (qubit) to another by means of quantum correlations (including quantum teleportation) has been given by O.N. Gadomsky and K.K. Altunin in the paper mentioned above [3]. In this paper the damping of states is considered to do not affect the process of excitation exchange between atoms. Such consideration is justified if the characteristic time of excitation transfer is much smaller than the atom lifetime Γ_n^{-1} in excited state. The given paper is a step to generalization of traditional statement of the problem of quantum teleportation and information resonance transfer from one two-level atom to another in two directions: a) only one of levels is considered to be stationary, and another possess the specified damping, b) the broadening of energy levels induced by the retarded interatomic interaction is taken into account in addition to radiative one.

The energy of resonance interaction of atoms located at arbitrary distances

Let us write the operator of electric dipole-dipole interaction of two electrons located at an arbitrarily large distance from each other, near different nuclei [23, 24]:

$$\hat{V}_{dip}^{(\pm)} = \left\{ \frac{\vec{d}_1 \vec{d}_2 - 3(\vec{n}_R \vec{d}_1)(\vec{n}_R \vec{d}_2)}{R^3} \pm \frac{e}{2mc} \left[\frac{\vec{d}_1 \hat{p}_2 - 3(\vec{n}_R \vec{d}_1)(\vec{n}_R \hat{p}_2)}{R^2} - \frac{\hat{p}_1 \vec{d}_2 - 3(\vec{n}_R \hat{p}_1)(\vec{n}_R \vec{d}_2)}{R^2} \right] - \frac{e^2}{m^2 c^2} \frac{\hat{p}_1 \hat{p}_2 - 3(\vec{n}_R \hat{p}_1)(\vec{n}_R \hat{p}_2)}{R} \right\} \exp\left(\frac{i}{c} \omega_0 R\right), \quad (1)$$

where $\vec{n}_R = \vec{R}/R$, $\omega_0 \equiv \omega_{n0} = (E_n - E_0)/\hbar$ is the resonance frequency in the spectrum of the atoms, $\vec{d}_1 = e\vec{r}_1$ and $\vec{d}_2 = e\vec{r}_2$ are the operators of the electric dipole

moments of single-electron atoms, \vec{p}_1 and \vec{p}_2 are the operators of the momenta of the first and the second atomic electron, respectively.

For the system of two interacting atoms having one electron, the Hamiltonian \hat{H} can be represented in the form of sum of Hamiltonians of isolated atoms, $\hat{H}_0 = \hat{H}_1(\vec{r}_1) + \hat{H}_2(\vec{r}_2)$, and operator of electric dipole-dipole interaction between them, $\hat{V}_{dip}^{(\pm)}$:

$$\hat{H} = \hat{H}_0 + \hat{V}_{dip}^{(\pm)} = \hat{H}_1(\vec{r}_1) + \hat{H}_2(\vec{r}_2) + \hat{V}_{dip}^{(\pm)}(\vec{r}_1, \vec{r}_2; R). \quad (2)$$

Let $E_{n_1 n_2} = E_{n_1} + E_{n_2}$ and $|n_1 n_2\rangle$ are the eigenvalue and eigenfunction of "unperturbed" operator of energy $\hat{H}_0 = \hat{H}_1 + \hat{H}_2$ without interatomic interaction, respectively. The unperturbed wave function of the initial state of two atoms has the form

$$|00\rangle = \varphi_0(1)\varphi_0(2) \exp(-iE_0 t_1/\hbar) \exp(-iE_0 t_2/\hbar) \equiv \tilde{\varphi}_0(1)\tilde{\varphi}_0(2). \quad (3)$$

Here E_0 is the energy of the initial states of the first and second atoms, the indices 1 and 2 correspond to the coordinates and times for the first and the second atom, respectively. For atoms having no constant dipole moment in the first order of perturbation theory, the energy correction is zero, i.e., $\langle 00|\hat{V}_{dip}^{(\pm)}|00\rangle = 0$. The interaction between the atoms distorts the wave functions of the atomic states, and the disturbed wave function for a system of two atoms in the ground state has the form [8, 1]):

$$\Psi_0(1)\Psi_0(2) = \tilde{\varphi}_0(1)\tilde{\varphi}_0(2) + \sum_{n_1 n_2} \frac{\langle \tilde{\varphi}_{n_1}(1)\tilde{\varphi}_{n_2}(2)|\hat{V}_{dip}^{(\pm)}|\tilde{\varphi}_0(1)\tilde{\varphi}_0(2)\rangle}{2E_0 - E_{n_1} - E_{n_2}} \tilde{\varphi}_{n_1}(1)\tilde{\varphi}_{n_2}(2), \quad (4)$$

where the indices n_1 and n_2 correspond to intermediate states of the atoms. We consider the state $\Psi_0(1)\Psi_0(2)$ as initial state of atoms interacting with field of real photons.

Let the final state $|n0\rangle$ of two resonant atoms corresponds to the excited state $|n\rangle$ of atom $A(1)$ with wave function $\tilde{\varphi}_n(1) = \varphi_n(1) \exp(-iE_n t_1/\hbar)$ and energy E_n and ground state $|0\rangle$ of atom $A(2)$ with wave function $\tilde{\varphi}_0(2) = \varphi_0(2) \exp(-iE_0 t_2/\hbar)$ and energy E_0 .

In the first approximation, to the stationary state of the system corresponds to two wave functions:

$$\Phi_s(1, 2) = \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) + \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)], \quad (5)$$

$$\Phi_a(1, 2) = \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) - \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)]. \quad (6)$$

In order to obtain corrections to the energy of symmetrical (42) and antisymmetrical (43) states of system of two resonant atoms in the first order of perturbation theory one has calculate the mean value of the perturbation operator $\hat{V}_{dip}^{(\pm)}$ (38) in these states, i.e.,

$$\Delta E_s = \langle \Phi_s|\hat{V}_{dip}^{(\pm)}|\Phi_s\rangle, \quad \Delta E_a = \langle \Phi_a|\hat{V}_{dip}^{(\pm)}|\Phi_a\rangle. \quad (7)$$

Substituting the expressions (38), (42) and (43) into the matrix elements (7) we obtain that

$$\begin{aligned} \Delta E_s(R) &= -\Delta E_a(R) = \omega_0^3 e^{i\omega_0 R/c} |\langle n | \vec{d} | 0 \rangle|^2 \times \\ &\times \left[\left(\frac{1}{\omega_0^3 R^3} - \frac{i}{c\omega_0^2 R^2} \right) \Phi(1, 2) - \frac{\Phi'(1, 2)}{c^2 \omega_0 R} \right] \equiv \Delta E_{AA}(R). \end{aligned} \quad (8)$$

Here

$$\begin{aligned} \Phi(1, 2) &\equiv \cos \theta_1^x \cos \theta_2^x + \cos \theta_1^y \cos \theta_2^y - 2 \cos \theta_1^z \cos \theta_2^z, \\ \Phi'(1, 2) &\equiv \cos \theta_1^x \cos \theta_2^x + \cos \theta_1^y \cos \theta_2^y \end{aligned}$$

are the geometric factors, θ_i^x , θ_i^y , θ_i^z ($i = 1, 2$) are the angles between one of corresponding axes and the direction of dipole transition in i -th atom $A(i)$.

Note that the quantity ΔE_s (ΔE_a) is complex shift of energy E_s (E_a) of symmetrical (antisymmetrical) state Φ_s (Φ_a) of atoms:

$$\Delta E_s = \delta E_s - \frac{i}{2} \hbar \gamma_s, \quad \Delta E_a = \delta E_a - \frac{i}{2} \hbar \gamma_a. \quad (9)$$

First-order corrections to the energy $E_n + E_0$ of states (42) and (43) gives only real parts δE_s and δE_a of complex shifts:

$$\delta E_{s,a}(R) = \pm \text{Re} \Delta E_{AA}(R) = \pm e^2 |\langle n | \vec{r} | 0 \rangle|^2 F(1, 2; R). \quad (10)$$

Here

$$F(1, 2; R) = \left[\frac{\Phi(1, 2)}{R^3} - \frac{\omega_0^2 \Phi'(1, 2)}{c^2 R} \right] \cos \left(\frac{\omega_0 R}{c} \right) + \frac{\omega_0 \Phi(1, 2)}{c R^2} \sin \left(\frac{\omega_0 R}{c} \right), \quad (11)$$

f_{n0} are observable strengths of oscillators of corresponding transitions.

The expressions for δE_s and δE_a can be simplified at $\omega_0 R/c \rightarrow 0$, i.e., when size of two-atomic quantum system is much smaller than characteristic wave length $\lambda_0 = 2\pi c/\omega_0$ in spectrum of interacting atoms ($R \ll \lambda_0$). In this case one can neglect retardation of dipole-dipole interaction of atoms, that makes it possible to substitute $\cos(\omega_0 R/c) = 1$, $\sin(\omega_0 R/c) \approx \omega_0 R/c$ in the expression (11) for F and to omit terms containing velocity of light c . Then for δE_s and δE_a we obtain

$$\delta E'_{s,a} = \pm \frac{e^2}{R^3} |\langle n | \vec{r} | 0 \rangle|^2 \Phi(1, 2). \quad (12)$$

As one would expect, this expression coincides with the known formula [8] for the energy of resonant exchange of excitations between two neutral atoms located closely one from other.

Therefore, we see that at account of dipole-dipole interatomic interaction (38) symmetrical and antisymmetrical states of atoms are characterized by the energies

$$E_s = E_n + E_0 + \delta E_s = E_n + E_0 + \frac{e^2 \hbar f_{n0}}{2m\omega_0} F(1, 2; R), \quad (13)$$

$$E_a = E_n + E_0 + \delta E_a = E_n + E_0 - \frac{e^2 \hbar f_{n0}}{2m\omega_0} F(1, 2; R), \quad (14)$$

and wave functions

$$\Psi_s = \Phi_s \exp(-i\delta E_s t_1/\hbar), \quad \Psi_a = \Phi_a \exp(-i\delta E_a t_1/\hbar), \quad (15)$$

where t_1 is the local time for the atomic pair associated with the position of the atom $A(1)$.

It is apparent that states (15) are entangled states of a pair of interacting atoms because in these states separate atom-qubit has not certain energy.

In the first order of perturbation theory the contribution of dipole interaction in complete width Γ_s (Γ_a) is determined by imaginary part γ_s (γ_a) of doubled complex shift ΔE_s (ΔE_a):

$$\gamma_s(R) = -\gamma_a(R) = -2\text{Im}\Delta E_{AA}(R) = -\frac{3}{2}\gamma_0 c^3 \tilde{F}(1, 2; R), \quad (16)$$

where

$$\tilde{F}(1, 2; R) = \left[\frac{\Phi(1, 2)}{\omega_0^3 R^3} - \frac{\Phi'(1, 2)}{c^2 \omega_0 R} \right] \sin\left(\frac{\omega_0 R}{c}\right) - \frac{\Phi(1, 2)}{c \omega_0^2 R^2} \cos\left(\frac{\omega_0 R}{c}\right), \quad (17)$$

γ_0 is the radiation width of an excited state E_n of the isolated atom.

The general expression (16) for γ_s and γ_a can be essentially simplified in the limit cases of "large" ($R \gg \lambda_0$) and "small" ($a < R \ll \lambda_0$) interatomic distances. For example, when $\omega_0 R/c \ll 1$ and orientations of dipole transition in the both atoms are along the system axis \vec{R} formula (16) becomes

$$\gamma_s^{(\parallel)}(R) = -\gamma_a^{(\parallel)}(R) = \gamma_0 \left(1 - \frac{\omega_0^2 R^2}{10c^2} \right), \quad (18)$$

where the leading term of expansion $\gamma_s^{(\parallel)}(0) = -\gamma_a^{(\parallel)}(0) = \gamma_0$ coincides with he radiation width γ_0 of exited state of the isolated atom. Similar expressions for width $\gamma_{s(a)}$ one can find when the orientations of dipole transitions in both atoms are perpendicular to the axis \vec{R} :

$$\gamma_s^{(\perp)}(R) = -\gamma_a^{(\perp)}(R) = \gamma_0 \left(1 - \frac{\omega_0^2 R^2}{5c^2} \right). \quad (19)$$

Therefore the complete widths of symmetrical and antisymmetrical states are of the form:

$$\Gamma_s^{(\parallel)} = \gamma_0 + \gamma_s^{(\parallel)} \approx 2\gamma_0, \quad \Gamma_a^{(\parallel)} = \gamma_0 + \gamma_a^{(\parallel)} \approx \frac{\gamma_0 \omega_0^2 R^2}{10 c^2}, \quad (20)$$

$$\Gamma_s^{(\perp)} = \gamma_0 + \gamma_s^{(\perp)} \approx 2\gamma_0, \quad \Gamma_a^{(\perp)} = \gamma_0 + \gamma_a^{(\perp)} \approx \frac{\gamma_0 \omega_0^2 R^2}{5 c^2}. \quad (21)$$

We can see that under the condition $R \ll \lambda_0$ the retarded interaction of atoms in the symmetrical state Ψ_s (15) leads to doubling of natural width of atomic levels. For antisymmetrical state the widths γ_a and γ_0 compensate each other almost completely, and the complete width Γ_a is small when compare with γ_0 .

It means that when $R \ll \lambda_0$ retarded interaction of atoms completely suppresses radiation decay of excited states of atoms. This property of mutual influence of closely located atoms was interpreted in [10] as near-field effect.

In paper [11] the near-field effect was proposed as a basis of writing the quantum information on separate two-level atoms of two-qubit quantum computer by means of intensive quairesonant radiation at a modification of an angle of incidence of external wave. Reading of the quantum information (after calculations) can be performed by means of weakly intensive probe radiation using solutions of equations system for dipole oscillators obtained in [10].

Quantum teleportation and resonant transfer of quantum information between two atom-qubits

Let us assume that at the initial point in time $t_1 = 0$ the system of two atoms under consideration is in the state $\Psi_m^{(0)} = \Psi_0(1)\Psi_0(2)$ (see (41)). Let us consider the quantum transition $\Psi_m^{(0)} \rightarrow \Psi_n^{(0)} = \Psi_{s(a)}$, where the wave functions $\Psi_{s(a)}$ are given by formulas (15). We define the transition energy for this case as

$$E_m^{(0)} - E_n^{(0)} = E_0 - E_n \mp \Delta E_s = -\hbar(\omega + \Delta), \tag{22}$$

where the upper sign corresponds to the symmetric state Ψ_s of the atomic pair, while the lower sign refers to its antisymmetric state Ψ_a , ω is the frequency of a real photon, Δ is the resonance off-tuning. Let us assume that the quantum transition $\Psi_m^{(0)} \rightarrow \Psi_{s(a)}$ corresponds to the event of annihilation of the photon. With that, we shall consider the transitions $\Psi_m^{(0)} \rightarrow \Psi_a$ and $\Psi_m^{(0)} \rightarrow \Psi_s$ separately from one another. For the symmetric channel $\Psi_m^{(0)} \rightarrow \Psi_s$ we have the following system of equations which determine the probability amplitudes a_m and a_n with the proviso that $\Gamma_s t_1 \rightarrow 0$:

$$\left. \begin{aligned} i\hbar \frac{da_m}{dt_1} &= F_{mn} \exp[i(\omega_{mn} + \omega)t_1] a_n = F_{mn} \exp[i(\varepsilon_+ + i\gamma_n)t_1] a_n, \\ i\hbar \frac{da_n}{dt_1} &= F_{nm} \exp[-i(\varepsilon_+ + i\gamma_n)t_1] a_m, \end{aligned} \right\} \tag{23}$$

where $\varepsilon_+ = (\delta E_s - \hbar\Delta)/\hbar$; $\gamma_n = \Gamma_n/2$, $\Gamma_n = \Gamma_s$, the matrix element of the transition is given by

$$F_{mn} = -\frac{i}{c} \sqrt{\frac{n_\omega}{2}} \omega_0 \vec{A}_0 \vec{d}_{n0}^{\vec{e}ff}(R) \exp(i\vec{k}\vec{R}), \tag{24}$$

\vec{A}_0 is the amplitude of the vector potential, \vec{k} is the wave vector of the real photon which is absorbed at the location of the second atom with the radius vector \vec{R} . The matrix element $\vec{d}_{n0}^{\vec{e}ff}(R)$ in (24) takes into account all terms of the function (41) of the initial state. The system of equations (23) corresponds to the two-level approximation for a system of atoms, such that the most significant contribution is from those terms of the Schrödinger wave equation in which the dependence on

time is determined by a low frequency ($\omega_0 - \omega$). A similar system of equations will also take place for the antisymmetric channel if in (23) we replace ε_+ with ε_- , defined as $\varepsilon_- = -(\delta E_s + \hbar\Delta)/\hbar$. After making the substitution $\tilde{a}_n(t_1) = a_n(t_1)e^{-\gamma_n t_1}$ the system (23) becomes as following:

$$\left. \begin{aligned} i\hbar \frac{da_m}{dt_1} &= F_{mn} \exp[i(\omega_{mn} + \omega)t_1] \tilde{a}_n = F_{mn} \exp(i\varepsilon_+ t_1) \tilde{a}_n, \\ i\hbar \frac{d\tilde{a}_n}{dt_1} &= -i\gamma_n \hbar \tilde{a}_n + F_{nm} \exp(-i\varepsilon_+ t_1) a_m. \end{aligned} \right\} \quad (25)$$

It is well known that the exchange by excitations between atoms through their resonance interaction is determined by the characteristic exchange time $\hbar/\delta E'_s$ [8]. If, at a certain point in time, a system consisting of two identical dipole atoms is in a state where one atom is excited, this excitation, as shown in [8], will be transferred to the other atom in a time $\tau' = \hbar/\delta E'_s$ through resonance interaction. With that, the time τ' required to transfer the excitation is considerably shorter than the lifetime $\Gamma_{s(a)}^{-1}$ of atoms in the symmetrical and (antisymmetrical) states, therefore, the energy $\delta E'_s$ (12) is independent of time. In this paper, we consider a situation where the time t_1 for which a system of two interacting atoms exists is also short compared to $\Gamma_{s(a)}^{-1}$. However, in contrast to [8], we take into account, besides the interaction of the atoms with one another, their interaction with the field of real photons.

Solving system of equations (23) at $a_m(0) = 1$, $a_n(0) = 0$ and following [1], we obtain a wave function which corresponds to a symmetric channel for the interaction of a pair of atoms with the field of real photons:

$$\begin{aligned} \tilde{\Psi}_s(t_1) &= \frac{1}{\sqrt{2}} \exp\left(-\frac{\Gamma_s}{4} t_1 + i\frac{\varepsilon_+}{2} t_1\right) \left[\cos((\Omega_+ + i\beta_+)t_1) - \right. \\ &\quad \left. - \frac{i(\varepsilon_+ + i\Gamma_s/2)}{2(\Omega_+ + i\beta_+)} \sin((\Omega_+ + i\beta_+)t_1) \right] \tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle - \\ &\quad - \frac{iF_{nm}}{\hbar(\Omega_+ + i\beta_+)} \exp\left(-\frac{\Gamma_s}{4} t_1 - i\frac{\varepsilon_+}{2} t_1\right) \sin((\Omega_+ + i\beta_+)t_1) \times \\ &\quad \times \frac{1}{\sqrt{2}} [\tilde{\varphi}_n(1)\tilde{\varphi}_0(2) + \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)] \exp(-i\delta E_s t_1/\hbar) |n_\omega - 1\rangle, \end{aligned} \quad (26)$$

where

$$\Omega_+ + i\beta_+ = \sqrt{\frac{|F_{nm}|^2}{\hbar^2} + \frac{(\varepsilon_+ + i\gamma_n)^2}{4}}. \quad (27)$$

For an antisymmetric interaction channel we replace ε_+ , β_+ , Γ_s , Ω_+ with ε_- , β_- , Γ_a , Ω_- in (26). Here we have

$$\varepsilon_\pm = (\pm\delta E_s - \hbar\Delta)/\hbar, \quad \Omega_\pm + i\beta_\pm = \sqrt{\frac{|F_{nm}|^2}{\hbar^2} + \frac{(\varepsilon_\pm + i\Gamma_{s(a)}/2)^2}{4}}. \quad (28)$$

Therefore, the wave function of the final state of the compound-system "A(1) + A(2) + F" being a superposition of $\tilde{\Psi}_s(t_1)$ and $\tilde{\Psi}_a(t_1)$ is of the form

$$\tilde{\Psi}(t_1) = A_1 \tilde{\varphi}_0(1)\tilde{\varphi}_0(2)|n_\omega\rangle + A_2 \tilde{\varphi}_n(1)\tilde{\varphi}_0(2)|n_\omega - 1\rangle + A_3 \tilde{\varphi}_0(1)\tilde{\varphi}_n(2)|n_\omega - 1\rangle, \quad (29)$$

where the probability amplitudes for possible states of the system are given by

$$A_1 = \frac{1}{2} \left\{ \exp\left(-\frac{\Gamma_s}{4}t_1 + i\frac{\varepsilon_+}{2}t_1\right) \left[\cos((\Omega_+ + i\beta_+)t_1) - \frac{i(\varepsilon_+ + i\Gamma_s/2)}{2(\Omega_+ + i\beta_+)} \sin((\Omega_+ + i\beta_+)t_1) \right] + \exp\left(-\frac{\Gamma_a}{4}t_1 + i\frac{\varepsilon_-}{2}t_1\right) \left[\cos((\Omega_- + i\beta_-)t_1) - \frac{i(\varepsilon_- + i\Gamma_a/2)}{2(\Omega_- + i\beta_-)} \sin((\Omega_- + i\beta_-)t_1) \right] \right\}, \quad (30)$$

$$A_2 = -\frac{iF_{nm}}{2\hbar} \left[\frac{1}{\Omega_+ + i\beta_+} \exp\left(-\frac{\Gamma_s}{4}t_1 - i\frac{\varepsilon_+}{2}t_1\right) \times \exp(-i\delta E_s t_1/\hbar) \sin((\Omega_+ + i\beta_+)t_1) + \frac{1}{\Omega_- + i\beta_-} \exp\left(-\frac{\Gamma_a}{4}t_1 - i\frac{\varepsilon_-}{2}t_1\right) \sin((\Omega_- + i\beta_-)t_1) \exp(i\delta E_s t_1/\hbar) \right], \quad (31)$$

$$A_3 = \frac{iF_{nm}}{2\hbar} \left[-\frac{1}{\Omega_+ + i\beta_+} \exp\left(-\frac{\Gamma_s}{4}t_1 - i\frac{\varepsilon_+}{2}t_1\right) \times \sin((\Omega_+ + i\beta_+)t_1) \exp(-i\delta E_s t_1/\hbar) + \frac{1}{\Omega_- + i\beta_-} \exp\left(-\frac{\Gamma_a}{4}t_1 - i\frac{\varepsilon_-}{2}t_1\right) \sin((\Omega_- + i\beta_-)t_1) \exp(i\delta E_s t_1/\hbar) \right]. \quad (32)$$

The probability amplitudes (30)-(32) are normalized by the condition $|A_1|^2 + |A_2|^2 + |A_3|^2 = 1$. As follows from (30)-(32), at $t_1 = 0$ a system of two atoms is in a state where both atoms have an energy E_0 , i.e., $A_1 = 1$, $A_2 = A_3 = 0$. Let us consider the behavior of the function $\tilde{\Psi}(t_1)$ at subsequent points in time.

A. True resonance

A1. Large interatomic distances

Consider the case of the true resonance when $\Delta = 0$, i.e. the frequency of real photon ω coincides with the transfer frequency $(E_m^{(0)} - E_n^{(0)})/\hbar$. At $R \gg \lambda_0$ the contribution of $\gamma_{s(a)}$ caused by the retarded dipole-dipole interaction of atoms in $\Gamma_{s(a)}$ can be neglected ($\Gamma_s = \Gamma_a = \gamma_0$), and the quantities ε_{\pm} , Ω_{\pm} , β_{\pm} take the form $\varepsilon_+ = -\varepsilon_- = \delta E_s/\hbar$,

$$\Omega_{\pm} = \Omega = \frac{1}{\sqrt{2}} \left\{ \sqrt{\left[\frac{|F_{nm}|^2}{\hbar^2} + \frac{(\delta E_s)^2 - \hbar^2 \gamma_0^2/4}{4\hbar^2} \right]^2 + \frac{\gamma_0^2 (\delta E_s)^2}{16\hbar^2}} + \frac{|F_{nm}|^2}{\hbar^2} + \frac{(\delta E_s)^2 - \hbar^2 \gamma_0^2/4}{4\hbar^2} \right\}^{1/2}, \quad (33)$$

$$\beta_{\pm} = \beta = \frac{1}{\sqrt{2}} \left\{ \sqrt{\left[\frac{|F_{nm}|^2}{\hbar^2} + \frac{(\delta E_s)^2 - \hbar^2 \gamma_0^2/4}{4\hbar^2} \right]^2 + \frac{\gamma_0^2 (\delta E_s)^2}{16\hbar^2}} - \frac{|F_{nm}|^2}{\hbar^2} - \frac{(\delta E_s)^2 - \hbar^2 \gamma_0^2/4}{4\hbar^2} \right\}^{1/2}. \quad (34)$$

In true resonance ($\Delta = 0$) when $R \gg \lambda_0$ the amplitudes are equal to

$$A_1 = \exp\left(-\frac{\gamma_0}{4}t_1\right) \left\{ \cos((\Omega + i\beta)t_1) \cos\left(\frac{\delta E_s t_1}{2\hbar}\right) + \frac{\delta E_s \sin((\Omega + i\beta)t_1)}{2\hbar(\Omega + i\beta)} \sin\left(\frac{\delta E_s t_1}{2\hbar}\right) + \frac{\gamma_0 \sin((\Omega + i\beta)t_1)}{4(\Omega + i\beta)} \cos\left(\frac{\delta E_s t_1}{2\hbar}\right) \right\}, \quad (35)$$

$$A_2 = -\frac{iF_{nm} \sin((\Omega + i\beta)t_1)}{\hbar(\Omega + i\beta)} \exp\left(-\frac{\gamma_0}{4}t_1\right) \cos\left(\frac{3\delta E_s t_1}{2\hbar}\right), \quad (36)$$

$$A_3 = -\frac{F_{nm} \sin((\Omega + i\beta)t_1)}{\hbar(\Omega + i\beta)} \exp\left(-\frac{\gamma_0}{4}t_1\right) \sin\left(3\frac{\delta E_s t_1}{2\hbar}\right). \quad (37)$$

From (37) one can see the probability amplitude A_2 (A_3) reaches a maximum (minimum) at a time given by

$$t'_1 = 2\pi\hbar/3\delta E_s. \quad (38)$$

After absorption of real photon by $A(2)$ atom the probability of remaining the excitation near this atom is equal to $|A_3|^2$, and the probability of excitation transfer to $A(1)$ atom is equal $|A_2|^2$. If it does not matter in which of the atoms the excitation remains then the probability of photon trapping by the system of two resonance atoms is determined as follows

$$|A_2|^2 + |A_3|^2 = \exp\left(-\frac{\gamma_0}{2}t_1\right) \frac{|F_{nm}|^2}{2\hbar^2(\Omega^2 + \beta^2)} (\operatorname{ch} 2\beta t_1 - \cos 2\Omega t_1).$$

A2. Small interatomic distances

For the case of true resonance ($\Delta = 0$) and small distances $\omega_0 R/c \ll 1$ the antisymmetric state is found to be stable as to radiation damping, and the system decay goes by symmetric channel with $\Gamma_s = 2\gamma_0$. When $t_1 \ll \Gamma_s^{-1}$ one can put $\Gamma_{s(a)} = \beta_{\pm} = 0$, $\varepsilon_+ = -\varepsilon_- = \delta E_s/\hbar$, $\Omega_{\pm} = \Omega_0 = \sqrt{[|F_{nm}|^2 + (\delta E_s)^2]}/\hbar$ for a time which is short compared to $\Gamma_{s(a)}^{-1}$ and obtain the following formulas for the probability amplitudes:

$$A_1 = \cos(\Omega_0 t_1) \cos\left(\frac{\delta E_s t_1}{2\hbar}\right) + \frac{\delta E_s \sin(\Omega_0 t_1)}{2\hbar \Omega_0} \sin\left(\frac{\delta E_s t_1}{2\hbar}\right), \quad (39)$$

$$A_2 = -\frac{iF_{nm} \sin(\Omega_0 t_1)}{\hbar \Omega_0} \cos\left(\frac{\delta E_s t_1}{2\hbar}\right), \quad A_3 = -\frac{F_{nm} \sin(\Omega_0 t_1)}{\hbar \Omega_0} \sin\left(\frac{\delta E_s t_1}{2\hbar}\right). \quad (40)$$

The amplitude A_2 reaches a maximum at a time given by $t'_1 = 2\pi\hbar/3\delta E_s$. With that, the amplitude A_3 is a minimum.

B. Nonresonance absorption of a photon

B1. Large interatomic distances

Let us consider the case where $\Delta \neq 0$, i.e., the frequency of the real photon does not coincide with the frequency of the transition $(E_m^{(0)} - E_n^{(0)})/\hbar$. Moreover, we

assume that the conditions $\Delta \gg \delta E_s/\hbar$ and $R \gg \lambda_0$ are fulfilled. Then

$$\Omega_{\pm} = \Omega = \frac{1}{\sqrt{2}} \left[\sqrt{\left(\frac{|F_{nm}|^2}{\hbar^2} + \frac{\Delta^2 - \gamma_0^2/4}{4} \right)^2 + \frac{\gamma_0^2 \Delta^2}{16}} + \frac{|F_{nm}|^2}{\hbar^2} + \frac{\Delta^2 - \gamma_0/4}{4} \right]^{1/2}$$

$$\beta_{\pm} = \beta = \frac{1}{\sqrt{2}} \left[\sqrt{\left(\frac{|F_{nm}|^2}{\hbar^2} + \frac{\Delta^2 - \gamma_0^2/4}{4} \right)^2 + \frac{\gamma_0^2 \Delta^2}{16}} - \frac{|F_{nm}|^2}{\hbar^2} - \frac{\Delta^2 - \gamma_0/4}{4} \right]^{1/2},$$

and the probability amplitudes (30)-(32) are equal to

$$A_1 = \exp\left(-\frac{\gamma_0}{4}t_1 - i\frac{\Delta}{2}t_1\right) \left[\cos((\Omega + i\beta)t_1) + \frac{i\Delta + \gamma_0/2}{2(\Omega + i\beta)} \sin((\Omega + i\beta)t_1) \right], \quad (41)$$

$$A_2 = -\frac{iF_{nm}}{\hbar(\Omega + i\beta)} \sin((\Omega + i\beta)t_1) \exp\left[-\frac{\gamma_0}{4}t_1 + i\frac{\Delta}{2}t_1\right], \quad A_3 = 0. \quad (42)$$

It is seen that at nonresonance photon absorption only two of the three states of compound-system are realized. The state with amplitude A_3 is realized as a result of the absorption of a photon at the position of $A(2)$ atom (polarizer atom) and electric dipole transition of $A(1)$ atom (observer atom) into the excited one. The amplitude A_2 of this state reaches a maximum at a time given by $t_1'' = \pi/2\Omega$.

B2. Small interatomic distances

In the case when $\Delta \gg \delta E_s/\hbar$ and $R \ll \lambda_0$ one can neglect the damping of states ($\Gamma_s = \Gamma_a = \beta_{\pm} = 0$) and put $\Omega_{\pm} = \Omega_0 = \sqrt{(\Delta^2/4) + |F_{nm}|^2/\hbar}$. Then the amplitudes (30)-(32) become

$$A_1 = \exp\left(-i\frac{\Delta}{2}t_1\right) \left[\cos(\Omega_0 t_1) + \frac{i\Delta}{2\Omega_0} \sin(\Omega_0 t_1) \right], \quad (43)$$

$$A_2 = -\frac{iF_{nm}}{\hbar\Omega_0} \sin(\Omega_0 t_1) \exp\left(i\frac{\Delta}{2}t_1\right), \quad A_3 = 0. \quad (44)$$

Similarly to the case of large interatomic distances, here two of the three possible states are realized. The state with the amplitude A_2 is realized as a result of the absorption of a photon at the position of the second atom (polarizer atom) and the transition of the first atom (observer atom) into the excited state. The amplitude A_2 of this state reaches a maximum at a time given by $t_1'' = \pi/2\Omega$.

C. Resonance absorption of a photon. Large interatomic distances

Let us consider $\Delta = 0$, $\delta E_s = 0$, and $\gamma_s = \gamma_a = 0$. Then $\Gamma_s = \Gamma_a = \gamma_0$, $\beta_+ = \beta_- = 0$, and the amplitudes (30)-(32) become

$$A_1 = \exp\left(-\frac{\gamma_0}{4}t_1\right) \left[\cos(\Omega t_1) + \frac{\gamma_0}{4\Omega} \sin(\Omega t_1) \right], \quad (45)$$

$$A_2 = -\frac{iF_{nm}}{\hbar\Omega} \sin(\Omega t_1) \exp\left(-\frac{\gamma_0}{4}t_1\right), \quad A_3 = 0. \quad (46)$$

At the initial point in time $t_1 = 0$, as follows from (45), (46), both atoms are in the ground state. However, even at an infinitely close subsequent point in time, the state $\tilde{\Phi}_n(1)\tilde{\Phi}_0(2)$ from the superposition (29) starts being realized. At the time $t_1''' = \pi/2\Omega$ the probabilities become $|A_2|^2 = 1$ and $|A_1|^2 = 0$, i.e., one quantum bit of information is transferred from one atom to another at an infinitely large interatomic distance in the time t_1''' . We refer to this effect as quantum teleportation in a system of two resonance atoms. This effect is quantum in nature, being related to the superposition principle of quantum mechanics, and corresponds to the quantum means of information transfer whose characteristic time is defined as $t_1''' = \pi/2\Omega$, in contrast to the classical means of information transfer with the characteristic time t_p being the time of flight of a photon through a distance R .

If to neglect the states damping ($\gamma_0 = 0$) one can use the following simpler expression for time of information transfer $t_1''' = \pi\hbar/2|F_{nm}|$. In accordance with the sense of this approximation, the condition $t_1''' \ll \Gamma_{s(a)}^{-1}$ should be fulfilled. This condition is fulfilled the more precisely, the greater the amplitude of the vector potential in (24). Using the notion of the Rabi frequency [12], we obtain $t_1''' = \hbar/|\vec{d}_{n0}|\mathcal{E}_0$, where \mathcal{E}_0 is the amplitude of the electric field of the light wave acting on the $A(2)$ atom.

Thus, say we have been able to create entangled states $\Psi_{s(a)}$ (42), (43) for two widely spaced atoms. Bouweester et al. [13] could do this using one photon source which distributes in a random manner photons randomly polarized in two different directions. Krein and Zeilinger [14] used for this purpose an electron source. In our case, one possible way of creating entangled states may be the use of two beams of double-level atoms irradiated in a random manner with a light field.

In accordance with the sense of the obtained solution (45), (46) for a system of two resonance atoms in the field of actual photons, we represent the operation of the information system as follows: let us assume that the system operation begins at a time $t_2 = R/c$ at the location of the $A(2)$ polarizer atom when this atom is irradiated with the field of a light wave with a frequency $\omega = \omega_0$. At a local point in time, $t_1 = 0$, both atoms are in the ground state with an energy E_0 , and at a point in time t_1''' the $A(1)$ observer atom goes into an excited state with an energy $E_n = E_0 + \hbar\omega$. As this takes place, the $A(2)$ atom remains in the former state with the energy E_0 , and the average energy of the system, calculated with the help of the wave function (29) with the amplitudes (45), (46), will be determined by the formula

$$\langle E \rangle = |A_1|^2(2E_0 + \hbar\omega_0) + |A_2|^2(E_0 + E_n + \hbar\omega(n_\omega - 1)). \quad (47)$$

For an interatomic distance $R = 30$ km, the time it takes for a photon to cover this distance with the classical means of information transfer will be $t_p = 10^{-4}$ sec. The time of information transfer due to the effect of quantum teleportation is given by $t_1''' = \hbar/|\vec{d}_{n0}|\mathcal{E}_0$. For $d_{n0} = 4.8 \times 10^{-21}$ units of the cgs electrostatic system and $\mathcal{E}_0 = 10^{-2}$ units of the cgs electrostatic system, we obtain $t_1''' = 2,1 \times 10^{-5}$ sec. Thus, the velocity of the information transfer due to the effect of quantum teleportation, $v_{QT} = R/t_1''' = 1,4 \times 10^6$ km/sec, is substantially higher

than the velocity of light. Nevertheless, this does not imply any contradiction with the relativity principle, since $v_{QT} = R/t_1'''$ is a phase velocity.

References

- [1] K.A. Valiev, *Usp. Fiz. Nauk* **175**, 3 (2005).
- [2] D.J. Wineland, *Usp. Fiz. Nauk* **184**, 1089 (2014).
- [3] O.N. Gadosky and K.K. Altunin, *Russian Physics Journal* **43**, 893 (2000).
- [4] O.N. Gadosky and Yu.Yu. Voronov, *Zh. Eksp. Teor. Fiz.* **121**, 1028 (2002).
- [5] O.N. Gadosky and Yu.Ya. Kharitonov, *Kvantovaya Elektronika* **34**, 249 (2004).
- [6] V.Yu. Lazur, S.I. Myhalyna, and O.K. Reity, *Phys. Rev. A* **81**, 062707 (2010).
- [7] V.Yu. Lazur, O.F. Pavlyk, and O.K. Reity, *Teor. Mat. Fiz.* **165**, 70 (2010).
- [8] A.S. Davydov, *Quantum Mechanics* [in Russian], (Fizmatgiz, Moscow, 1963).
- [9] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Nonrelativistic Theory* [in Russian] (Course of Theoretical Physics, Vol. 3), (Nauka, Moscow, 1989); English transl. of prev. ed. (Oxford Univ. Press, Oxford, 1975).
- [10] O.N. Gadosky, *Usp. Fiz. Nauk* **170**, 1145 (2000).
- [11] O.N. Gadosky and Yu.Yu. Voronov, *Pisma Zh. Eksp. Teor. Fiz.* **69**, 750 (1999).
- [12] L. Allen, J.H. Eberly, *Optical Resonance and Two-level Atoms* (John Wiley & Sons, Inc., New York, 1975).
- [13] D. Bouweester *et al.*, *Nature* **390**, 575 (1997).
- [14] G. Krein and A. Zeilinger, *Phys. Rev. A* **54**, 1793 (1996).