IMPROVED DEFORMATION MODEL OF THE REINFORCED CONCRETE BAR STRUCTURE FOR THE GENERAL CASE OF STRESSED STATE

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The presented calculation model enables to estimate a change in the stress-strain behaviour of a reinforced concrete bar during a simple proportional loading at all stages of its performance, including destruction, with due account of the actual properties of materials.

Представлена розрахункова модель дозволяє оцінювати зміну напружено-деформованого стану залізобетонного стержня в процесі простого пропорційного навантаження на всіх стадіях його роботи, включаючи руйнування, з урахуванням реальних властивостей матеріалів.

Keywords: *stress, strain, bending, tcip, tension, torsion*

Introduction. Rational design of building structures encounters the well-known obstacles as concrete is a composite, non-elastic, heterogeneous and anisotropic material susceptible both to crack formation and brittle fracture as well as to demonstration of plastic behaviour, creeping, shrinkage and swelling.

Resistance of reinforced concrete elements to external loads under complex stress-strain condition which is characterized by origination of transverse and axial forces, bending and twisting moments has been understudied. Consequently, a semi-empirical approach to their calculation is practiced.

Therefore, the research in this line is important and relevant. It is closely related to the research mix of the Academy, is systemic by its nature and makes an integral component of the state-funded topic No.0108U000559 of the Ministry of Education, Science, Youth and Sports of Ukraine.

Analysis of prior research. Pioneer research of physical non-linearity of concrete and reinforced concrete was conducted by A.F. Loleyta [1] and V.I. Murashev [2]. Their works have created prerequisites for the development of engineering methods to calculate the reinforced concrete structures subjected to bending. However, these theories describe the nature of stress distribution across the height of element section just at certain stages of their behaviour and do not allow of tracing the actual stress-strain condition until the boundary condition is reached, as a rule of normal sections.

As it turned out, in order to study the process of strain of the considered complex stressed reinforced concrete elements, it is necessary to apply a theory of plasticity and methods used in mechanics to describe deformation and destruction of a solid body.

First studies of plasticity of materials subjected to stress-strain condition were conducted by L. Prandtl, Ye. Reiss, O.A. Ilyushin et al. However, quite soon it was found that said classic theories used for describing concrete plasticity are inadequate as concrete shows different strength at compression and tension and can crack causing strain-induced anisotropy and dilatation under three-axial compression.

G.Geniyev, V.M.Kissyuk and G. A. Tyupin [3] were the first who proposed to take into account all above mentioned peculiarities of concrete strain and to consider concrete as a non-linear elastic isotropic material, and reinforced concrete – as a transversely isotropic material both prior to and after cracks have been formed.

The works by A.I. Kozachevskyi, V.M. Kruglov [4], S.F. Klovanych [5] and V.I.Korsun [6] substantially developed the plasticity theory for concrete and reinforced concrete that had been proposed in [3].

In their studies [7, 8, 9] M.I. Karpenko and his disciples developed a theory of small elastic-plastic deformations which considers concrete, both before and after the cracks appear, as an anisotropic material with discrete arrangement of reinforcement.

Modern concepts of the theory of strength of concrete under three-axial stress condition were laid by M.M. Filonenko- Borodych [10], G.O. Geniyev, V.M. Kissyuk, G.A. Tyupin [3], G.S. Pysarenko, A.A. Lebedev [11], T.A. Balan, S.F. Klovanych [12], M.I. Karpenko [7] and his disciples - Dei Poli [13], K.H. Gerstle [14], H.B. Kupfer [15] and others.

Emergence of modern high-performance computers with large memory has made it possible to engage numerical methods for solving problems with the aid of complex computation models. Under such circumstances the main point is to select an effective numerical method.

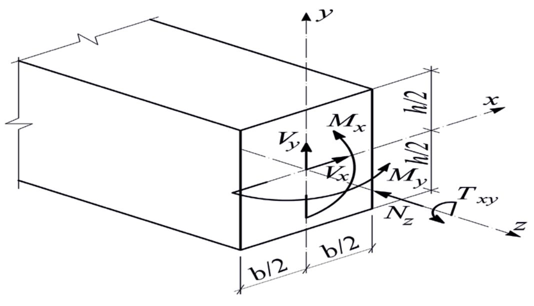

Figure 1. Diagram of internal forces in a bar cross-section for the general case of its stress-strain behaviour

Formulation of the problem and basic premises. We consider a rectangular cross-section reinforced concrete bar (Fig. 1) which has a fixed rigidity along its length and is characterized by a general case of stress condition in the sections to be calculated.

The bar is made of a heavy concrete which was hardened under normal natural environment. Its reinforcement was made conventional as a system of orthogonally oriented bars of the working and handling reinforcements arranged along z-axis and transverse vertical (along y-axis) and horizontal (along x-axis) axes. The load applied to the bar is simple and proportional.

The task of this research stage is to determine the bearing capacity of the reinforced concrete bar with due account of its central compression (tension), skew bending with free or constrained torsion, influence of structural factors and the ambient effects of non-linear properties of concrete and reinforcement. Main symbols, indices and legend are used in this model in conformity with the recommendations of the existing standards [16, 17].

Basic prerequisites:

- the reinforced concrete bar is stiff;
- the links between stresses and relative strain in concrete and reinforcement are established with the aid of complete compression/ tension and shear diagrams;
- the calculated sections are normal to the longitudinal axis;
- distribution of the general linear relative strains along the calculated section height meets the plane section hypothesis when the concrete is subjected to compression (tension) and bending;
- tangential stresses in the calculated section of the element that arise at free bending are determined in accordance with the recommendations [18, 19];
- tangential and normal stresses in the calculated sections of the bar under constrained torsion condition are determined with due account of the solution proposed by M.I. Bezukhov [20] and the recommendations by Yu.O. Shkola [21];
- concrete and bars of the axial reinforcement are subjected to normal σx, σy, σz and tangential τzx, τzy, τxy stresses;
- the bars of the transverse reinforcement are subjected to tangential stresses τzx and τzy only. Their distribution along the bar lengths is assumed non-uniform;
- phenomenological condition of strength proposed by V.M. Kruglov [4] or M.I. Karpenko [7] and his disciples may be accepted as a concrete destruction criterion (macrocrack formation);
- prior to macrocrack origination, the condition of compatibility of strains in concrete and reinforcement is considered correct. After the cracks appear, concrete ceases to work and all forces in the cracked section are taken by reinforcement only;
- bar rods cease to work when yield begins. As a criterion, the Huber-Mises-Hencky flow rule [20, 22] is accepted;
- when transiting from stresses to the generalized internal forces, a procedure for numerical integration of elementary internal force factors across the entire area of the calculated section is applied. In doing so, the calculated section of the bar element is conventionally subdivided into individual tiny elements – parts – wherein the stresses are considered equal.

In accordance with [3, 7, 23], concrete strength in the main stress coordinate system $\sigma_1\sigma_2\sigma_3$ is described by a surface, which is continuous, convex, symmetrical as to the octahedral normal stress σ_0 and equally inclined to said coordinate axes, that is constructed in accordance with the method developed by M.M. Filonenko-Borodych with the use of the equation:

$$f\left(\sigma_{oc},\tau_{oc},\theta_{c},\right)=\tau_{oc}-\tau_{o1c}\left(\sigma_{oc}\right)\cdot\rho\left(\theta_{c}\right)=0 \qquad (1)$$

where σ_{oc} τ_{oc} — octahedral normal and tangential stresses;
Θ — a stress state kind;
$\rho\ \Theta_c$ — interpolation function between $\tau_{o1}\left(\theta_c=60°\right)$ and $\tau_{o2}\left(\theta_c=0°\right)$ [23]:

$$\rho\left(\theta_c\right)=\left[2a_c\cos\theta_c+b_c\sqrt{a_c\left(4\cos^2\theta_c-1\right)+b_c^2}\right]\Big/\left(4a_c\cos^2\theta_c+b_c^2\right) \qquad (2)$$

where $a_c=1-c_c^2,\ b_c=2c_c-1,\ c_c=\tau_{o2c}/\tau_{o1c}$.

The relation between octahedral stresses at a stress state pattern angles will be $\Theta_c=60°$ and $\Theta_c=0°$ can be represented in accordance with [23] as:

$$\sigma_{oc}=A_1\tau_{o1c}^2+B_1\tau_{o1c}+C_1,\ \ \sigma_{oc}=A_2\tau_{o2c}^2+B_2\tau_{o2c}+C_1 \qquad (3)$$

Coefficients $A_1 A_2 B_1 B_1 C_1$ were obtained by referencing to the characteristic points on the surface of concrete strength. Using the dependences experimentally obtained by V.M. Bondarenko and V.I. Kolchunov [24] and taking [16, 23] into account, it is proposed to determine the coefficients by the following simplified formulae:

$$A_1=4{,}14/\left(f_{ck}-f_{ctk}\right);$$
$$B_1=\left(5{,}38f_{ck}^2+f_{ck}f_{ctk}-6{,}38f_{ck}^2\right)\Big/\left[4{,}24\left(f_{ck}-f_{ctk}\right)^2\right];$$
$$A_2=\left(4{,}09f_{ck}-4{,}16f_{ctk}\right)\Big/\left(1{,}20f_{ck}^2-2{,}20f_{ck}f_{ctk}+f_{ctk}^2\right);$$
$$B_2=\left(4{,}46f_{ck}^2-2{,}04f_{ck}f_{ctk}-0{,}73f_{ctk}^2\right)\Big/\left(4{,}32f_{ck}^2-7{,}92f_{ck}f_{ctk}+3{,}60f_{ctk}^2\right);$$
$$C_1=-H=-\left(0{,}82f_{ck}f_{ctk}\right)\Big/\left(f_{ck}-f_{ctk}\right), \qquad (4)$$

where f_{ck}, f_{ctk} is a characteristic strength of concrete (or a calculated strength f_{cd}, f_{ctd} at design) at compression and tension, accordingly. With the aid of formula (1) it is possible to uniquely describe the surface of concrete strength because it includes, due to coefficients (4), five independent strength parameters of concrete that correspond to the individual cases of the stress pattern:

at uniaxial compression f_{ck} and tension f_{ctk},
at biaxial compression $1{,}2f_{ck}$ and tension f_{ctk},
as well as at a triaxial uniform tension H

The angle of the studied stress pattern of the bar in concrete can be determined by means of [20, 23] with due account of $\sigma_x=\sigma_y=0$

$$\theta_c=\frac{1}{3}\arccos\left(\frac{3\sqrt{3D_3}}{2\sqrt{D_2^3}}\right)=\frac{1}{3}\arccos\left[\frac{\sqrt{\sigma_{zc}\left[2\sigma_{zc}^2+9\left(\tau_{xyc}^2+\tau_{zyc}^2+\tau_{zxc}^2\right)\right]}}{2\sqrt{\left(\sigma_{zc}^2/3+\tau_{xyc}^2+\tau_{zyc}^2+\tau_{zxc}^2\right)^3}}\right] \qquad (5)$$

where $D_2 D_3$ - mean the second and third invariants of the stress deviator.

Taking (1) and (3) into consideration,

$$\sigma_{oc} = \frac{A_1}{\rho^2(\theta_c)}\tau_{oc}^2 + \frac{B_1}{\rho(\theta_c)}\tau_{oc} + C_1 \quad (6)$$

The boundary values of concrete strength (on the strength "surface) expressed through $\widehat{\sigma}_{oc}$ and $\widehat{\tau}_{oc}$ are determined by solving a system of equations:

$$\begin{cases} \widehat{\tau}_{oc} - \tau_m = m_\sigma\left(\widehat{\sigma}_{oc} - \sigma_m\right); \\ \widehat{\sigma}_{oc} = \frac{A_1}{\rho^2(\theta_c)}\widehat{\tau}_{oc}^2 + \frac{B_1}{\rho(\theta_c)}\widehat{\tau}_{oc} + C_1, \end{cases} \quad (7)$$

Where σ_m and τ_m mean the stresses at the previous loading level (at simple proportional loading $\sigma_m = \tau_m = 0$);

m_σ is a coefficient that characterizes the stress-strain condition of concrete. For instance, at the uniform triaxial tension $m_\sigma = 0$, at biaxial tension - $m_\sigma = \pm\sqrt{2}/2$ and at uniaxial tension/compression $m_\sigma = \pm\sqrt{2}$ (the sign «+» corresponds to tension strain, and the sign «-» - to compression).

The Huber-Mises-Hencky flow rule for the reinforcement steel [20, 22] equals, at $\sigma_x = \sigma_y = 0$:

$$\sigma_{zs}^2 + 3\tau_{xys}^2 + 3\tau_{zxs}^2 + 3\tau_{zys}^2 = \tilde{f}_{yd}^2, \quad (8)$$

where $\tilde{f}_{yd}^2$ is a calculated strength of reinforcement at the yield limit with due account of its reduction because of the complex stress condition as compared with the central tension/compression.

In the general case of the complex stress strain behaviour this criterion is of the form:

$$\sigma_{xs}^2 + \sigma_{ys}^2 + \sigma_{zs}^2 - \sigma_{xs}\sigma_{ys} - \sigma_{ys}\sigma_{zs} - \sigma_{zs}\sigma_{xs} + 3\tau_{xys}^2 + 3\tau_{zys}^2 + 3\tau_{zxs}^2 = \tilde{f}_{yd}^2. \quad (9)$$

To construct the shear diagram, [21] makes use of the hypothesis advanced in the elastic-plastic deformation theory which tells that the stress intensity is linked to the deformation intensity and is described by the same dependence for all stress patterns. For the uniaxial tension case M.M. Malinin suggested [25] to describe the intensities of stresses and strains as:

$$\sigma_i = \sigma; \qquad \varepsilon_i = \varepsilon\left(1 - 2\nu\right)/3E, \quad (10)$$

where σ are normal stresses;

ε - relative axial strains.

At pure shear the intensity of stresses and strains can be found out according to the formulae:

$$\sigma_i = \sqrt{3}\tau; \qquad \varepsilon_i = \gamma / \sqrt{3}, \quad (11)$$

where τ are tangential stresses and γ – angle strains.

Making use of the above hypothesis and based on expressions (10) and (11), M.M. Malinin [25] established the expressions:

$$\sigma_i = \frac{\sigma}{\sqrt{3}}; \qquad \gamma = \sqrt{3}\left[\varepsilon - \frac{(1-2\nu)\sigma}{3E}\right]. \quad (12)$$

Thus, the diagram illustrating shear of a material can be obtained from the diagram illustrating its axial tension. Hence, the modulus of elasticity of a material at shear equals:

$$G = \frac{\tau}{\gamma} = \frac{\sigma}{3}\left(\varepsilon - \frac{1-2\nu}{3E}\sigma\right)^{-1}. \quad (13)$$

In accordance with M.I. Karpenko [7] recommendations the diagram of concrete deformation at compression (tension) with due account of [16] can be represented as follows:

$$\varepsilon_b = \frac{\sigma_b}{E_b^0 \nu_b} = \frac{\sigma_c}{E_{cm}\zeta_c} = \varepsilon_c, \quad (14)$$

where $\varepsilon_b = \varepsilon_c$ means the relative linear strains of concrete;

$\sigma_b = \sigma_c$ – normal stresses in concrete;

$E_b 0 = E_{cm}$ – the initial modulus of elasticity of concrete;

$\nu_b = \zeta_c$ – the coefficient accounting for a change of the secant modulus of elasticity of concrete.

Strain dependencies for the concrete subjected to a complex stress-strain condition should also be formulated as a link between octahedral stresses and deformations [23]. At this, the following hypotheses [7] are assumed to be fair:

- a link between octahedral stresses τ_{oc} and shears in octahedral areas γ_{oc} is non-linear: $\tau_{oc} = G_c(\gamma_{oc}) \cdot \gamma_{oc}$, where $G_c(\gamma_{oc})$ is a secant (octahedral) shear modulus of concrete;
- a link between octahedral normal stresses σ_{oc} and medium deformations ε_{oc} is also non-linear and looks like $\sigma_{oc} = K(\gamma_{oc}) \cdot (\varepsilon_{oc} - \rho_c \gamma_{oc}^2)$, where ρ_c is a dilatation modulus (according to G.O. Geniyev [3] - g_{oc}); $K(\gamma_{oc})$ is a modulus of volume elasticity.

To determine the secant moduli by analogy with the hypothesis [3, 7] about the "unified curve of strain", it is expedient to use the hypothesis [23] stating that the form of the link between stresses and strains does not depend on the kind of stress state, i.e. the link between τ_{oc} and γ_{oc} can be assumed the same as at axial compression, and the secant modulus of shear can be adopted (Fig. 2) based on diagram which is used in Eurocode and was suggested by Sayence - $G_c(\gamma_{oc}) = G_{oc} \cdot f(\gamma_o)$, where:

$$f(\gamma_{oc}) = \frac{1}{1 + A\eta + B\eta^2 + C\eta^3} \quad (15)$$

where $C=\lambda\left(1-\xi_r\right)/\left[\xi_r\left(\eta_r-1\right)^2-1/\eta_r\right]$; $B=1-2C$; $A=C+\lambda-2$; $\xi_r=\bar{\sigma}_r/f_{ck}\approx 0{,}85$ and

$\eta_r=\gamma/\bar{\gamma}_r\approx 1{,}41$; $\xi=\sigma_{oc}/f_{ck}$; $\eta=\gamma_{oc}/\bar{\gamma}_{oc}$; $\lambda=\xi/\eta$;

the initial shear modulus:

$$G_{oc}=G_{cm}=E_{cm}/\left[2\left(1+v_c\right)\right];\; \sigma_{oc}=\left(\sigma_{xc}+\sigma_{yc}+\sigma_{zc}\right)/3;\; \varepsilon_{oc}=\left(\varepsilon_{xc}+\varepsilon_{yc}+\varepsilon_{zc}\right)/3;$$

$$\tau_{oc}=1/3\sqrt{\left(\sigma_{xc}-\sigma_{yc}\right)^2+\left(\sigma_{zc}-\sigma_{yc}\right)^2+\left(\sigma_{zc}-\sigma_{xc}\right)^2+6\left(\tau_{xyc}^2+\tau_{zyc}^2+\tau_{zxc}^2\right)};$$

$$\gamma_{oc}=2/3\sqrt{\left(\varepsilon_{xc}-\varepsilon_{yc}\right)^2+\left(\varepsilon_{zc}-\varepsilon_{yc}\right)^2+\left(\varepsilon_{zc}-\varepsilon_{xc}\right)^2+3/2\left(\gamma_{xyc}^2+\gamma_{zyc}^2+\gamma_{zxc}^2\right)}.$$

Taking into account $\sigma_{xc}=\sigma_{yc}=0$ for the analysed bar: $\sigma_{oc}=\sigma_{zc}/3$;

$$\varepsilon_{oc}=\varepsilon_{zc}/3;\; \tau_{oc}=\frac{1}{3\sqrt{2\sigma_{zc}^2+6\left(\tau_{xyc}^2+\tau_{zyc}^2+\tau_{zxc}^2\right)}};$$

$$\gamma_{oc}=\frac{2}{3\sqrt{2\varepsilon_{zc}^2+3/2\left(\gamma_{xyc}^2+\gamma_{zyc}^2+\gamma_{zxc}^2\right)}}.$$

It is recommended that the boundary (maximum possible) shears $\bar{\Upsilon}r$ on octahedral areas are determined by means of the regression equations [23] that were obtained by A.V. Yashyn and M.D. Kotsovos when processing the known experimental data related to triaxial compression:

$$\bar{\gamma}_r=7{,}97\left(\tau_{oc}/f_{ck}\right)^2+15{,}22\left(\tau_{oc}/f_{ck}\right)-3{,}713 \quad (16)$$

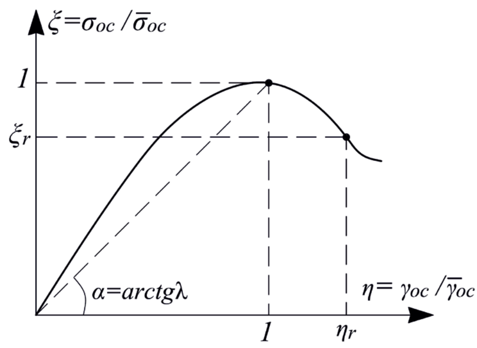

Figure 2. Diagram illustrating concrete deformation at complex loading conditions

The concrete dilatation modulus can be determined with due account of [14, 12] according to the formula:

$$\rho_c=g_{oc}=-\theta_c/\Gamma_c^2= \\ =-\left(\varepsilon_{xc}+\varepsilon_{yc}+\varepsilon_{zc}\right)G_{oc}/4f_{bk} \quad (17)$$

where Θ_c, Γ_c are, respectively, boundary volume strains and the intensity of shear strain in concrete at pure shear; f_{bk} means a characteristic (at design it is the calculated f_{bd}) value of boundary stresses of cohesion [17] which equals, approximately, $R_{b,sh} = 0,7\sqrt{R_b R_{bt}}$ according to [28].

The modulus of volume elasticity is determined, according to [23] in a similar way: $K_c(\gamma_{oc}) = K_{oc} \cdot f(\gamma_{oc})$, where $K_{oc} = \frac{E_{cm}}{1-2v_c}$ is the initial modulus of volume elasticity.

Taking into consideration the above, the secant elasticity modulus E_c and the transverse strain coefficient v_c of the complex stressed concrete are determined in accordance with [8] as:

$$E_c = 3K_c(\gamma_{oc})G_c(\gamma_{oc}) / \left[G_c(\gamma_{oc}) + K_c(\gamma_{oc})\right], \quad (18)$$

$$v_c = \frac{K_c(\gamma_{oc}) - 2G_c(\gamma_{oc})}{2\left[G_c(\gamma_{oc}) + K_c(\gamma_{oc})\right]}.$$

By similarity with the expressions for concrete, it is possible to obtain the formulae applicable for the secant modulus of elasticity at shear for the reinforcement steel as well as the dependencies for the diagram of its shear:

$$G_s = \frac{E_{sk}\vartheta_s}{\left[2(1+v_s)\right]}; \qquad \tau_s = \frac{E_{sk}\vartheta_s}{\left[2(1+v_s)\right]}\gamma_s, \quad (19)$$

where ϑ_s is a coefficient describing a change in the secant elasticity modulus.

Axial deformation in the transversal reinforcement bars and the relative shearing strain in the adjacent concrete can be calculated according to [26]:

$$\varepsilon^*_{sw} = \gamma^*_c = \gamma_c\left[1 + d_{sw}E_{sw}v_{sw}(1+v_c) / (2l_{sw}E_{cm}\vartheta_c)\right]^{-1} \quad (20)$$

When calculating reinforced concrete elements, combined behaviour of the axial and transverse reinforcement is taken into account by reducing the theoretical value of the axial reinforcement yield limit according to [26, 28]:

$$\tilde{f}_{yd} = f_{yd}\sqrt{1 - 3s^2\kappa_1\left(ctg^2\alpha / l^2_{sw,x} + ctg^2\beta / l^2_{sw,y}\right) / \left[4(1+v_s)^2\right]}, \quad (21)$$

where κ_1 - is a reduction factor which was established experimentally, $\kappa_1 = 0,08...0,10$.

Calculated cross-section of the element. The concrete part of the bar cross-section is conventionally divided into tiny parts of rectangular shape (Fig. 3) which size is coordinated with the fineness of the biggest fraction of the concrete [26]. Each tiny part is assigned the appropriate number [26]. For each *n-th* part of the concrete in the calculated section records are taken of the coordinates of its centre of gravity relative to the centre of the axes of symmetry of the section x_{cn}, y_{cn}, area A_{cn}, characteristic (standardized) compression strength of concrete f_{ck}, tension f_{ctk} and initial elasticity modulus E_{cm}.

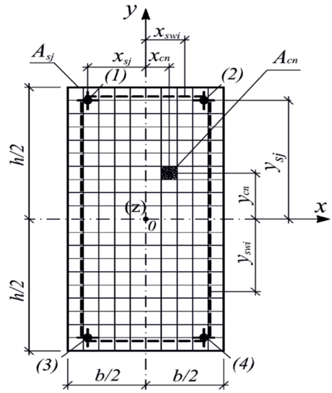

Figure 3 Components of the calculated cross-section of a bar

Poisson ratio $(v_c=0,2)$ is assumed to be constant. This way of recording strength and strain parameters of concrete for each element enables to calculate reinforced concrete bars in the reinforced concrete section, which concreting or reinforcement was made in several stages, and with concrete of different strength and strain, as well as the reinforced concrete elements damaged with corrosion and/or thermal and other effects.

Arrangement of axial reinforcement bars is assumed to be discreet. Each axially oriented bar is assigned its number j, and its following characteristics are indicated: the diameter d_{sj}, location of the centre of gravity of element x_{sj}, y_{sj} relative to the centre of the axes of symmetry of the section, characteristic value of strength at the yield limit f_{ykj} (or $f_{0,2kj}$), characteristic values of the relative strain of rebar or pre-stressed steel at maximum loading ε_{uk}, the initial elasticity modulus E_{skj} and the reinforcement class. Poisson ratio v_s is assumed to be constant for all bars of the axial reinforcement and equals 0,25. Arrangement of the transversely oriented bars in the calculated section is also assumed to be discreet. Horizontal and vertical bars of the transverse reinforcement (stirrups) are conventionally divided into separate sites, each site being given its number i, and records are taken of its diameter d_{swi}, cross-sectional area A_{swi}, surface of the contact area with concrete A_{cswi} and the coordinates of its centre of gravity in the calculated cross-sectional area x_{swi}, y_{swi} relative to the symmetry axes. Strength and deformation parameters of all transverse bars located in the section area are preset: characteristic value of strength at the yield limit f_{ywk}, characteristic value of the tension strength f_{twk}, elasticity modulus E_{sw}, Poisson ratio $v_{sw}=0,25$, characteristic values of relative strains εuwk, the limit or level of elasticity and the class of the transverse reinforcement.

The transverse reinforcement for the section indicated in Fig.3 is accounted for along the length of the reinforcement element (along z-axis) as a layer of area per unit length divided on the section plane in accordance with the recommendations [26].

$$A_{zswi} = \pi\, d_{swi}^2 / (4 s_i), \quad (22)$$

where s_i is a spacing of the transverse bars in the axial direction.

Equation of equilibrium. With due account of the above assumptions and pre-requisites, the equation of equilibrium for the calculated section of the spanned reinforce concrete element can be represented as:

$$N_z = \sum_{n=1}^{k} A_{cn}\sigma_{zcn} + \sum_{j=1}^{m} A_{sj}\sigma_{zsj},\ M_y = \sum_{n=1}^{k} A_{cn}\sigma_{zcn}X_{cn} + \sum_{j=1}^{m} A_{sj}\sigma_{zsj}X_{sj},$$

$$M_x = \sum_{n=1}^{k} A_{cn}\sigma_{zcn}Y_{cn} + \sum_{j=1}^{m} A_{sj}\sigma_{zsj}Y_{sj},\ V_x = \sum_{n=1}^{k} A_{cn}\tau_{zxcn} + \sum_{j=1}^{m} A_{sj}\tau_{zxsj} + \sum_{i=1}^{l_{xsw,1-2,3-4}} A_{xswi}\sigma_{xswi},$$

$$V_y = \sum_{n=1}^{k} A_{cn}\tau_{zycn} + \sum_{j=1}^{m} A_{sj}\tau_{zysj} + \sum_{i=1}^{l_{ysw,1-4,2-3}} A_{yswi}\sigma_{yswi}, \qquad (23)$$

$$T_{xy} = \sum_{n=1}^{k} A_{cn}\left(\tau_{zycn}X_{cn}^{tor} - \tau_{zxcn}Y_{cn}^{tor}\right) + \sum_{j=1}^{m} A_{sj}\left(\tau_{zysj}X_{sj}^{tor} - \tau_{zxsj}Y_{sj}^{tor}\right) +$$

$$+ \sum_{i=1}^{l_{xysw,1...4}} A_{swi}\left(\sigma_{yswi}X_{swi}^{tor} - \sigma_{xswi}Y_{swi}^{tor}\right)$$

where σ_{zcn} is a normal stress in the *n-th* part of the concrete section;

σ_{zsj} is a normal stress in the *j-th* axial bar;

τ_{zxcn}, τ_{zycn} are tangential stresses in the *n-th* part of the concrete section;

τz_{xsj}, τ_{zysj} are tangential stresses in the *j-му* axial bar;

σ_{xswi}, σ_{yswi} are normal stresses arising in the *i-th* site of the horizontal and vertical transverse reinforcement, accordingly.

Normal and tangential stresses in equations (23) are determined with the aid of complete diagrams illustrating strain of concrete and deformation of the reinforcement [7, 24, 26, 30] on the basis of the assumed hypotheses in accordance with the formulae below:

$$\sigma_{zml} = E_{ml}\zeta_{zml}\varepsilon_{zml};\ \tau_{zxml} = G_{ml}\vartheta_{zxml}\gamma_{zxml};$$
$$\tau_{zyml} = G_{ml}\vartheta_{zyml}\gamma_{zyml};\ \tau_{xyml} = G_{ml}\vartheta_{xyml}\gamma_{xyml}; \qquad (24)$$
$$\sigma_{yswi} = E_{swi}\zeta_{yswi}\varepsilon^{*}_{yswi};\ \sigma_{xswi} = E_{swi}\zeta_{xswi}\varepsilon^{*}_{xswi},$$

where ς - a coefficient reflecting a change of the secant modulus of elasticity $E_{ml,}$ ϑ - a coefficient reflecting changes of the secant modulus of elasticity at shear G_{ml}; *m=c* for the parts of concrete section, *m=s* for the bars of the axial reinforcement; *m=sw* for the bars of transverse reinforcement; *l* – a number of the part of concrete or bar.

Generalized linear and angular strains are determined with due account of the plane section hypothesis, solutions of the elasticity theory [31] at cross bending as well as of the stress distribution function at compressed [29] and free [32] torsion. These can be represented as:

$$\varepsilon_{zml} = \varepsilon_0 + \chi_x X_{ml} + \chi_y Y_{ml} + \beta_z \theta_z \varphi\left(X_{ml}^{tor}, Y_{ml}^{tor}\right), \quad (25)$$
$$\gamma_{zxml} = K_x g_{xml} + K_y h_{yml} + \theta_z f_{zxml},$$
$$\gamma_{zyml} = K_y g_{yml} + K_x h_{xml} - \theta_z f_{zyml},$$
$$\gamma_{xyml} = -\theta_z f_{xyml},$$

where ε_0 - the axial relative deformation of an element along the line of the longitudinal z-axis; X_x, X_y - curvatures that reflect bending in the bending moment planes M_x, M_y, respectively. They can be found with the aid of average strains of the tensioned reinforcement and compressed concrete; K_x, K_y – curvatures of shear in the planes of transverse forces V_x, V_y, respectively; Θ_z a relative length angle of twist of the bar length unit (rad/m); $\varphi\left(X_{ml}^{tor}, Y_{ml}^{tor}\right)$ - a Saint-Venant torsion function with respect of the torsion centre; β_z - a twist factor of the section which is determined for the case of constrained torsion in accordance with the formula $\beta_z = \eta e^{-\eta z}$; η - a compression ration [21]; *z* – a distance to the nearest rigid fixture taken along the element axis. At free torsion of bars $\beta_z = 1$; $g_{xml}, g_{yml}, h_{xml}, h_{yml}$ are the distribution functions of angular deformations in case of cross-bending [31];

$f_{zxml} = \tau_{zxml}/\theta_z G_{ml}$; $f_{zyml} = \tau_{zyml}/\theta_z G_{ml}$; $f_{xyml} = \tau_{xyml}/\theta_z G_{ml}$ are the distribution functions at free [32] and constrained torsion [29].

General physical relations. Using the equilibrium equation (23), generalized linear and angular strains (25), strain diagrams of materials [30] and the general physical relations for the calculated cross-section of a reinforced concrete bar can be represented as:

$$\begin{Bmatrix} N_z \\ M_y \\ M_x \\ V_x \\ V_y \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & D_{16} \\ D_{21} & D_{22} & D_{23} & 0 & 0 & D_{26} \\ D_{31} & D_{32} & D_{33} & 0 & 0 & D_{36} \\ 0 & 0 & 0 & D_{44} & D_{45} & D_{46} \\ 0 & 0 & 0 & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \chi_y \\ \chi_x \\ K_x \\ K_y \\ \theta \end{Bmatrix}, \quad (26)$$

$$\text{or } \{N\} = [D]\{\varepsilon\}, \quad (27)$$

where D_{11} is the axial stiffness of an element:

$$D_{11} = \sum_{n=1}^{k} A_{cn} E_{cmn} \zeta_{cn} + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} / \psi_{sj}; \quad (28)$$

D_{22}, D_{33} are bending stiffnesses in planes z_{ox}, z_{oy}:

$$D_{22} = \sum_{n=1}^{k} A_{cn} E_{cmn} \zeta_{cn} X_{cn}^2 + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} X_{sj}^2 / \psi_{sj}, \quad (29)$$

$$D_{33} = \sum_{n=1}^{k} A_{cn} E_{cmn} \zeta_{cn} Y_{cn}^2 + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} Y_{sj}^2 / \psi_{sj}; \quad (30)$$

D_{23} is a stiffness illustrating mutual influence of bending in two planes:

$$D_{23} = D_{32} = \sum_{n=1}^{k} A_{cn} E_{cmn} \zeta_{cn} X_{cn} Y_{cn} + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} X_{sj} Y_{sj} / \psi_{sj}; \quad (31)$$

D_{12}, D_{13} – are the stiffness values reflecting the influence of the axial force on bending, and of bending moments on elongation of an element:

$$D_{12} = D_{21} = \sum_{n=1}^{k} A_{cn} E_{cmn} \zeta_{cn} X_{cn} + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} X_{sj} / \psi_{sj}, \quad (32)$$

$$D_{13} = D_{31} = \sum_{n=1}^{k} A_{cn} E_{cmn} \zeta_{cn} Y_{cn} + \sum_{j=1}^{m} A_{sj} E_{sj} \zeta_{zsj} Y_{sj} / \psi_{sj}; \quad (33)$$

D_{44}, D_{55} – are shear siffness values in planes zox, zoy originating due to transverse forces:

$$D_{44} = \sum_{n=1}^{k} \frac{A_{cn} E_{cmn} \vartheta_{zxcn} g_{xcn}}{[2(1+\nu_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zxsj} g_{xsj}}{[2(1+\nu_s)]} + \sum_{i=1}^{l_{sw,x}} A_{swi} E_{sw} \zeta_{xswi} g_{xswi,c}, \quad (34)$$

$$D_{55} = \sum_{n=1}^{k} \frac{A_{cn} E_{cmn} \vartheta_{zycn} g_{ycn}}{[2(1+\nu_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zysj} g_{ysj}}{[2(1+\nu_s)]} + \sum_{i=1}^{l_{sw,y}} A_{swi} E_{sw} \zeta_{swi} g_{yswi,c}; \quad (35)$$

D_{45}, D_{54} – are the stiffness values reflecting the mutual influence of bending in planes zox, zoy:

$$D_{45} = \sum_{n=1}^{k} \frac{A_{cn} E_{cmn} \vartheta_{zxcn} h_{xcn}}{[2(1+\nu_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zxsj} h_{xsj}}{[2(1+\nu_s)]} + \sum_{i=1}^{l_{sw,x}} A_{swi} E_{sw} \zeta_{xswi} h_{xswi,c}, \quad (36)$$

$$D_{54} = \sum_{n=1}^{k} \frac{A_{cn} E_{cmn} \vartheta_{zycn} h_{ycn}}{[2(1+\nu_c)]} + \sum_{j=1}^{m} \frac{A_{sj} E_{sj} \vartheta_{zysj} h_{ysj}}{[2(1+\nu_s)]} + \sum_{i=1}^{l_{sw,y}} A_{swi} E_{sw} \zeta_{swi} h_{yswi,c}; \quad (37)$$

D_{16}, D_{26}, D_{36} – are the stiffness values reflecting the influence of the twisting moment T_{xy} on elongation and curvature of bending in z_{ox}, z_{oy} planes, and of the axial force N and bending moments M_x, M_y on the shear in xoy plane:

$$D_{16}=D_{61}=\sum_{n=1}^{k}A_{cn}E_{cmn}\zeta_{cn}\beta_z\varphi\left(X_{cn}^{tor},Y_{cn}^{tor}\right)+\sum_{j=1}^{m}A_{sj}E_s\zeta_{zsj}\beta_z\varphi\left(X_{sj}^{tor},Y_{sj}^{tor}\right)/\psi_{sj}, \quad (38)$$

$$D_{26}=D_{62}=\sum_{n=1}^{k}A_{cn}E_{cmn}\zeta_{cn}X_{cn}^{tor}\beta_z\varphi\left(X_{cn}^{tor},Y_{cn}^{tor}\right)+\sum_{j=1}^{m}A_{sj}E_{sj}\zeta_{zsj}X_{sj}^{tor}\beta_z\varphi\left(X_{sj}^{tor},Y_{sj}^{tor}\right)/\psi_{sj}, \quad (39)$$

$$D_{36}=D_{63}=\sum_{n=1}^{k}A_{cn}E_{cmn}\zeta_{cn}Y_{cn}^{tor}\beta_z\varphi\left(X_{cn}^{tor},Y_{cn}^{tor}\right)+\sum_{j=1}^{m}A_{sj}E_{sj}\zeta_{zsj}Y_{sj}^{tor}\beta_z\varphi\left(X_{sj}^{tor},Y_{sj}^{tor}\right)/\psi_{sj}; \quad (40)$$

D_{46}, D_{56} are the stiffness values reflecting the influence of the twisting moment T_{xy} on shear in z_{ox}, z_{oy} planes, and of transverse forces V_x, V_y on the shear in xoy plane:

$$D_{46}=D_{64}=\sum_{n=1}^{k}\frac{A_{cn}E_{cmn}\vartheta_{zxcn}f_{zxcn}}{\left[2\left(1+\nu_c\right)\right]}+\sum_{j=1}^{m}\frac{A_{sj}E_{sj}\vartheta_{zxsj}f_{zxsj}}{\left[2\left(1+\nu_s\right)\right]}+\sum_{i=1}^{l_{sw,x}}A_{swi}E_{sw}\zeta_{xswi}f_{zxswi,c}, \quad (41)$$

$$D_{56}=D_{65}=\sum_{n=1}^{k}\frac{A_{cn}E_{cmn}\vartheta_{zycn}f_{zycn}}{\left[2\left(1+\nu_c\right)\right]}+\sum_{j=1}^{m}\frac{A_{sj}E_{sj}\vartheta_{zysj}f_{zysj}}{\left[2\left(1+\nu_s\right)\right]}+\sum_{i=1}^{l_{sw,y}}A_{swi}E_{sw}\zeta_{yswi}f_{zyswi,c}; \quad (42)$$

D_{66} is the stiffness at twisting the rod in $x\,y$ plane:

$$D_{66}=\sum_{n=1}^{k}\frac{A_{cn}E_{cmn}\vartheta_{xycn}}{\left[2\left(1+\nu_c\right)\right]}\left(f_{zycn}X_{cn}^{tor}-f_{zxcn}Y_{cn}^{tor}\right)+\sum_{j=1}^{m}\frac{A_{sj}E_{sj}\vartheta_{xysj}}{\left[2\left(1+\nu_s\right)\right]}\left(f_{zysj}X_{sj}^{tor}-f_{zxsj}Y_{sj}^{tor}\right)+ \quad (43)$$
$$+\sum_{i=1}^{l_{sw,y}+l_{sw,x}}A_{swi}E_{sw}\zeta_{xyswi}\left(f_{zyswi}X_{swi}^{tor}-f_{zxswi}Y_{swi}^{tor}\right)$$

where ψ_{sj} is V.I. Murashev coefficient which can be determined in accordance with Russian standards by the formula:

$$\psi_{sj}=1-\omega\sigma_{sj,cr}/\sigma_{sj}, \quad (44)$$

where $\sigma_{sj,cr}$ reflects the stress in j-th bar at the moment when a crack originates; σ_{sj} is a current stress in the *j-th* bar of the axial reinforcement at the considered level of loading; ω is a coefficient illustrating completeness of the tensioned concrete diagram which can be adopted, in accordance with the recommendations [3], as $\omega=0,7$. It is considered [30] that the physical relations (26)…(43) are fair at all stages of the stress-strain behaviour of rectangular cross-section reinforced bar elements in case their loading is simple and proportional. These relations make a part of the algorithm which determines strength and deformation properties of individual calculated sections (Fig. 4) of these elements.

Conclusions. The applied approach makes it possible, by introducing secant moduli, to take into account a discreet arrangement of the axial and transverse reinforcement and non-linear properties of materials at tension (compression) and shear, and a non-uniform distribution of stresses along the length of transverse reinforcement, as well as to analyse the general case of complex stress behaviour under the influence of constrained or free torsion, central or eccentric compression (tension) with small or great eccentricity, and skew bending.

The mentioned physical relations can also be applied to other reinforced concrete elements under stress-state condition which are subject to complex stress-strain behaviour and have arbitrary shape of their cross-section provided the appropriate functions of tangential stress distribution are available.

With the constant relation of external forces at any loading stage it is possible to find a vector of strains using the physical relations (26)…(43):

$$\{\varepsilon\}=[D]^{-1}\{N\}. \quad (45)$$

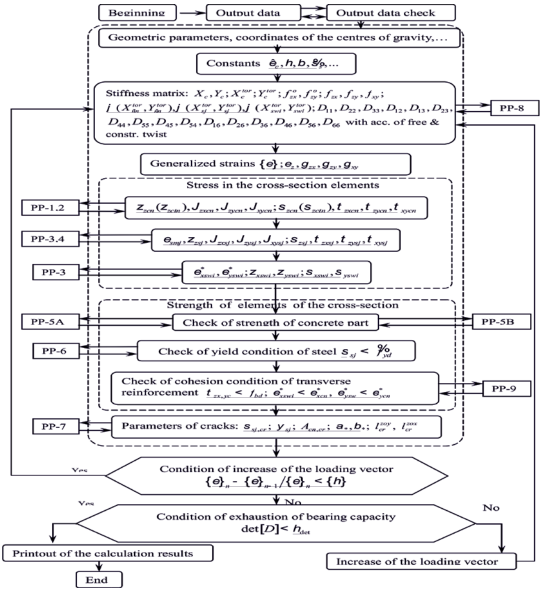

Figure 4. Block diagram illustrating the algorithm for determination of the calculated section bearing capacity of the reinforced concrete rod subject to a complex stress

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